Advanced Macroeconomics 1

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October 9, 2019

Lecture 10:
Schumpeterian growth

Schumpeterian growth: basic model

(ref: Aghion, Akcigit and Howit (2014, Handbook of Economic Growth))

Setup

- Closely connected to the theoretical IO
- The model is Schumpeterian in that
  - it is about growth generated by innovations
  - innovations result from entrepreneurial investments that are motivated by monopoly rents
  - new innovations replace old technologies
- Time is continuous
- No population growth

Setup: agents

- A mass L of risk neutral (linear preferences) infinitely lived workers with discount rate ρ (in EQM $\rho = r(t)$).
- Each agent is endowed with one unit of labor per unit of time, which she can allocate between production and research.
- In equilibrium agents are indifferent with these two activities.

Setup: final good

- There is a final good, which is also the numeraire
- Final good at time t is produced competitively using an intermediate input,

$$Y(t) = A(t)y(t)^{\alpha}, \tag{1}$$

where $0 < \alpha < 1$, $y(t)$ is the amount of intermediate good currently used in the production of final good and $A(t)$ is the productivity (quality) of the currently used intermediate input.

Setup: intermediate good

- The intermediate good is produced one for one with labor.
- That is, $y(t)$ denotes both the production of intermediate input and labor employed in manufacturing.
- Growth in this model results from innovations that improve the quality of intermediate input.
- Quality ladders. If the previous state-of-the-art intermediate good was of quality A, then an innovation will introduce a new intermediate input of quality γA, where $\gamma > 1$.

- Creative destruction. Bertrand competition will allow the new innovator to drive the firm producing the intermediate good of quality A out of the market.
- Growth in this model involves both positive and negative externalities
    - A positive externality (”knowledge spillover effect”): Any new innovation raises productivity, A, forever.
    - A negative externality (”business-stealing effect”) : A new innovation destroys the rents of previous innovator.
- If $z(t)$ units of labor are currently used in R&D, then a new innovation arrives during the current unit of time at the Poisson rate $\lambda z(t)$.
- From now on we concentrate on the balanced growth path EQM and drop the ”time index”.

The research arbitrage and labor market clearing equations

- In BGP the allocation of labor between production (y) and R&D (z) remains constant over time.
- The stationary equilibrium (BGP) is summarized by two basic equations.
- The first one is the labor market clearing condition

$$L = y + z. \tag{2}$$

  The total flow of labor supply is fully absorbed by the demands for manufacturing and R&D labor.
- Research arbitrage condition. The second key equation states that in equilibrium individuals have to be indifferent between working in the intermediate good sector or engaging in R&D.

- Let w_k denote the current wage conditional on there having already been $k \in \mathbb{Z}_{++}$ innovations, from time 0 until current time t.
- Since innovation is the only source of change in this model (no population growth, no exogenous technological change), all variables remain constant constant during the time interval between two successive innovations.
- Let V_{k+1} denote the net present value of becoming the next innovator.

- During a small time interval dt, between the kth and (k+1)th innovations, an agent can either
  - work in manufacturing at the current wage $w(t)$ earning $w(t)dt$
  - devote her flow unit of labor to R&D in which case she will innovate during the current time period with probability λdt and then get V_{k+1}, whereas she gets nothing if she does not innovate.
- The research arbitrage condition

$$w_k = \lambda V_{k+1} \tag{3}$$

- We are implicitly assuming that previous innovators are not candidates for being new innovators.

- But we do not know V_{k+1}?
- During a small time interval dt, a firm collects $\pi_{k+1}dt$ profits.
- At the end of the interval, it is replaced by a new entrant with probability $\lambda z dt$, otherwise the firm continues.
- Hence the stationary value function can be written as

$$V_{k+1} = \pi_{k+1}dt + (1 - rdt)(\lambda z dt * 0 + (1 - \lambda z dt) * V_{k+1}) \quad (4)$$

- Rearranging, dividing both sides by dt, taking the limit as $dt \to 0$ and using the fact that in EQM $\rho = r$, we get

$$\rho V_{k+1} = \pi_{k+1} - \lambda z V_{k+1}. \tag{5}$$

  That is, the annuity value of a new innovation is equal to the current flow profit minus the expected capital loss due to creative destruction.
- We can rewrite (5) as

$$V_{k+1} = \frac{\pi_{k+1}}{\rho + \lambda z} \tag{6}$$

- Eq (6) states that the value of new innovation is equal to the "sum of properly discounted flow payoffs", i.e,

$$V_{k+1} = \int_0^\infty e^{-(\rho+\lambda z)t} \pi_{k+1} dt.$$

Equilibrium profits

- Suppose that k innovations have already occurred until time t.
- As the final good sector is competitive, the monopoly will face the following inverse demand curve

$$p_k(y) = \frac{\partial(A_k y^\alpha)}{\partial y} = A_k \alpha y^{\alpha-1} \tag{7}$$

- The monopoly will choose y to

$$\pi_k = \max_y p_k(y)y - w_k y \tag{8}$$

subject to (7).

- The equilibrium price is

$$p_k(y) = \frac{w_k}{\alpha}, \tag{9}$$

  a constant markup over the marginal cost.
- We can solve y by equating (7) and (9)
- Then the equilibrium profits are given by

$$\pi_k = \frac{1-\alpha}{\alpha} w_k y \tag{10}$$

Equilibrium aggregate R&D

- Combining (10), (6) and (3), we can rewrite the research arbitrage condition as

$$w_k = \lambda \frac{\frac{1-\alpha}{\alpha} w_{k+1} y}{\rho + \lambda z} \tag{11}$$

- Using the labor market clearing condition ($y = L - z$) and the fact that on the BGP all variables are multiplied by γ each time a new innovation occurs, we can solve z from (11)

$$z = \frac{\frac{1-\alpha}{\alpha}\gamma L - \frac{\rho}{\lambda}}{1 + \frac{1-\alpha}{\alpha}\gamma}. \tag{12}$$

- To have positive R&D in the (steady state) EQM, we need $\frac{1-\alpha}{\alpha}\gamma L > \frac{\rho}{\lambda}$.

- Comparative static analysis:
  - a higher productivity of the R&D technology measured by λ, a higher step size γ or a higher size of the population L have a positive effect on aggregate R&D.
  - More elastic inverse demand (a higher α) or a higher discount rate ρ discourage R&D.

Equilibrium expected growth

- Once we know the equilibrium aggregate R&D, z, we can compute the expected growth rate.
- During a small interval there will be a new innovation with probability $\lambda z dt$. If this happen, the output will be multiplied by γ each time there is a new innovation
- Thus

$$Y(t+dt) = (1-\lambda z dt)Y(t) + \lambda z dt Y(t)\gamma \tag{13}$$

Subtract $Y(t)$ from both sides, divide by dt and take the limit as $dt \to 0$

$$\dot{Y}(t) = \lambda z(\gamma - 1)Y(t)$$

$$\frac{\dot{Y}(t)}{Y(t)} = \lambda z(\gamma - 1) \tag{14}$$