Advanced Macroeconomics 1

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September 3, 2019

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Course materials

- Slides and the problem set can be downloaded at Moodle (https://moodle.helsinki.fi/course/view.php?id=35351)
- Enrolment key: Daron
- The main reference:
  - Introduction to Modern Economic Growth by Acemoglu (2008)
- Additional reading:
  - Recursive Macroeconomic Theory by Ljungqvist and Sargent, 3rd edition (2012) (or 2nd edition, 2004)
  - The New Dynamic Public Finance by Kocherlakota (2010)
  - Dynamic General Equilibrium Modeling: Computational Methods and Applications by Heer and Maussner, Springer 2nd edition (2010)

Course requirements

- Four problem sets
  - Handle it either by email to the TA, before the exercise session, or on paper, at the beginning of the exercise session.
  - Achieving 40% of points is required for taking the exam
- The exam counts for 100% of the final grade

The road map for the autumn

- Adv Macroeconomics 1: introduction to
    - growth theory
    - dynamic optimization
    - workhorse models of macro
- Adv Macroeconomics 2
    - the intertemporal savings decision under uncertainty
    - incomplete markets and industry equilibrium models
    - search and unemployment

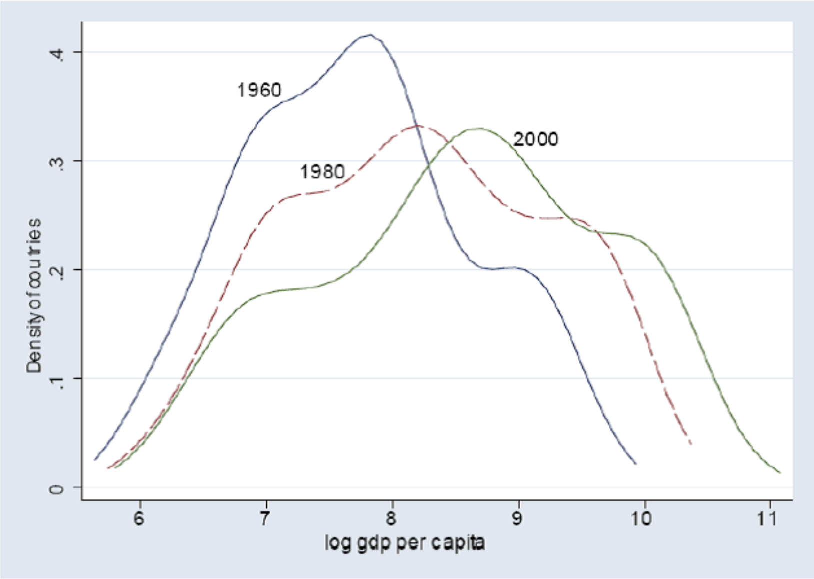
Overview of the course

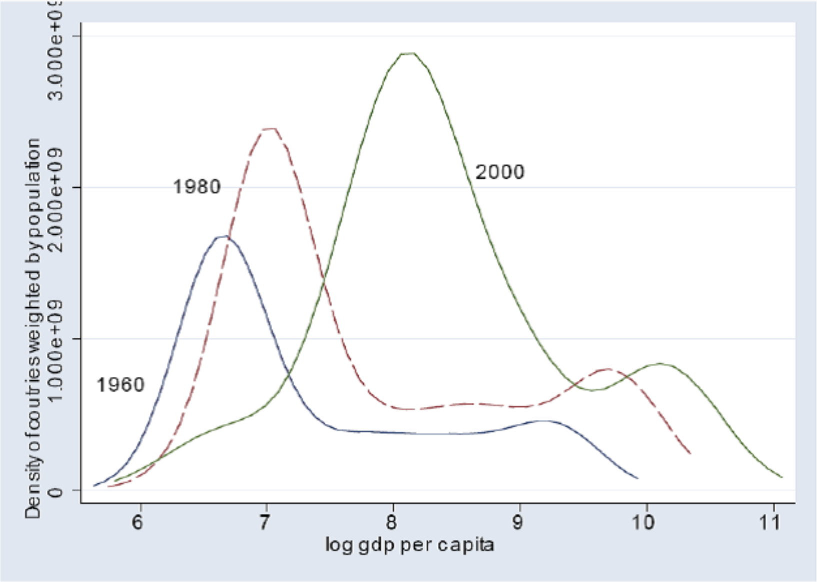
- Economic growth and data, Solow model
- Neoclassical growth model in discrete time
- Dynamic programming and optimal control
- Neoclassical growth model in continuous time
- Overlapping generations and growth
- Introduction to endogenous technological change

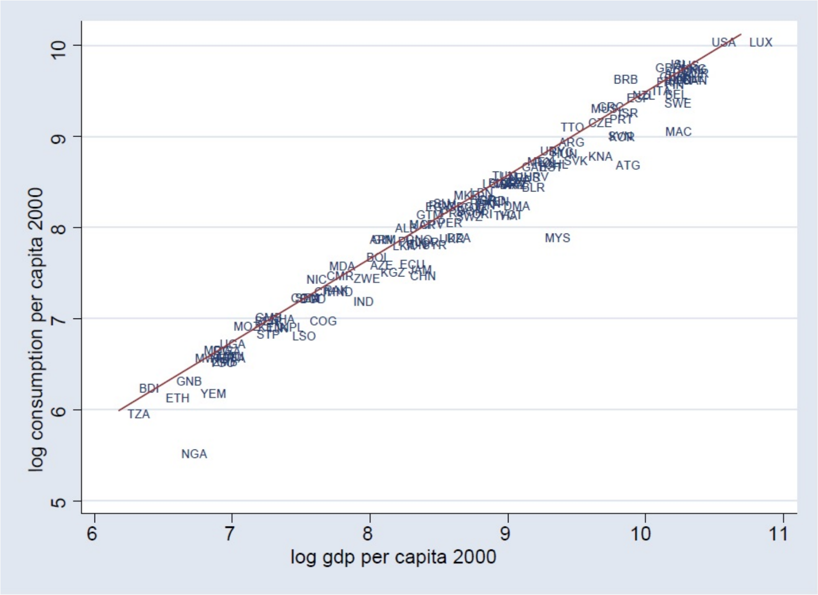
Lecture 1:
A look at the data and the Solow model

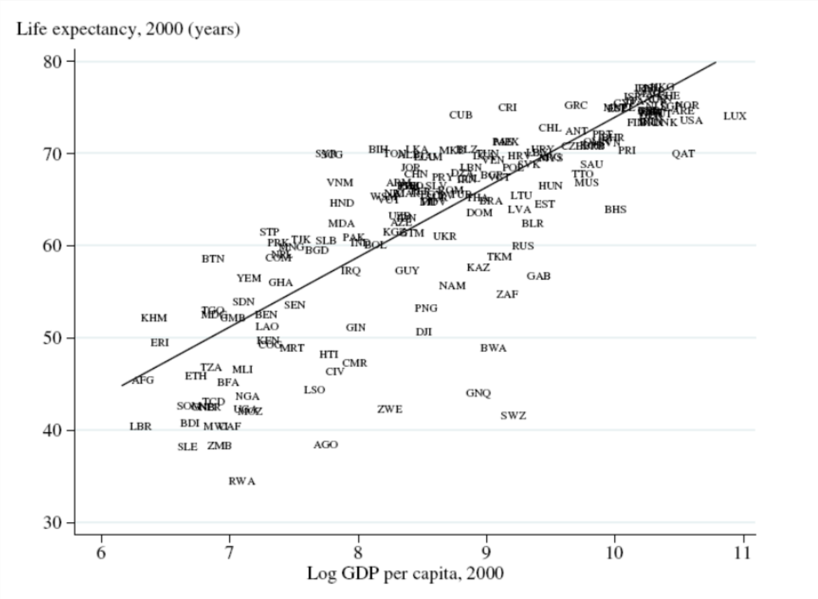
Cross-country data (ref: Acemoglu, ch 1)

Cross-country income differences




Density of coutries weighted by population
0
1.000e+09
2.000e+09
3.000e+09
1960
1980
2000
6
7
8
9
10
11
log gdp per capita


log consumption per capita 2000
log gdp per capita 2000
5
6
7
8
9
10
11
USA
LUX
BRB
SWE
MAC
TTO
CZE
PRT
ISR
ARG
KNA
ATG
BLR
DMA
MYS
PER
BOL
MDA
NIC
ZWE
KGZ
CHN
IND
COG
STP
LSO
UGA
GNB
BDI
ETH
YEM
TZA
NGA


Life expectancy, 2000 (years)
80
70
60
50
40
30
6
7
8
9
10
11
Log GDP per capita, 2000
LUX
USA
QAT
PRI
GRC
CRI
CUB
CHL
ANT
SAU
TTO
MUS
HUN
BHS
LTU
EST
LVA
BLR
VNM
HND
MDA
DOM
UKR
RUS
STP
SLB
BGD
BTN
IRQ
GUY
TKM
KAZ
GAB
YEM
GHA
NAM
ZAF
SDN
SEN
PNG
KHM
BEN
LAO
GIN
DJI
ERI
MRT
HTI
CMR
BWA
TZA
MLI
ETH
AFG
BFA
CIV
LSO
NGA
GNQ
ZWE
SWZ
LBR
BDI
SLE
ZMB
AGO
RWA

- Comparisons between the top 20% countries and the poorest 20%
    - infant rate mortality: 4/1000 and 200/1000
    - daily calorie intake one third lower in the poor countries
- 2.1 billion lack access to safe water at home
- 4.5 billion, lack proper sanitation

Economic growth and income differences (Scheidel, 2009)

Date	Location	Wheat wage (in liters)
18th c. BCE	Southern Mesopotamia (prescriptive/conventional)	(6.2-9.4?)
17th c. BCE	Southern Mesopotamia	4-7.5
15th/14th c. BCE	Nuzi	6-6.4
7th/5th c. BCE	Southern Mesopotamia	4-17.3 **(8-12?)**
late 5th c. BCE	Athens	**8.7**
320s BCE	Athens	**13-15.6**
321 BCE	Babylon	(1.3*)
3rd/early 2nd c. BCE	Delos	3.2-11.1 (4.6-8.6?)
260s/250s BCE	Egypt	3.4-4.2
210-180s BCE	Egypt	3.2-6.2
160s-120s BCE	Egypt	1.6-1.9
120s-90s BCE	Egypt	1.3-5.8 (2.6-3.1?)
93 BCE	Babylon	(2-2.4*)
1st c. BCE/CE	Rome	(>5.9?? <11.8-17.2??)
1st c. CE	Pompeii	(4.6-11.5??)
1st/2nd c. CE	Palestine	(8.6?? 4.3-17.2?)
100-160s CE	Egypt	4.9
190s-260s CE	Egypt	4.9
301 CE	Roman Empire	4.7
314 CE	Egypt	(1.9[-3.2]*)
315 CE	Egypt	(5.1[-8.5]?*)
5th/6th c. CE	Eastern Mediterranean	2.8-3.9
570s/early 8th c. CE	Egypt	**>7.7-13.4**
760s CE	Mesopotamia	3.6-5.3
late 8th/early 9th c.	Egypt	(3.2-10?)
*c.*1000 CE	Constantinople	(5.6??)
*c.*1000-1050 CE	Egypt	(>?)4.3-5.3
11th c. CE	Mesopotamia	6.1
11th/13th c. CE	Cairo	**7.5-13.5**
12th c. CE	Egypt	6.4-10.6
12th/13th c. CE	Constantinople	(4.2-9.3?)
13th c. CE	Mesopotamia	**9**

Note: "?"= uncertain; "??"= highly uncertain; "*"= single source. All uncertain estimates are in parentheses. Underlined figures or ranges fall within, overlap with, or encompass a "core" range of 3.5-6.5 liters while bold figures or ranges in their entirety exceed the upper limit of this "core" range.

Table 3. *Estimates of wages in litres of wheat or rye in 15 cities/regions, 1500–20 to 1780–1800.*

	1500–20	1600–20	1680–1700	1780–1800
In litres of *rye*				
Warsaw	–	38.5	25.3	22.8
Cracow	45.1	31.1	21.1	18.3
Lvov	41.3	10.5	–	–
Danzig	15.1*	10.6	13.1	8.5
Ausburg	8.1	5.7	7.2	5.9
Leipzig	–	6.4	12.1	–
Stockholm	–	14.4	17.0	8.7
In litres of *wheat*				
Vienna	18.5**	8.8	8.1	5.7***
Holland	14.0	11.4	16.6	9.9
Ghent	15.0	8.6	7.5	6.9
Southern England	13.2	6.0	8.1	8.1
Paris	11.9	9.1	7.9	9.9
Valencia/Seville	13.7	10.0	16.2	6.2
Florence/Milan	6.6	5.3	9.3	6.0
Coefficient of variation ****	0.67	0.70	0.43	0.44
Excl. Cracow	0.28	0.27	0.36	0.23

Notes: * 1530–39; ** 1520–29; *** 1770–79; **** excl. Leipzig, Lvov, Stockholm and Warsaw.

Table 1: *

Van Zanden (1999)

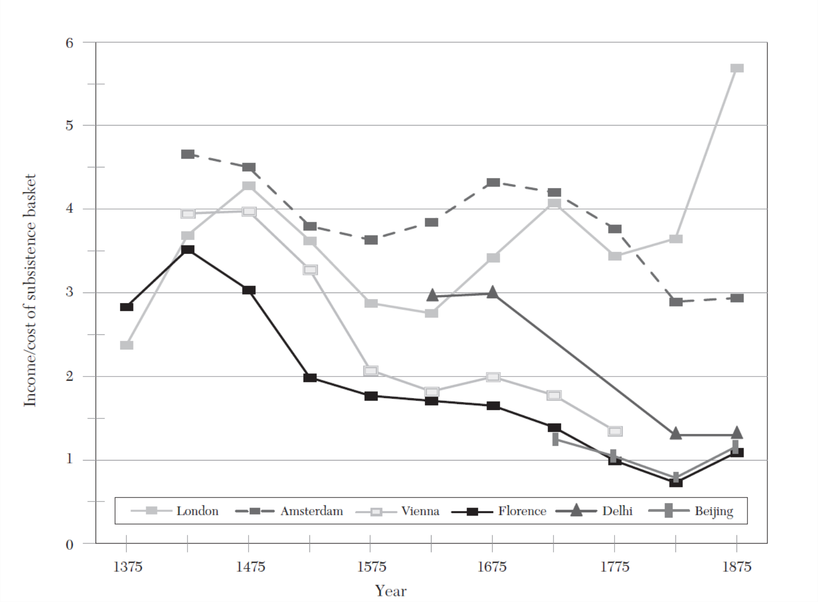
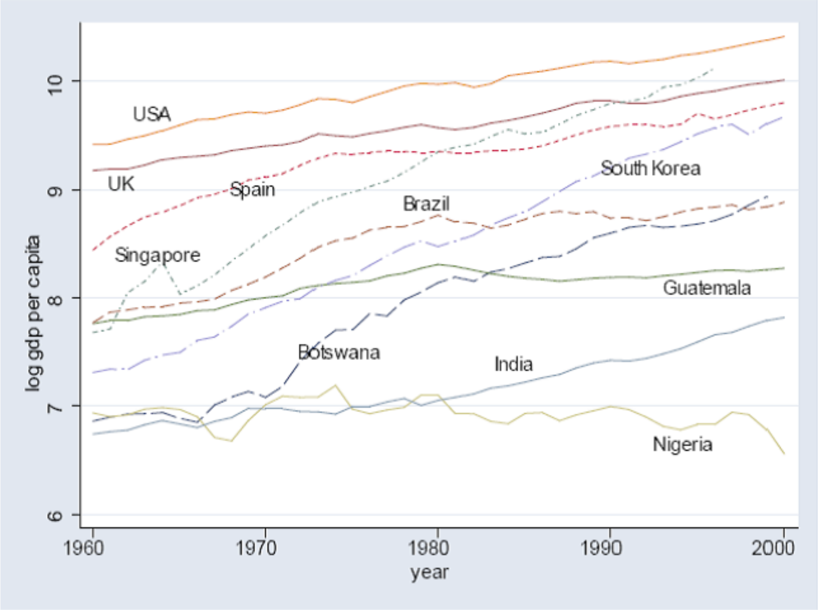
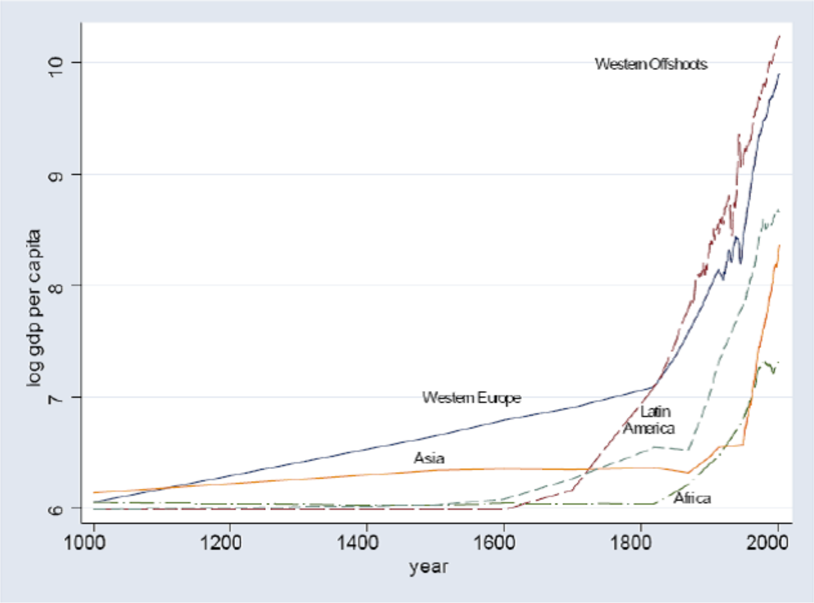


Figure 1: Income/cost of subsistence basket (Allen, 2008)


USA
UK
Spain
Singapore
Brazil
South Korea
Guatemala
Botswana
India
Nigeria
log gdp per capita
6
7
8
9
10
1960
1970
1980
1990
2000
year

Origins of Income differences and Economic growth



Western Offshoots
Western Europe
Latin America
Asia
Africa
log gdp per capita
6
7
8
9
10
1000
1200
1400
1600
1800
2000
year

Growth and poverty (Easterly, 2001)

	% change in avg income per year	% change in poverty rate per year
Strong contraction	-9.8	23.9
Moderate contraction	-1.9	1.5
Moderate expansion	1.6	-0.6
Strong expansion	8.2	-6.1

data source: Ravallion and Chen (1997)

Questions

- What explains the large differences in income across countries?
- Why do some countries grow rapidly while others do not (the poorest countries tend to have the lowest growth rates)?
- What sustains economic growth?
- What is the relationship between economic policies and growth?

The Solow model (ref: Acemoglu, ch 2)

- A simple dynamic model which can be used to analyze
    - growth
    - cross-country differences
- Developed by Solow (1956) and Swan (1956)
- Essentially a neoclassical growth model with exogenous growth

Environment

- Closed economy with a unique final good
- The economy is inhabited by a large number of identical households
- HHs save a constant exogenous fraction $s \in (0, 1)$ of their income.
- A representative firm with a production function

$$Y(t) = F(K(t), L(t), A(t)), \tag{1}$$

where Y(t) is the aggregate output, K(t) is the capital stock, L(t) is employment and A(t) is technology at time t.

- Technology is nonexcludable and nonrival.
- The production function F
    - is twice differentiable
    - satisfies $F_K > 0$, $F_L > 0$, $F_{KK} < 0$ and $F_{LL} < 0$
    - is homogeneous of degree 1, .i.e., $F(\lambda K, \lambda L, A) = \lambda F(K, L, A)$
    - satisfies the Inada conditions, i.e.,

$$\lim_{K \to 0} F_K(K, L, A) = \infty,$$

$$\lim_{K \to \infty} F_K(K, L, A) = 0,$$

$$\lim_{L \to 0} F_L(K, L, A) = \infty,$$

$$\lim_{L \to \infty} F_L(K, L, A) = 0,$$

for all A. Moreover, $F(0, L, A) = 0$ for all L and A.

- Markets are competitive (agents take prices as given)
- The HHs own the factors of production
- They inelastically supply $\bar{L}(t)$ units of labor.
- Thus, labor market clearing requires that for some non-negative wage rate

$$L(t) = \bar{L}(t) \tag{2}$$

  where L(t) is the demand for labor.
- The HHs rent capital to the firms. The rental rate of capital is $R(t)$ and the capital market clearing condition is

$$K(t) = \bar{K}(t),$$

- Capital depreciates at rate δ. Thus, the HHs face interest rate $r(t) = R(t) - \delta$

Firm optimization and Equilibrium

- Firm's (static) problem

$$\max_{K\geq 0, L\geq 0} F(K(t), L(t), A(t)) - R(t)K - w(t)L \tag{3}$$

- A competitive equilibrium requires that firms maximize profits and factor markets clear ($\longrightarrow$ zero profits).
- Factor prices must satisfy the following FOCs

$$w(t) = F_L(K(t), L(t), A(t)) \tag{4}$$

$$R(t) = F_K(K(t), L(t), A(t)) \tag{5}$$

- The law of motion for capital is

$$K(t+1) = (1-\delta)K(t) + I \tag{6}$$

- Aggregate investment is equal to saving

$$I(t) = S(t) = Y(t) - C(t) = sY(t), \tag{7}$$

  where we have used the assumption of a constant saving rate.
- Combining eqs (6) and (7) gives the fundamental law of motion of the Solow model

$$K(t+1) = sF(K(t), L(t), A(t)) + (1-\delta)K(t). \tag{8}$$

Equilibrium

Definition

In the discrete time Solow model for a given $\{L(t), A(t)\}_{t=0}^{\infty}$ and $K(0)$, an equilibrium path is a sequence $\{K(t), Y(t), C(t), w(t), R(t)\}_{t=0}^{\infty}$ such that K(t) satisfies (8), Y(t) is given by (1), $C(t) = (1 - s)Y(t)$ and $w(t)$ and $R(t)$ are given by (4) and (5).

EQM without population growth and technological progress

- Let's denote the capital-labor ratio as

$$k(t) \equiv \frac{K(t)}{L(t)} \tag{9}$$

- Using the assumption of constant returns to scale

$$y(t) = F(\frac{K(t)}{L}, 1, A) \equiv f(k(t)), \tag{10}$$

  where we have normalized $A = 1$
- Moreover, $R(t) = f'(k(t))$ and $w(t) = f(k(t)) - k(t)f'(k(t))$

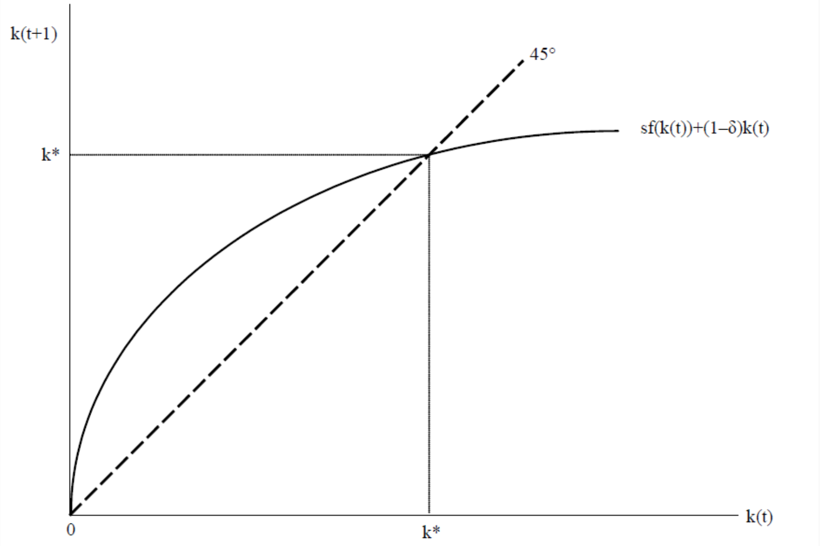
- Dividing both sides of (8) by L gives

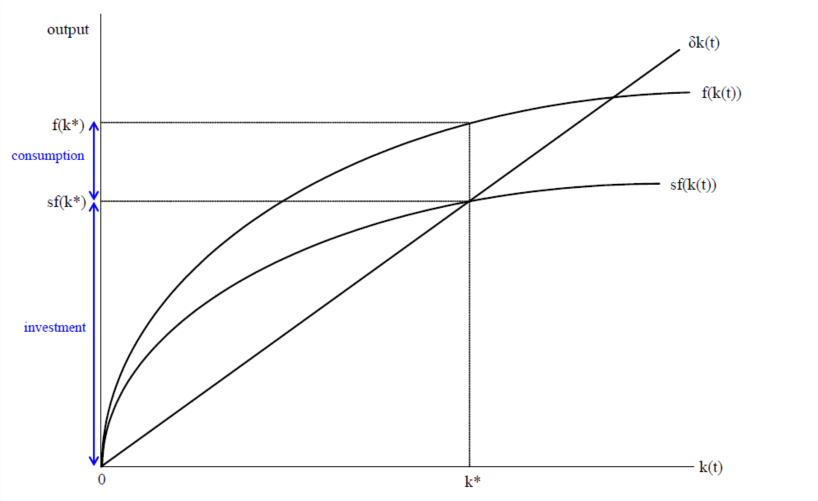
$$k(t+1) = sf(k(t)) + (1-\delta)k(t) \tag{11}$$

- This equation pins down the key equilibrium object of the model, the capital labor ratio.
- A steady state equilibrium is an equilibrium path such that $k(t) = k^*$ for all t.
- Plugging $k(t) = k^*$ into (11) gives

$$\frac{f(k^*)}{k^*} = \frac{\delta}{s} \tag{12}$$

- Note that $k = 0$ is also a (non-stable) steady state.


k(t+1)
45°
sf(k(t))+(1–δ)k(t)
k*
0
k*
k(t)


output
δk(t)
f(k(t))
f(k*)
consumption
sf(k*)
sf(k(t))
investment
0
k*
k(t)

- What happens to k^* if s is altered?
- Steady state consumption
  $c^*(s) = (1 - s)f(k^*(s)) = f(k^*(s)) - \delta k^*(s)$
- There exists a saving rate s_{gold} that maximizes steady state consumption
- Using implicit function theorem one gets

$$\frac{\partial c^*(s)}{\partial s} = [f'(k^*(s)) - \delta]\frac{\partial k^*}{\partial s} \tag{13}$$

- Thus, k^*_{gold} is such that

$$f'(k^*_{gold}) = \delta \tag{14}$$

Towards continuous time

- $x(t+1) - x(t) = g(x(t))$
- We can approximate the growth between periods as

$$x(t + \Delta t) - x(t) = \Delta t * g(x(t)) \tag{15}$$

- Divide both sides by Δt and take limits

$$\lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} = \dot{x} \simeq g(x(t)) \tag{16}$$

The fundamental equation in continuous time

- Assume that the labor force grows as

$$L(t) = exp(nt)L(0) \tag{17}$$

- Note that $\dot{k}(t) = \frac{\dot{K}(t)L(t) - \dot{L}(t)K(t)}{L(t)^2}$.
- Thus, $\frac{\dot{k}(t)}{k(t)} = \frac{\dot{K}(t)}{K(t)} - \frac{\dot{L}(t)}{L(t)} = \frac{\dot{K}(t)}{K(t)} - n$.
- Using eq (16), we get the law of motion of the capital

$$\dot{K}(t) = sF(K(t), L(t), A(t)) - \delta K(t). \tag{18}$$

- Using the previous two equations, we get the fundamental law of motion of the Solow model (in relative terms)

$$\frac{\dot{k}(t)}{k(t)} = \frac{sf(k(t))}{k(t)} - \delta - n \tag{19}$$

- This can also be written as

$$\dot{k}(t) = sf(k(t)) - (n + \delta)k(t) \tag{20}$$

Steady state

- As before, steady state equilibrium is an equilibrium path such that k(t) does not change, i.e., $\dot{k(t)} = 0$ for all t.
- From eq (20), it is easy to see that the steady state is given by

$$sf(k*) = (n + \delta)k* \tag{21}$$

- New capital is needed to replace the depreciated capital and to keep pace with population growth.
- This steady state is globally asymptotically stable and, starting from any $k(0) > 0$, $k(t)$ converges to k^*
- Thus, no sustained growth.

Transitions

- We can use (21) in order to express the fundamental law of motion as

$$\frac{\dot{k}(t)}{k(t)} = (n + \delta)\left(\frac{f(k(t))/k(t)}{f(k^*)/k^*} - 1\right) \tag{22}$$

- If all countries converge to the same steady state, poor countries should grow faster.

The AK model

- Assume that $F(K(t), L(t), A(t) = AK(t)$ and A is a constant
- Plugging this into (19) gives

$$\frac{\dot{k}(t)}{k(t)} = sA - \delta - n \tag{23}$$

- If $sA - \delta - n > 0$, there will be sustained growth in the capital-labor ratio (and output per capita)
- This would imply that the share of national income going to capital should increase.

Balanced growth

- While output per capita has increased (more or less) at a constant rate,
    - the capital-output ratio
    - the interest rate
    - the distribution of income between capital and labor

  have remained constant.
- A balanced growth path refers to an allocation that is consistent with these four facts.

Types of neutral technological progress

- Hicks-neutral

$$\tilde{F}(K(t), L(t), A(t)) = A(t)F(K(t), L(t))$$

- Solow-neutral

$$\tilde{F}(K(t), L(t), A(t)) = F(A(t)K(t), L(t))$$

- Harrod-neutral

$$\tilde{F}(K(t), L(t), A(t)) = F(K(t), A(t)L(t))$$

- Balanced growth in the long run is only possible if technological progress is Harrod-neutral

Uzawa's theorem

Consider a growth model with aggregate production function

$$Y(t) = \tilde{F}(K(t), L(t), \tilde{A}(t)),$$

where $\tilde{F} : \mathbb{R}^2_+ \times \mathcal{A} \to \mathbb{R}_+$ and $\tilde{A}(t) \in \mathcal{A}$ represents technology at time t (where $\mathcal{A}$ is an an arbitrary set). Suppose that $\tilde{F}$ exhibits constant returns to scale in K and L. The aggregate resource constraint is

$$\dot{K}(t) = Y(t) - C(t) - \delta K(t)$$

Moreover, suppose that there is a constant growth rate of population, $L(t) = \exp(nt)L(0)$ and that there exists $T\infty$ such that for all $t \geq T$, $\dot{Y}(t)/Y(t) = g_Y > 0$, $\dot{K}(t)/K(t) = g_K > 0$ and $\dot{C}(t)/C(t) = g_C > 0$. Then...

1. $g_Y = g_K = g_C$
2. For any $t > T$, there exists a function $F : \mathbb{R}^2_+ \to \mathbb{R}_+$ homogeneous of degree 1 in its two arguments, such that the aggregate production function can be represented as

$$Y(t) = F(K(t), A(t)L(t)), \tag{24}$$

where $A(t) \in \mathbb{R}_+$ and

$$\frac{\dot{A}(t)}{A(t)} = g = g_Y - n. \tag{25}$$

The Solow model with technological progress

- $Y(t) = F(k(t), A(t)L(t))$
- $\frac{\dot{A}(t)}{A(t)} = g > 0$
- As before, population grows at rate n
- Capital accumulates according to

$$\dot{K}(t) = sF(K(t), A(t)L(t)) - \delta K(t) \tag{26}$$

- We can stationarize the economy by dividing everything by $A(t)L(t)$, i.e.,

$$k(t) = \frac{K(t)}{A(t)L(t)}$$

  and

$$\hat{y}(t) = \frac{Y(t)}{A(t)L(t)} = f(k(t))$$

- Income per capita, $y(t)$, is given by $\hat{y}(t) * A(t)$
- Differentiating $k(t)$ with respect to time gives

$$\dot{k}(t) = \frac{\dot{K}(t)A(t)L(t) - K(\dot{A}(t)L(t) + A(t)\dot{L}(t)}{(A(t)L(t))^2}$$

and so

$$\frac{\dot{k}(t)}{k(t)} = \frac{\dot{K}(t)}{K(t)} - g - n$$

- Substituting for $\dot{K}(t)$ from (26) into (27) gives

$$\frac{\dot{k}(t)}{k(t)} = \frac{sF(K(t), A(t)L(t))}{K(t)} - \delta - g - n$$

- The fundamental equation can be written as

$$\frac{\dot{k}(t)}{k(t)} = \frac{sf(k(t))}{k(t)} - \delta - g - n \tag{27}$$

- This differential equation converges into a steady state which is given by

$$\frac{f(k^*)}{k^*} = \frac{\delta + g + n}{s} \tag{28}$$

The Solow model and the data (ref: Acemoglu, ch 3)

Growth accounting

- Take the aggregate production of the form

$$Y(t) = F(k(t), L(t), A(t))$$

and differentiate it with respect to time (and divide by $Y(t)$ in order to get

$$\frac{\dot{Y}}{Y} = \frac{F_A A}{Y}\frac{\dot{A}}{A} + \frac{F_K K}{Y}\frac{\dot{K}}{K} + \frac{F_L L}{Y}\frac{\dot{L}}{L} \tag{29}$$

- Denote $g \equiv \frac{\dot{Y}}{Y}$, $g_K \equiv \frac{\dot{K}}{K}$, $g_L \equiv \frac{\dot{L}}{L}$, $x \equiv \frac{F_A A}{Y}\frac{\dot{A}}{A}$, $\varepsilon_k \equiv \frac{F_K K}{Y}$ and $\varepsilon_l \equiv \frac{F_L L}{Y}$

- Equation (29) implies

$$x = g - \varepsilon_k g_K - \varepsilon_l g_L \tag{30}$$

- As factor prices in competitive markets are given by $w = F_L$ and $R = F_K$, the elasticities ε_k and ε_l are equal to factor shares $\alpha_K = \frac{RK}{Y}$ and $\alpha_L = \frac{wL}{Y}$.
- Plugging these into (30), we get

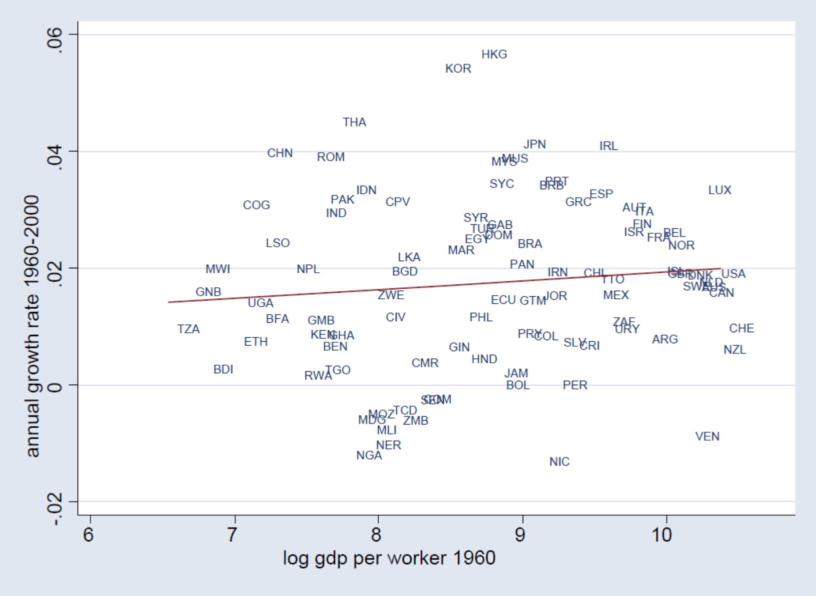
$$x = g - \alpha_K g_K - \alpha_L g_L \tag{31}$$

- Solow (1957) applied this framework to the US data. His results highlighted the importance of technological progress
- Critique: x is just a measure of ignorance

Convergence

- If two countries have the same fundamental parameters and access to the same technology, these countries should converge to the same steady state.
- Moreover, a country with a lower ”initial wealth” should grow at a faster rate.

Unconditional convergence (core OECD countries)



Conditional convergence

Income differences and regression analysis

- The steady state of the Solow model with technological progress was given by

$$\frac{f(k^*)}{k^*} = \frac{\delta + g + n}{s} \tag{32}$$

- Assume that the production function takes the Cobb-Douglas form $F(K(t), L(t), A(t)) = K(t)^{\alpha}(L(t)A(t))^{1-\alpha} = (\frac{K(t)}{L(t)A(t))})^{\alpha} L(t)A(t)$
- Thus, the steady state level of capital per effective unit of labor

$$k^* = (\frac{s}{\delta + g + n})^{\frac{1}{1-\alpha}} \tag{33}$$

- At the balanced growth path

$$Y(t)/L(t) = y(t) = (k^*)^{\frac{\alpha}{1-\alpha}} A(t) = (\frac{s}{\delta + g + n})^{\frac{\alpha}{1-\alpha}} A(0) e^{gt}$$

$$\log(y(t)) = \frac{\alpha}{1-\alpha} \log(s) - \frac{\alpha}{1-\alpha} \log(\delta + g + n) + \log(A_0) + gt \tag{34}$$

- MRW estimate equation (34) using cross-country data on s and n, i.e,

$$\log\left(y(t)_i\right) = b + \frac{\alpha}{1-\alpha}\log(s_i) - \frac{\alpha}{1-\alpha}\log\left(\delta + g + n_i\right) + gt + \varepsilon_i$$

- They assume that
    - each country is in its steady state (i.e. in its BGP)
    - $A_{0i} = A * \epsilon$, where ϵ is part of the error term.
    - $\delta + g = 0.05$ for all countries
    - s and n do not correlate with the error term (and ϵ)
- If differences in technology are important, one would assume that variation in the error term is large.

Estimates of the Basic Solow Model

	MRW	Updated data	
	1985	1985	2000
$\ln(s_k)$	1.42	1.01	1.22
	(.14)	(.11)	(.13)
$\ln(n+g+\delta)$	-1.97	-1.12	-1.31
	(.56)	(.55)	(.36)
Adj R^2	.59	.49	.49
Implied β	.59	.50	.55
No. of observations	98	98	107