

Extensive Games with Randomly Disturbed Payoffs: A New Rationale for Equilibrium Refinements

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Abstract

Refinements of Nash equilibrium hinge on the question: what inferences does a player draw about her opponent's future behavior or his type at an information set that has zero prior probability under the equilibrium. We address this question by adding, in the spirit of Harsanyi (*IJGT* 1973), shocks to the payoffs of each player i at every terminal node w that are independent across players and across nodes, that have a sufficiently large support, and have a continuous distribution F^{iw} . A behavior strategy profile b of the unperturbed game is *purifiable* if there exists some sequence of distributions $F_n := (F_n^{iw})_{i \in \mathcal{I}, w \in W}$, converging weakly to Dirac measures on 0, with a sequence of Bayes Nash equilibria σ_n whose aggregates \tilde{b}_n converge to b . Strategy profile b is *strongly purifiable* if it is purifiable for *every* converging sequence of distributions F_n . If the shocks are restricted to be also identically distributed for each player (i.e. they are i.i.d), this yields the notions of *symmetric purification* and *symmetric strong purification*.

First we consider finite games of perfect information with generic payoffs, with a unique backwards induction (BI) strategy profile. If each player moves at most once along any path, then the backwards induction strategy profile is strongly purifiable, and no other strategy profile is purifiable. However, if a player moves more than once along some path, as in the centipede game, then there may exist purifiable Nash equilibria that are not subgame perfect. For example, in the perturbed centipede game, if player 1 does not play the backwards induction action at her initial node, then player 2 cannot conclude that player 1 will play her backward induction action with high probability at a subsequent node, even if the payoff shocks are independent and arbitrarily small.

Furthermore, the backwards induction strategy profile is not strongly purifiable. However, every purifiable profile induces the backwards induction outcome. Furthermore, the backwards induction strategy profile is uniquely symmetrically purifiable and is strongly symmetrically purifiable.

We next consider signaling games, and on symmetric purification. In the beer-quiche game, pooling on beer is symmetrically purifiable, while pooling on quiche is not. In general, symmetric purification is neither stronger nor weaker than D1. Also, symmetric purification cannot justify forward induction arguments.

Keywords: Equilibrium refinements, Purification, backward induction, signaling, D1, forward induction.