Information requirement for efficient decentralized screening

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Abstract

We establish new efficiency results for decentralized markets with quality uncertainty. Buyers observe a constant flow of passing trade opportunities and related asset information, which allows buyers to screen asset quality by conditioning pricing on informative signals. We link key equilibrium properties with the intensity of screening and provide a new measure for the information required for efficient trade in an extensive class of frictional markets. Trade dynamics can be *standard* or *reversed*. Limit payoffs are neither *Walrasian* nor *Coasian*.

Keywords: *Decentralized lemons market; Buyer signals; Trade dynamics; Screening; Efficient equilibrium; Equilibrium existence; Non-Walrasian payoffs; Non-Coasian payoffs.* **JEL-codes:** D82, D83.

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1 Introduction

Asset markets are large and global. Trades are regularly executed *over-the-counter* in multiple *decentralized* exchanges. Some assets are clearly "lemons" as defined by Akerlof (1970), e.g., a traded firm might have issues with information security or customer management, just waiting to surface. However, even these assets often generate positive value for their owner, new trade opportunities arrive continuously, and buyers can inspect assets before trading. Indeed, the law requires *due diligence* in acquisitions and *caveat emptor* applies.

How do such decentralized markets with informative signals fare? Will the lemons problem resolve on its own with time, i.e., does the market settle to an efficient equilibrium? Or, will the payoff limits remain *Walrasian* (as suggested by, e.g., Gale (1986a,b, 1987); Serrano (2002), and Moreno and Wooders (2002, 2010, 2016))? What are the effects of frictions and the information content of signals on market performance? Which dynamic trade patterns, as characterized by Kaya and Kim (2018), are sustained in the long run?

In this article, we reply to these questions by investigating the effect of signals in a decentralized lemons market, where (i) traders are small, numerous and anonymous, (ii) trade frictions are vanishingly small, and (iii) trading has settled to a stationary equilibrium.[1] The setup adheres loosely to the seminal model of dynamic trade by Moreno and Wooders (2010): asset sellers enter the market with different asset qualities, meet a sequence of random buyers, and exit the market upon trading. To follow the current standard practice in the literature, we embed this model kernel in a continuous time environment. Moreover, to incorporate asset information in the model, we also introduce the assumption that a buyer can obtain a signal of a seller's asset quality before making the seller a price offer. This provides an updated version of canonical models for decentralized trade[2] where traders face not only a constant flow of trade opportunities, as in the previous literature, but also an incessant flow of asset information. The approach emulates information-abundant financial markets.

We establish new efficiency results for this formerly neglected class of markets that has more recently garnered great interest from financial economists.[3] In particular, this article observes that all key properties of an equilibrium – existence, efficiency, and dynamics – derive from the screening intensities of different asset qualities, i.e., the difficulty of obtaining a high price offer for low quality *versus* high quality. In the model, signal distributions differ between sellers, i.e., a lower signal suggests a lower asset quality. As a

[1]This case is particularly interesting as the decentralized counterpart of the *Walrasian* equilibrium; Gale (1986a,b, 1987); Rubinstein and Wolinsky (1985) and Binmore and Herrero (1988).

[2]See Wolinsky (1990); Serrano and Yosha (1993, 1996); Blouin and Serrano (2001); Blouin (2003)

[3]For examples of recent high impact work, see Rostek and Yoon (2021) for *imperfectly competitive trade* and Azevedo and Gottlieb (2017) for *perfect competition and adverse selection.*

result, it is possible for buyers to screen the quality of assets by offering high prices only for high enough signals. Furthermore, assuming that signals are sufficiently informative relative to frictions of trading, low quality can be screened more strongly than high as frictions become negligible. To equate the costs of waiting with those of paying too much, a buyer could thus make obtaining a high price offer, e.g., either equally hard for both qualities, or infinitely harder for low quality. This insight permits us to characterize stationary equilibria by focusing on screening.

Our first main result is that a market settles to an efficient stationary equilibrium for an extensive range of parameter values as trade frictions disappear. The range is partly characterized by *the severity of the lemons problem* and partly by *the relative trade surpluses* among different asset qualities, which is novel. Specifically, we show that an efficient limit equilibrium exists (i) if the trade surplus of low quality is larger, i.e., if $\Delta_l \geq \Delta_h$, or (ii) if the static lemons problem is not severe, i.e., if $\Delta_h \geq \Delta_g$; Δ_l (Δ_h) denotes the surplus of trading low (high) quality assets and Δ_g the gap between the value of buying low quality and selling high quality. As it turns out, efficiency hinges on adjusting screening to market conditions: In the former case (i), trade dynamics are standard (low quality trades faster) and the screening intensity of low quality is strong enough to make the seller accept a low price and not wait for a high signal. In the latter case (ii), trade dynamics are reversed (high quality trades faster) and the screening intensities of both qualities stay relatively low, encouraging a low quality seller to wait for a high price offer. Our efficiency results contrast with the persistence of trading problems in the literature (e.g., Blouin and Serrano (2001); Camargo and Lester (2014); Guerrieri and Shimer (2014); Moreno and Wooders (2010)).[4]

The analysis admits to quantify neatly the information requirements of efficient trading, basically by inverting the related screening mapping to uncover the information needs associated with the frictions. As our second key literature contribution, we can thus demonstrate that our findings, derived in a model with highly informative signals, immediately transfer to any markets where signals are *sufficiently informative relative to the prevailing trade frictions*. In general, because the low quality sellers' costs of waiting become smaller, the information required to separate assets elevates as frictions decrease. If trade frictions are relatively small, (approximately) efficient decentralized screening is thus shown to rely jointly on (i) the existence of small positive trade frictions and (ii) the availability of sufficiently informative signals. This revamps and verifies Moreno and Wooders (2010)'s hypothesis that "decentralized trade mitigates the lemons problem".

Our third major result is the observation that, if there exists no *efficient* equilibrium, there exists *no* equilibrium in the market. This occurs when trading high quality is both

[4]Camargo et al. (2020) find that non-stationary equilibria with aggregate uncertainty become efficient as frictions vanish. For other positive efficiency results in decentralized markets, see Golosov et al. (2014) for divisible assets and aggregate uncertainty and Asriyan et al. (2017) for correlated values and information spillovers. In our model, all learning happens through private quality screening.

more difficult (i.e., the lemons problem is severe) and more valuable (i.e., the trade surplus is larger), that is, for $\Delta_l < \Delta_h < \Delta_g$. This finding derives basically from a discrepancy between the required trade dynamics and the presumed trade surpluses. On one hand, we can show that, when the static lemons problem is severe, only standard trade dynamics may prevail.[5] This alleviates the lemons problem by increasing the average quality of assets. On the other hand, elevating asset quality with vanishing trade frictions also means that the opportunity cost of trading increases. This intensifies screening and boosts the quality of unsold assets. Thereby, we find that buyers only offer high prices when they are almost certain about high asset quality, which increases their payoffs up to Δ_h.[6] However, this implies that buyers cannot agree on a price with low quality sellers under lower expectations, because the trade surplus is smaller Δ_l – contradicting the assumed standard dynamics. The existence and efficiency of an equilibrium thus depend not only on the *severity of the lemons problem*, as known since Akerlof (1970), but also on the *relative trade surpluses* across traded assets.[7]

This article contributes to the rapidly growing literature that studies adverse selection in decentralized market environments with random sequential search. There is also a large literature about dynamic trading with incomplete information in directed search markets, e.g., Inderst and Müller (2002); Inderst (2005); Guerrieri et al. (2010); Camargo and Lester (2014), and in competitive lemons markets, e.g., Janssen and Roy (2002, 2004); Daley and Green (2012); Fuchs and Skrzypacz (2019).

A voluminous literature studies whether decentralized trade results in equal payoffs as its centralized counterpart if trade frictions are small. Gale (1986a,b, 1987) and Binmore and Herrero (1988) investigate the question under complete information, finding efficient Walrasian payoffs. Moreno and Wooders (2010) extend the analysis to markets with a lemons problem where no efficient Walrasian equilibrium may exist. They find that payoff limits remain Walrasian, i.e., inefficient *if and only if* the lemons problem is severe.[8] Unlike our current case, buyers can only separate sellers by randomizing between different prices, which leaves the surplus to low quality sellers and screens all assets with the same intensity – thereby fostering inefficient outcomes.

Our work contributes to this literature by showing that efficient decentralized screening

[5]Otherwise, buyers should only offer high prices and only trade for high signals but, then, average asset quality decreases so much that buyers only offer low prices – a contradiction.

[6]Here, buyers obtain positive trade surplus for at least the highest signals, unlike in Moreno and Wooders (2010), where buyers mix between high and low prices and receive no payoffs.

[7]As stressed already by Wilson (1980), different stable equilibria can exist. For example, if $\Delta_l \geq \Delta_h \geq \Delta_g$, an inefficient limit equilibrium with standard trade dynamics exists in addition to the two previously described efficient ones. In this case, intensive screening of both assets erodes payoffs, however, the payoffs may exceed the static Walrasian payoffs.

[8]For inefficient or non-Walrasian payoffs, see Rubinstein and Wolinsky (1985, 1990); De Fraja and Sakovics (2001); Blouin and Serrano (2001); Serrano (2002); Blouin (2003); Shneyerov and Wong (2010). As the key differences, our model allows buyers to choose their prices, has stationary exogenous entry, and does not rely on coordinated punishments.

can outperform inefficient centralized trade.[9] Previously, efficient trade mechanisms in a lemons market have been related to sorting. In Hendel et al. (2005), observed asset vintages allow the establishment of approximately efficient rental markets for all assets. In Inderst and Müller (2002), different assets are traded in separate markets with distinct prices and liquidity conditions. Interestingly, in Inderst and Müller (2002) the expected quality in markets adjusts to support the Riley separating equilibrium outcome whereas, here, only the cutoff signal adjusts to support efficient trading while prices remain semi-pooling as in Moreno and Wooders (2010) or Cho and Matsui (2018).[10]

Another impressive literature considers dynamic trading with adverse selection. Due to the different time preferences of high and low quality sellers, standard dynamics are derived in almost all articles in the literature. The few exceptions that feature reversed dynamics (Taylor, 1999; Zhu, 2012; Kaya and Kim, 2018; Palazzo, 2017; Martel, 2018; Hwang, 2018; Martel et al., 2022) are characterized by a non-stationary dynamics and observable time-on-market. Kaya and Kim (2018) explore a dynamic model where an asset seller meets a sequence of buyers who offer prices after observing the marketing time and a private quality signal of the asset. Trade dynamics depend on exogenous prior beliefs. If the prior is low, dynamics are standard. However, reversed dynamics prevail when buyers have inflated prior beliefs about quality, which alleviates screening to the point that no seller accepts low prices. Our article changes the setup by focusing on markets where the average asset quality is endogenous and stationary.[11,12] Because assets exit the market upon trading, reversed dynamics mean that low quality remains in the market longer, thus decreasing the average market quality and buyers' quality expectations. As a consequence, because quality expectations at the cutoff are bounded above by prior, we can show that dynamics only reverse when the lemons problem is non-severe; this follows immediately from simple application of the monotone likelihood ratio property. Our work connects trade dynamics to efficient asset screening and delivers a measure of the information required in efficient trading for given trade frictions. Previous work remains mute about the relationship between efficiency and dynamics and the complementary roles of information and frictions in mitigating the lemons problem.

The article is organized as follows. The model is outlined in Section 2 and its basic features in Section 3. Section 4 describes limit equilibria first with unbounded information and later with bounded information. Section 5 concludes by discussing the effects of

[9]When frictions remain positive, Moreno and Wooders (2010) also demonstrate that the surplus created by trade can be higher in the decentralized equilibrium than in the centralized equilibrium. However, the described payoffs remain generally inefficient. Moreover, as noted by Kim (2017), the result does not survive extension to continuous time trading.

[10]Contrary to what we have, Inderst and Müller (2002) assume that buyers outnumber sellers, which erases buyers' payoffs fostering therefore an efficient outcome.

[11]In the sequential adverse selection experiment of Araujo et al. (2021) the majority of players applied stationary responses in contrast to the optimal time varying ones.

[12]We also dispense with the assumption in Kaya and Kim (2018) that time-on-market is observable, our focus being on decentralized markets where assets sell quietly.

alternative model assumptions and the role of commitment granted by signal information. Proofs are relegated to the Appendix.

2 Model

The model closely follows that of Moreno and Wooders (2010) except for (i) incorporating buyer signals into trade meetings and (ii) embedding the model in a continuous time framework.

Time t is continuous and horizon infinite. A unit mass of buyers and a unit mass of sellers enter the market at a rate normalized to unity. Every seller holds an indivisible asset generating a stream of dividends, whose arrival times follow the Poisson process with the rate fixed to unity.[13] Sellers value the stream lower than buyers. Gains from trade thus arise: each seller wants to sell an asset and each buyer wants to buy one.

Assets differ in quality, $\theta = h, l$, which could be high or low, and generate different streams of dividends for their owners. Quality is private information to the seller. To simplify notation, we assume that asset qualities enter the market in equal proportions.[14] The (flow) valuation of a dividend to a buyer is given by u_θ whereas the (flow) valuation to a seller is denoted by c_θ, which is lower: $u_\theta > c_\theta$. It is further assumed that high quality assets are strictly more valuable to both buyers and sellers: $u_h > u_l$ and $c_h > c_l$.

Both sellers and buyers discount future payoffs at rate $r > 0$. Hence, if a seller with an asset of quality θ trades with a buyer for price p, evaluated at the moment of trading, the seller's payoff is $p - \frac{c_\theta}{r}$ whereas the buyer's payoff is $\frac{u_\theta}{r} - p$, because the present discounted value of the asset equals $\int_0^\infty e^{-rt} c_\theta dt = \frac{c_\theta}{r}$ to the seller and $\int_0^\infty e^{-rt} u_\theta dt = \frac{u_\theta}{r}$ to the buyer. Later, we refer to these (stock) valuations with capital letters: $U_\theta := \frac{u_\theta}{r}$ and $C_\theta := \frac{c_\theta}{r}$.

Opportunities to trade arise at a Poisson rate normalized to unity at which a buyer is randomly matched with one seller in the market.[15] In a meeting between a buyer and a seller, the buyer obtains a signal s of the seller's asset quality and, thereafter, makes the seller a take-it-or-leave-it-offer p about the price.

If the seller accepts the price p, the asset is traded to the buyer, and both traders exit the market. Otherwise, the buyer and seller separate and wait in the market until a trade opportunity with someone else appears at a rate one. The market is so large that the same buyer and seller are almost never matched again.

Signals s are distributed according to distribution functions $F_\theta : [0, 1] \to [0, 1]$, which are continuous and supported on the unit interval $[0, 1] = \mathrm{cl}\{s | f_\theta(s) > 0\}$, where f_θ

[13] This is without loss because changing the arrival rate from one to λ is equivalent with changing the dividend yield from c_θ to λc_θ for sellers (from u_θ to λu_θ for buyers).

[14] Thus, the low quality sellers enter at rate half and high quality sellers enter at the same rate. We relax this later in Section 5, showing the innocence of the assumption.

[15] Thus, frictions are captured by trade delay and the time cost from discounting r.

denotes the density function related to F_θ.[16] To allow a buyer to separate qualities by adjusting screening, we assume here that higher signals indicate higher quality and extreme signals are perfectly revealing.[17] The following assumption captures these ideas:

Assumption 1

$$\begin{aligned}
&\frac{f_h(s)}{f_l(s)} \in (0,\infty)\,, \ \textit{for all } s \in (0,1)\,,\\
&\frac{\partial}{\partial s}\frac{f_h(s)}{f_l(s)} \in (0,\infty)\,, \ \textit{for all } s \in (0,1)\,,\\
&\lim_{s\to 0}\frac{f_h(s)}{f_l(s)} = 0,\\
&\lim_{s\to 1}\frac{f_h(s)}{f_l(s)} = \infty.
\end{aligned}$$

The first two lines just state that signals $s \in (0,1)$ satisfy the standard monotone likelihood ratio property (MLRP). The two latter lines entail more specifically that any likelihood ratio $\frac{f_h(s)}{f_l(s)} \in (0,\infty)$ is attainable for an appropriate signal $s \in (0,1)$.

To focus on decentralized environments and simple trading strategies, we further assume that (i) the signals and actions in a pairwise meeting are not observable by outsiders and (ii) the signals observed in earlier meetings are not part of the trading history.

We study stationary Markov equilibria in behavioral strategies $\boldsymbol{\sigma} = (p, a_h, a_l)$ defined as follows: The strategy of a buyer is a function $p : [0,1] \to \Delta\,[0,\infty)$ mapping a signal s to the probability distribution $G(s)$ of offers $p(s)$. The strategy of a seller is a function $a_\theta : [0,\infty) \to [0,1]$ that maps a price p to the probability of acceptance $a_\theta(p)$. We focus on a steady-state market. The proportions of high and low quality assets thus remain constant. This enables us to endogenize buyers' expected asset quality naturally. The solution concept is a perfect Bayesian equilibrium (PBE). A PBE is a pair $(\boldsymbol{\sigma}, \boldsymbol{\pi})$ consisting of a strategy profile $\boldsymbol{\sigma}$ and a belief system $\boldsymbol{\pi}$ such that (i) the strategy profile $\boldsymbol{\sigma}$ is consistent with sequential rationality given the belief system $\boldsymbol{\pi}$, and (ii) the belief system $\boldsymbol{\pi}$ is derived from the strategy profile $\boldsymbol{\sigma}$ with Bayes' rule whenever possible.

As it turns out later, the properties of equilibrium will depend on the relative trade surpluses of different asset qualities,

$$\Delta_\theta = U_\theta - C_\theta > 0,$$

and the benefit of selling a low quality asset for a high price, i.e., the gap between the seller's payoff C_h and the buyer's utility U_l:

$$\Delta_g = C_h - U_l > 0.$$

[16] The set closure cl$A$ is the smallest closed set which contains the original set A.

[17] As F_θ is continuous, the likelihood of observing a revealing signal is almost zero.

Trading high quality assets becomes harder if the gap is larger:

Definition 1 *The static lemons problem is severe if* $\Delta_g > \Delta_h$.

Namely, if the gap is larger than the high surplus, the expected buyer valuation of assets is smaller than the valuation of a high quality asset seller:

$$\bar{U} := \frac{U_h + U_l}{2} < C_h \Longleftrightarrow \Delta_h < \Delta_g.$$

This implies that there is no one price for which all assets could trade immediately. In other words, the static *Walrasian* market equilibrium is inefficient if the lemons problem is severe, whereas if the problem is not severe, any $p \in \left[C_h, \bar{U}\right]$ constitutes an efficient *Walrasian* equilibrium.[18]

Our dynamic model extends the static *Walrasian* setting in that (i) there could be trade at different prices for different signals in a meeting between a buyer and a seller, and (ii) trade could be postponed if the terms of trade in the ongoing meeting are not sufficiently attractive.[19]

3 Preliminaries

We proceed to characterize an equilibrium in a stationary lemons market with continuous quality signals. This section initiates the analysis by making some preliminary observations regarding trade strategies.

Any strategies define continuations values V_b for a buyer and V_θ for a seller.[20] Sequential rationality requires that the strategies $p(s)$ for a buyer and a_θ for a seller in a meeting are optimal given V_b and V_θ.

Given a buyer's price offer p, the problem of a matched seller becomes

$$V_\theta(p) = \max_{a_\theta} \quad a_\theta(p - C_\theta) + (1 - a_\theta)V_\theta,$$

which is simply the choice between whether to trade for the price that the current buyers offers, which gives $p - C_\theta$, or return to the market to obtain V_θ. The solution to the seller's problem remains unchanged regardless of whether we assume that the seller can also observe the signal s.

[18] In both of these cases, $p = U_l$ represents an inefficient *Walrasian* equilibrium for which the supply consists of all low quality assets and the demand consists of one half of the buyers, who are indifferent between purchasing and not: hence, supply equals demand.

[19] The notion of *Walrasian* equilibrium (price taking and market clearing) does not preclude trade at multiple prices, e.g., high quality trading at C_h and low quality trading at C_l - but then we must add a rationing rule. Here we study signal-based (efficient) rationing.

[20] These continuation values are derived in the Appendix.

Conditional on observing a signal s, the problem of a matched buyer is

$$\begin{aligned} V_b(s) = \max_p \quad & q(s)a_h(p)(U_h - p) + (1 - q(s))a_l(p)(U_l - p) + \\ & (q(s)(1 - a_h(p)) + (1 - q(s))(1 - a_h(p)))V_b, \end{aligned}$$

where $q(s)$ gives the probability of being matched with a high quality seller. If buyer chooses price p, the probability of trading high quality $(U_h - p)$ is $q(s)a_h(p)$ whereas the that of trading low quality $(U_l - p)$ is $(1 - q(s))a_l(p)$. Otherwise, the buyer returns to the market getting V_b.

3.1 Cutoff strategies

Lemma 1 (Seller's cutoffs) *There exist a seller's reservation price x_θ for both low and high quality asset sellers $\theta = h, l$, respectively, such that*

$$a_\theta(p) = \begin{cases} 1, & \textit{if } p \geq x_\theta, \\ 0, & \textit{if } p < x_\theta. \end{cases}$$

As is standard in related models of dynamic trading, a seller's optimal strategy is a cutoff strategy: a seller of quality θ accepts any offer above a reservation price x_θ but rejects smaller offers. The reservation price x_θ equals a seller's continuation value V_θ, i.e., the opportunity cost of selling the asset in the current meeting. As a consequence, a buyer's strategy of targeting a seller of quality θ is to offer the price x_θ exactly.

The value of not selling the asset *in the current meeting*, V_θ, is at least the value of not selling the asset *in any future meeting*, C_θ. Additionally, a high quality seller must have a higher reservation price than a low quality seller because the asset has a higher dividend yield: $x_h > x_l$. Specifically, we can show that $x_h = C_h > x_l \geq C_l$ a holdup problem of a high quality seller permits a buyer to reduce the price x_h unless it equals C_h.

Lemma 2 (Buyer's cutoffs) *There exist a buyers' reservation price x_0 for low quality and a reservation signal y for a high price such that*

$$p(s) = \begin{cases} x_h = C_h, & \textit{if } s \geq y, \\ x_l = V_l, & \textit{if } s < y \textit{ and } U_l \geq V_l + V_b. \\ x_0 = U_l - V_b, & \textit{if } s < y \textit{ and } U_l < V_l + V_b. \end{cases}$$

The optimal price strategy of a buyer is more subtle as it depends on the endogenous valuations V_b and V_θ. At this point, we can show that buyers offer a high price x_h above a cutoff y and a lower price $\min\{x_l, x_0\}$ below the signal y. This incorporates the possibility that, when skipping a meeting is optimal, a buyer makes an *empty offer* x_0 that no seller would accept. Optimally, the empty offer x_0 is made when the expected quality is too low to justify a offering x_h but, at the same time, neither is trading low quality optimal because a low quality seller's reservation price, $x_l = V_l$, is higher than a buyer's reservation price for low quality, $x_0 = U_l - V_b$. This occurs when the payoff for buying low quality U_l fails to cover the joint opportunity costs of trade $V_b + V_l$.[21]

3.2 Expected quality

Because gains from trade are positive with both qualities, buyers are willing to pay higher prices for higher quality, but are reluctant to do so if the expected quality remains low. Buyers' optimal price strategies hence depend on their beliefs. Specifically, a buyer will offer a high price x_h which both sellers will accept if and only if the probability q that the seller has a high quality asset reaches a cutoff, i.e., $q \geq q^\star$. The cutoff $q^\star$ solves the following equality, requiring that a buyer is indifferent between offering a high price x_h and the minimum of a low price x_l and an empty offer x_0:

$$q^\star \underbrace{(U_h - x_h)}_{>0} + (1 - q^\star) \underbrace{(U_l - x_h)}_{<0} = \max \left\{ (1 - q^\star) (U_l - x_l) + q^\star V_b, V_b \right\}. \tag{1}$$

If a buyer makes a price offer x_h, the buyer's payoff is positive $U_h - C_h$ if the seller has a high quality asset but, if the seller has a low quality asset, the buyer's payoff is negative $U_l - C_h$. Instead, the payoff for offering x_l is $(1 - q^\star)(U_l - x_l) + q^\star V_b$ (only a low quality seller accepts the offer) and the payoff for offering x_0 is V_b (neither of the sellers accepts the offer). If the seller does not accept a price, the buyer's continuation payoff is V_b.

In forming beliefs q, buyers take into account the endogenous market composition, i.e., how many assets of each quality circulate in the market. We call these prior beliefs, that only condition on the average asset quality in the market, buyers' *unconditional* beliefs q_u. Additionally, buyers' beliefs condition on the signal they obtain in the meeting with the current seller. These buyers' *conditional* beliefs are hereby denoted by $q_c(s)$.

Because each asset quality enters the market at rate half and leaves the market upon trading, the market composition is determined solely by the sellers' relative trade probabilities. In a stationary equilibrium, the inflow of assets to the market has to equal asset outflow. Thus, the following equality must hold for both high and low quality assets

$$1/2 = M_\theta (1 - G_\theta (x_\theta -)).$$

The left-hand-side (lhs) denotes the market inflow of quality θ assets and the right-hand-side (rhs) denotes the market outflow. On the rhs, M_θ refers to the mass of sellers of

[21]Without loss of generality, we thus assume that buyers offer their reservation value $x_0 = U_l - V_b$ when it lies below a low quality seller's reservation value $x_l = V_l$. Yet, letting the empty offer x_0 assume any value below x_l would obviously be outcome-equivalent to offering x_0.

quality θ in the market, and $1 - G_\theta(x_\theta -)$ to the flow trade rate of a seller of quality θ, i.e., the probability a buyer offers at least x_θ in a meeting that arrives at rate unity.[22]

Using Bayes' rule, q_u and $q_c(s)$ can hence be derived as follows

$$q_u = \frac{M_h}{M_h + M_l} = \frac{1}{1 + \frac{1-G_h(x_h-)}{1-G_l(x_l-)}}, \tag{2}$$

$$q_c(s) = \frac{M_h f_h(s)}{M_h f_h(s) + M_l f_l(s)} = \frac{1}{1 + \frac{1-G_h(x_h-)}{1-G_l(x_l-)} \frac{f_l(s)}{f_h(s)}}, \tag{3}$$

where $q_c(s)$ is derived from q_u by incorporating the information about the likelihood ratio $\frac{f_h(s)}{f_l(s)}$ of receiving the observed signal s from high *versus* low quality.

The unconditional beliefs q_u depend on the relative rates at which different assets are traded in the market. That is, if one asset quality is traded more slowly than the other asset quality, it amasses in the market in relative terms, increasing a buyer's expectation of meeting a seller with this quality.

This affects how willing a buyer is to make a high price offer, x_h, and whether a low quality seller accepts a low price, x_l – or prefers to wait until a buyer observes a sufficiently high signal to offer a high price.

We denote the signal for which $q_c(s) = q^\star$ by $s = y$. Because the likelihood ratio $f_h(s)/f_l(s)$ is by assumption increasing in s, buyers' conditional beliefs $q_c(s)$ are clearly increasing in the observed signal. Thus, a buyer will offer a high price if and only if the signal is above the cutoff, i.e., $q \geq q^\star$ *iff* $s \geq y$.

Our framework differs from most earlier approaches in that buyers observe continuous signals and have a positive probability of observing signals that are strongly informative. This results in purified strategies (Harsanyi, 1973) relative to the previous literature. Mixing between higher and lower prices occurs, for example, in a model without signals by Moreno and Wooders (2010) and in a model with binary signals by Kaya and Kim (2018). The reason for mixing is basically that, unlike here, the desirable screening may fail to be implementable without signals or with only two distinct signals. To adjust screening the best they can, buyers thus resort to random pricing, whereas here pricing follows pure strategies.

3.3 Trade dynamics

Whether trade dynamics are standard (i.e., low quality trades faster), reversed (i.e., high quality trades faster) or what we call "knife-edge" depends on the endogenous valuations V_b and V_l.

Lemma 3 *Feasible equilibrium dynamics can be classified into the following patterns:*

[22]Technically, $G_\theta(x-) = \lim_{p \to x-} G_\theta(p)$ denotes the left derivative of a buyer's unconditional (marginal) offer distribution G_θ to sellers of quality θ at x.

1. *Standard dynamics. If $V_b + (V_l - C_l) \leq \Delta_l$, $p(s) = C_h$ for $s \geq y$, $p(s) = x_l$ for $s < y$, and $q_u = \frac{1}{1+(1-F_h(y))} \geq 1/2$.*

2. *Reversed dynamics. If $V_b + (V_l - C_l) > \Delta_l$, $p(s) = C_h$ for $s \geq y$, $p(s) = x_0$ for $s < y$, and $q_u = \frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}} \leq 1/2$.*

3. *Knife-edge dynamics: If $V_b + (V_l - C_l) = \Delta_l$, $p(s) = C_h$ for $s \geq y$, $p(s) = x_l$ for $s \in (z, y)$ and $p(s) = x_0$ for $s \in (0, z)$ for $z \in (0, y)$, and $q_u = \frac{1}{1+\frac{1-F_h(y)}{1-F_l(z)}} \lesseqgtr 1/2$.*[23]

We focus on standard and reversed trade dynamics in the main text; the analysis of knife-edge dynamics is delegated to the Appendix. We can show immediately that, because the prior depends on the severity of the lemons problem, reversed dynamics cannot arise here under a severe lemons problem, unlike in Kaya and Kim (2018) where the prior is exogenous.[24]

Lemma 4 *A necessary condition for reversed dynamics is $\Delta_h \geq \Delta_g$.*

The result arises because reversed dynamics, where high quality leaves the market faster, decrease market quality. Thus, buyers' conditional beliefs at the cutoff $s = y$ are bounded above by the prior beliefs q_0

$$q_c(y) = \frac{1}{1 + \frac{1-F_h(y)}{1-F_l(y)} \frac{f_l(y)}{f_h(y)}} < q_0 = \frac{1}{2},$$

simply due to the MLRP. As a result, if buyers are not willing to offer x_h under q_0, under a severe lemons problem, buyers still remain unwilling to offer x_h under $q_c(y)$, as required to sustain an equilibrium.[25]

Intuitively, trading low quality at a lower rate $1 - F_l(y)$ and high quality at a higher rate $1 - F_h(y)$ reduces market quality so much below the prior that, under a severe lemons problem, offering high prices is only optimal for signals s that are strictly larger than the cutoff signal y. This contradicts the assumption that offering x_h is optimal at y.

4 Equilibrium

4.1 Positive frictions

To evaluate market welfare in a stationary equilibrium, we use the measure applied by Moreno and Wooders (2010),

[23] This construction defines a pure strategy equilibrium. However, a payoff-equivalent randomized equilibrium, where buyers mix between x_l and x_0 for $s < y$, always exists.

[24] This restricts the conditions for reversed dynamics from Kaya and Kim (2018) where the prior can deviate from the average market quality, allowing for arbitrary inflated beliefs.

[25] The details are in the Appendix.

$$W = V_b + \frac{1}{2}(V_h - C_h) + \frac{1}{2}(V_l - C_l) = V_b + \frac{1}{2}(V_l - C_l),$$

which gives the expected present discounted value of the trade surplus accruing to a cohort of buyers and sellers about to enter the market. The maximum welfare is given by the complete information benchmark, $\frac{\Delta_h + \Delta_l}{2}$, which the buyers and sellers would get if they traded without a delay. The maximum is generally unattainable when frictions are positive because the inevitable lag in meeting the first trading partner diminishes the trade surplus of buyers and sellers who discount future payoffs.

Another source of inefficiency is screening. Because buyers only offer high prices for high signals, all sellers cannot expect to trade at rate one, which is the meeting rate between a buyer and a seller. Indeed, according to our earlier results, high quality sellers trade at rate $1 - F_h(y)$ because they only accept high prices x_h. Low quality sellers trade at rate 1 with standard dynamics (also accepting low prices x_l) but only at rate $1 - F_l(y)$ with reversed dynamics (only accepting high prices x_h).

Lemma 5 *$y > 0$ for $r > 0$.*

According to Lemma 5 the cutoff y is always positive in dynamic markets. This shows that, although the trade surplus of both asset qualities is positive, some meetings are not conductive to trade as would be efficient. The result is notable in showing that screening reduces efficiency even when the lemons problem is not severe in the market. An endogenous lemons problem therefore arises.

It is noteworthy that, in the absence of signals, all sellers could trade in the first meeting for a common price C_h if the lemons problem is not severe and the average asset quality is thus high, i.e., $\bar{U} > C_h$. Trade would thus be efficient. Unfortunately, the pooling equilibrium becomes impossible to sustain when signals are introduced. In particular, we find that trade is always delayed with signals because, by Assumption 1, for any $\epsilon > 0$, there is a positive probability $F_h(\delta(\epsilon)) > 0$ of observing such a low signal $s < \delta(\epsilon)$ that buyers' beliefs collapse to $q_c(s) < \epsilon$. Convinced that the quality of the seller's asset is low, a buyer hence makes a low price offer, which a high quality seller rejects.

Previously, Daley and Green (2012) observe in a model with news that trade could also be delayed without a severe lemons problem because traders wait for news to accumulate in order to trade. The reason for trade delay is much like here, that information makes buyers' beliefs noisy. This will make it harder to agree on a price when the noise takes a buyer's belief about an asset far from its seller's belief.[26]

[26]This is akin to the so called Hirshleifer (1971) effect, which shows that information can destroy efficient pooling opportunities.

4.2 Vanishing frictions

We move to investigate the properties of equilibria in markets where trade frictions are negligible. Lemma 5 shows that equilibria are inefficient with positive frictions because the cutoff y at which trade is certain is positive. Lemma 6 proves further that the cutoff y approaches its upper bound as frictions disappear and meetings happen at exploding speed.

Lemma 6 $y \to 1$ *as* $r \to 0$.

Intuitively, the cutoff increases as frictions vanish because both a buyer's payoff and a low quality seller's payoff for waiting for higher signal elevates with patience. To terminate search by offering price x_h, a buyer thus needs to be more strongly convinced about high asset quality, whereas low quality sellers must be screened more strongly to accept x_l.

Based on Lemmata 5 and 6, the efficiency properties of equilibria as frictions disappear thus remain unclear. On the one hand, as $r \to 0$, buyers will obtain information at a lower wait cost, i.e., it costs less to wait for highly informative signals. On the other hand, as $y \to 1$, buyers will become more selective, which makes trading more difficult.

Interestingly, we find however that equilibrium properties are not governed by either of the limit properties alone but the *proportions* of r and y. Specifically, this article shows that there are three paths for $r(\frac{f_h(y)}{f_l(y)})^{-1}$ satisfying $(y, r) \to (1, 0)$ that may each converge to a possible limit equilibrium. The equilibrium candidates differ in dynamics, efficiency, and payoffs.

1. In the first tentative equilibrium, the likelihood ratio $\frac{f_h(y)}{f_l(y)}$ remains low relative to discounting r. The general ease of trading at high prices thus entails that dynamics are reversed and efficient pooling prevails.

2. In the second equilibrium candidate, $\frac{f_h(y)}{f_l(y)}$ stays higher for r. This guarantees that the time cost of obtaining a high price offer x_h is much lower for high quality than low quality assets. Dynamics are standard and screening efficient.

3. In the third possible equilibrium, $\frac{f_h(y)}{f_l(y)}$ gets even higher for r. All sellers thus face extremely high cost of waiting for a high price offer. This extreme screening is inefficient. Trade dynamics remain standard.

4.3 Screening with unbounded signal information

We proceed to describe the parametric conditions under which each candidate represents a stationary limit equilibrium. This is done by partitioning the signal set by screening, i.e., the ease of trading different qualities at a high price. Lemma 7 formalizes our notion of screening.

Lemma 7 *For any $M > 1$ and any $r_0 < 1/M$, there exist signals $0 < s_0 < s_l < s_h < 1$ and functions $\nu_h(s,r) < \nu_l(s,r)$ such that*

$$\nu_l(s_0, r_0) = \nu_h(s_l, r_0) = \frac{1}{M} < M \leq \nu_l(s_l, r_0) = \nu_h(s_h, r_0),$$

where

$$\begin{aligned} \nu_h(y,r) &:= \frac{r}{1 - F_h(y)} < \\ \nu_l(y,r) &:= \frac{r}{1 - F_l(y)}, \end{aligned}$$

and $s_0 \to 1$ as $M \to \infty$.

Above, Lemma 7 defines a mapping ν_θ for each quality θ, called here the *screening intensity*, which describes the time cost of selling this quality for a high price, x_h. The screening intensities ν_h and ν_l of both qualities are increasing in y and r because, all else being equal, trading for a high price is generally more difficult if either the discount factor r (representing frictions) or the cutoff signal y (representing screening) is higher. However, the screening intensity of high quality ν_h always stays below that of low quality ν_l because higher signals $s \geq y$ are more likely to come from high quality assets.

Moreover, Lemma 7 shows that when frictions are low, screening partitions the signal space as follows: First, if the cutoff y belongs to $I_0 = [0, s_0]$, it is very easy for all assets to trade for x_h. Next, if $y \in I_l = (s_0, s_l)$, obtaining a high price for low quality becomes hard (i.e., as hard as we want) whilst receiving a high price high quality remains easy (i.e., as easy as we want). Thereafter, presuming the cutoff reaches higher levels, $y \in I_h = [s_l, s_h)$, screening also intensifies for high quality. For $y \in I_1 = [s_h, 1]$, it hence becomes very hard to sell high quality, which never settles for a low price x_l.

Figure 1 illustrates this partitioning by mapping ν_h and ν_l as functions of y and showing the cutoffs s_0 and s_l corresponding to $M = 4$ for $r = 0.05$; s_h is so close to one that it cannot be discerned. Note that the cutoffs $s_0 < s_l < s_h$ increase if either M increases or r decreases. That is, to keep the relative screening of low quality above a certain level, $\frac{1}{M^2} \geq \frac{\nu_h}{\nu_l}(y,r)$, the cutoff y has to be increased if the frictions r are decreased.[27] These basic properties of screening with unboundedly informative signals make it easy to demonstrate existence and characterize equilibria by focusing on screening.

Generally, an equilibrium with standard trade dynamics is given by y and (V_b, V_l)

[27] By Lemma 6, this is what we find happening in equilibrium.

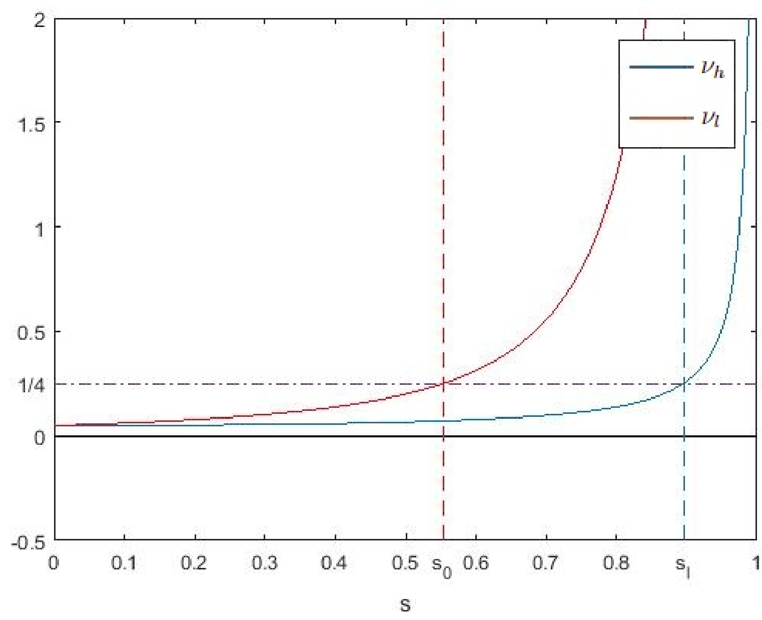


Figure 1: Illustration of Lemma 7.

satisfying the following system[28]

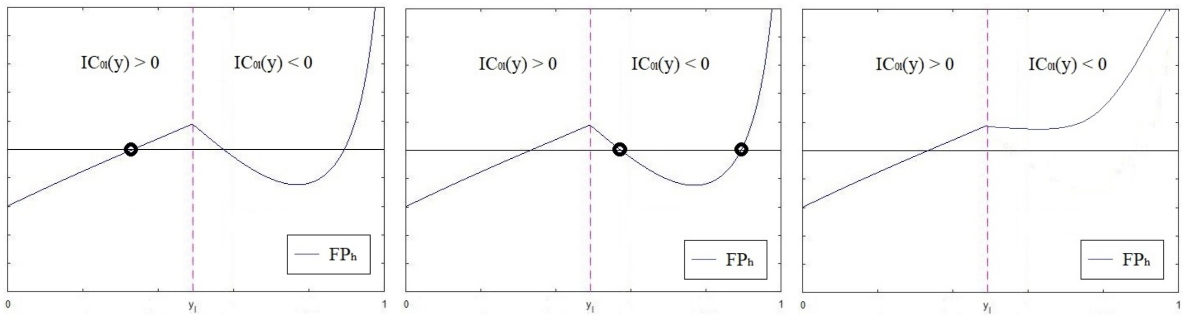
$$
\begin{aligned}
& q_c(s) = \frac{1}{1+\frac{1-F_h(y)}{1}\frac{f_l(s)}{f_h(s)}}, \text{ for } s \in [0,1] \\
& q_c(y)\left(U_h - C_h\right) + \left(1-q_c(y)\right)\left(U_l - C_h\right) = \left(1-q_c(y)\right)\left(U_l - V_l\right) + q_c(y)V_b, && (4)\\
& V_b = \frac{\Delta_h - (1-F_l(y))\Delta_g + F_l(y)(U_l - V_l)}{2+r+\nu_h(y,r)}, \\
& V_l = C_l + \frac{\Delta_g + \Delta_l}{1+\nu_l(y,r)}, \\
& V_b + (V_l - C_l) \le \Delta_l. && (5)
\end{aligned}
$$

Similarly, an equilibrium with reversed trade dynamics is given by y and (V_b, V_l) satisfying the system of conditions

$$
\begin{aligned}
& q_c(s) = \frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(s)}{f_h(s)}}, \text{ for } s \in [0,1] \\
& q_c(y)\left(U_h - C_h\right) + \left(1-q_c(y)\right)\left(U_l - C_h\right) = V_b, && (6)\\
& V_b = V_b(y) = \frac{\Delta_h - \Delta_g}{2+\nu_h(y,r)+\nu_l(y,r)}, \\
& V_l = C_l + \frac{\Delta_g + \Delta_l}{1+\nu_l(y,r)}, \\
& V_b + (V_l - C_l) > \Delta_l. && (7)
\end{aligned}
$$

[28] See the Appendix for the details and additional commentary.

In both systems the first line denotes the beliefs for the dynamics. The next line is a *fixed point condition* FP_h that defines the cutoff. Then come the continuation values formulated here as a function of screening ν_h and ν_l. The last line is an *incentive condition* IC_{0l} which ascertains that dynamics are as assumed. Notably, because FP_h and IC_{0l} are continuous in y, we can demonstrate existence and characterize equilibria by locating for the roots of $FP_h(y)$ and $IC_{0l}(y)$. The roots (dots) corresponding to different equilibria are shown in Figure 2.



(a) $\Delta_h \geq \Delta_g$ (b) $\Delta_l \geq \Delta_h$ (c) $\Delta_g > \Delta_h > \Delta_l$

Figure 2: FP_h for low r.

4.3.1 Reversed dynamics

The first equilibrium illustrated in Figure 2a has the most relaxed screening and thus reverse dynamics. By (6), the equilibrium cutoff signal y thus satisfies the following fixed point condition

$$\frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\Delta_h + \left(1 - \frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\right)(-\Delta_g) =$$
$$\frac{1}{2+\nu_h(y,r)+\nu_l(y,r)}\Delta_h + \frac{1}{2+\nu_h(y,r)+\nu_l(y,r)}(-\Delta_g),$$

where the lhs captures the payoff for offering x_h, $E(u|y) - C_h$ and the rhs that of offering x_0, V_b. Note that, by Lemma 4, reversed dynamics require that the lemons problem is not severe: i.e., $\Delta_h \geq \Delta_g$.

Presuming $\Delta_h \geq \Delta_g$, it is now easy to see that a fixed point exists for low r as the utility of offering x_h is $U_l - C_h$ at $y = 0$ and $\bar{U} - C_h$ at $y = 1$ whereas the utility from x_0 is $\bar{U} - C_h$ at $y = 0$ and 0 at $y = 1$.

Proposition 1 (Reversed dynamics) *If $\Delta_h \geq \Delta_g$, there exists an efficient limit equi-*

librium where $\nu_h \leq \nu_l \to 0$,

$$V_l - C_l \to \Delta_l + \Delta_g, V_b \to \frac{\Delta_h - \Delta_g}{2}$$
$$W = V_b + \frac{1}{2}(V_l - C_l) \to \frac{\Delta_h + \Delta_l}{2},$$

as $r \to 0$. *The equilibrium features reversed dynamics and prior average quality,* $q_u = 0$ *and* $q_c(y) = 1/2$.

The logic behind the result is quite simple. Although waiting for a higher signal raises the probability of trading high quality, the benefit of stronger *equilibrium* screening is limited under reversed trade dynamics. Reversed dynamics mean that high quality leaves the market faster. As a result, screening reduces the expected quality in the market, q_u, limiting the quality expectations at the cutoff signal, $q_c(y)$, by $q_0 = 1/2$ – Lemma 3.

Likewise, while the costs of screening decrease with frictions, stronger *equilibrium* screening erodes buyers' payoffs. Assuming no screening, buyers obtain a positive payoff for trading prior quality with no delay, $\frac{\Delta_h - \Delta_g}{2}$, whereas screening introduces delay and decreases the traded quality; prices x_h at which trade occurs remain intact. Therefore, although the cutoff will approach one – Lemma 6 – equilibrium screening remains limited.

Altogether, we therefore observe that the unique limit equilibrium with reversed dynamics is defined by a sequence of frictions $r \to 0$ and cutoffs $y \to 1$ such that screening remains negligible for all assets, i.e., $\nu_h \leq \nu_h \to 0$ as $(y, r) \to (1, 0)$. This gives buyers the payoff of $\frac{\Delta_h - \Delta_g}{2}$ and low quality sellers the (net) payoff of $\Delta_g - \Delta_l$. Despite $y \to 1$, both qualities are thus traded in the market at efficient rates *relative to frictions* $r \to 0$.

These are novel findings, extending the scope of reversed dynamics in markets with signals[29] described previously by Kaya and Kim (2018) from non-stationary environments of unknown efficiency properties to efficient stationary markets. A significant caveat to practitioners arising from our research is that, although Kaya and Kim (2018) show that reversed dynamics arise under flexible conditions assuming the prior is above the steady-state beliefs, we observe instead that reversed dynamics cannot be sustained in the long run in stationary markets under a severe lemons problem.

The restriction may seem unfortunate. There is ample evidence of reversed dynamics (Hendel et al., 2009; Lei, 2011; Tucker et al., 2013; Albertazzi et al., 2015; Jolivet et al., 2016; Aydin et al., 2019) whereas standard dynamics are rarer (Ghose, 2009). A solution is suggested by our Lemma 5. It shows that trade is delayed with signals irrespective of whether the lemons problem is severe. Hence, while the literature has concentrated on

[29] Without signals the standard dynamics in lemons markets derive straight from the *skimming property* (Fudenberg and Tirole, 1991), which states that all prices that are accepted by high quality sellers are also accepted by low quality sellers. If the same prices are offered to all sellers, this means that low quality is traded faster. By Lemmata 1 and 2, the skimming property holds also in this article. However, because signals enable buyers to target high prices to high quality sellers, as accurately as desirable, the property does not suffice to characterize trade dynamics with signals.

severe examples, applications may be dominated by non-severe lemons cases. The severity of the problem, i.e., Δ_h vs. Δ_g, is hardly known in practice.

4.3.2 Standard dynamics

The remaining equilibria pictured in Figure 2b have more intensive screening and thus standard dynamics. The equilibrium cutoff signal y should satisfy the following fixed point condition, which follows directly from (4) as we let $y \to 1$ and $r \to 0$ based on Lemma 6

$$\Delta_h = V_b = \frac{\Delta_h + \left(\Delta_l - \frac{\Delta_g + \Delta_l}{1+\nu_l(1,0)}\right)}{2 + \nu_h(1,0)}, \tag{8}$$

where $\nu_l(1,0)$ and $\nu_h(1,0)$ refer to the limit screening intensities, which by Lemma 7 can assume various values as (y, r) approach $(1, 0)$.

Despite not holding exactly outside the limit, this formulation of the equilibrium fixed point condition is very illuminating and hence a great tool for demonstrating the logic behind the model. First, (8) shows that when the costs of waiting for an unboundedly informative signal vanish, buyers only offer high prices when they are almost sure the asset is of high quality. This endogenous certainty allows buyers to capture the whole high trade surplus, $V_b = \Delta_h$, from the high price offers that are precisely targeted to high quality sellers only.

Second, because the payoff to buyers from trading low quality assets in a limit equilibrium under standard dynamics is captured by

$$\Delta_l - \frac{\Delta_g + \Delta_l}{1 + \nu_l(1,0)} \geq 0 \iff \nu_l(1,0) \geq \frac{\Delta_g}{\Delta_l}.$$

(8) also demonstrates that the incentives for trading low quality at low prices rely on strong enough screening of low quality assets.

Finally, we can also see in (8) a previously unrecognized strategic complementarity of low quality screening ν_l and high quality screening ν_h

$$\frac{\partial V_b}{\partial \nu_l} > 0, \frac{\partial V_b}{\partial \nu_h} < 0.$$

As it turns out, this gives rise to multiple limit equilibria with standard trade dynamics: one has lower ν_l and ν_h and the other higher ν_l and ν_h.

This complementarity originates generally from the fact that (i) ν_l furthers trade of low quality assets whereas (ii) ν_h impedes trade of high quality assets. Specifically, the effect of higher ν_l on V_b is positive because stronger screening reduces low quality sellers' payoff. This makes it, on the one hand, optimal for low quality sellers to accept low prices instead of waiting for high and, on the other hand, permits a buyer to capture a larger share of low quality trade surplus Δ_l. Moreover, notably, because low quality is always traded in the first match, ν_l has no effect on average market quality and, therefore, the frictions of trading high quality. This contrasts starkly with the negative effect of ν_h on V_b. A higher ν_h implies both a higher average quality in the market and a higher threshold for offering a high price. Consequently, more high quality assets remain unsold and more meetings involve high quality assets but, at the same time, exceeding numbers of meetings that involve high quality assets result in trade. Thus, by affecting *positively* the pool of traded assets, *increased* screening of high quality assets - surprisingly - *decreases* buyers' overall chances to trade. This reduces buyers' payoffs V_b.

In summary, this shows that we can either decrease ν_l or increase ν_h to reduce buyers' payoffs from the technical maximum

$$V_b = \frac{\Delta_h + \Delta_l}{2}, \tag{9}$$

which is given by $\nu_l \to \infty$ and $\nu_h \to 0$. Provided high quality is less valuable, $\Delta_l > \Delta_h$, two solutions to (9) hence exist.

Proposition 2 (Standard dynamics) *If $\Delta_l \geq \Delta_h$, there exist both an efficient limit equilibrium where $\nu_h \to 0 < \nu_l \to \frac{\Delta_g + \Delta_h}{\Delta_l - \Delta_h}$*

$$\begin{aligned} &V_l - C_l \to \Delta_l - \Delta_h, V_b \to \Delta_h \\ &W = V_b + \frac{1}{2}(V_l - C_l) \to \frac{\Delta_h + \Delta_l}{2}, \end{aligned}$$

as $r \to 0$, and an inefficient limit equilibrium where $\nu_h \to \frac{\Delta_l - \Delta_h}{\Delta_h} < \nu_l \to \infty$

$$\begin{aligned} &V_l - C_l \to 0, V_b \to \Delta_h \\ &W = V_b + \frac{1}{2}(V_l - C_l) \to \Delta_h < \frac{\Delta_h + \Delta_l}{2}. \end{aligned}$$

as $r \to 0$. These equilibria feature standard dynamics and high average quality $q_u = q_c(y) \to 1$.

Thus, the equilibrium with lower screening of both qualities is efficient and that with higher screening of both assets inefficient. As suggested previously, equilibria with different welfare properties arise because sustaining an equilibrium with standard dynamics only requires that $V_b = \Delta_h$ and $\nu_l \geq \frac{\Delta_g}{\Delta_l}$. That is, we need to find a cutoff $y \to 1$ as $r \to 0$ such that (i) buyers are willing to offer high prices for $s > y$ and (ii) low quality sellers accept low prices for $s < y$. This can be achieved either by rather mild quality screening, which allocates assets of both qualities efficiently, or by relatively strong screening, which results in wasteful delay in trading high quality.

The division of payoffs in equilibrium is noteworthy. First, buyers payoffs are always positive as they need to equal Δ_h. Second, we find that the intensified screening in the inefficient equilibrium, ultimately, drives low quality out of the market, i.e., $q_u \to 1$, and leaves the sellers with zero surplus, i.e., $V_l \to 0$. This contrasts with the literature, where private information enables low quality sellers to receive positive information rents. For example, the surplus of trading Δ_l is fully extracted by low quality sellers in Moreno and Wooders (2010). In their model, buyers and high quality sellers thus obtain nothing, i.e., limit payoffs as frictions vanish are *Walrasian*.[30]

4.3.3 Non-existence

Figure 2c points out that a non-existence of an equilibrium is also a possibility. This is already suggested by our previous analysis, which shows that the maximum for buyers' payoffs is

$$V_b = \frac{\Delta_h + \Delta_l}{2}. \tag{10}$$

If high quality is more valuable, $\Delta_h > \Delta_l$, no solution to (10) thence exists. Adding to this, if we assume the lemons problem is severe, $\Delta_g > \Delta_h$, no equilibrium with reverse dynamics exists.

Proposition 3 (Non-existence of equilibrium) *If $\Delta_g > \Delta_h > \Delta_l$, there exists no stationary limit equilibrium as $r \to 0$.*

The intuition for the non-existence of an equilibrium is given by a fundamental discrepancy between (i) the required screening to overcome a severe lemons problem and (ii) the higher payoff of trading high quality than low quality. In particular, although different qualities can in the limit be separated efficiently by signals, buyers cannot be simultaneously encouraged to trade both high quality, for the higher payoff of Δ_h, and low quality, for the lower payoff of Δ_l. In equilibria with standard dynamics, negligible information costs allow buyers to obtain the full surplus of high quality trade Δ_h. But this means that buyers are no longer interested in trading low quality for a lower trade surplus Δ_l, thus contradicting standard dynamics.[31]

Figure 3 summarizes the existence conditions of different equilibria in terms of welfare and dynamics. A multiplicity of equilibria with different dynamics and efficiency properties arises when low quality is more valuable to trade while either a unique equilibrium or no equilirium exists if low quality has smaller trade surplus. The relatively neglected trade value of "lemons" thus determines trade possibilities.

[30]Fuchs and Skrzypacz (2019) argue that limit payoffs in a lemons market are *Coasian*. We discuss this idea more closely in Section 5.

[31]See the Appendix for the derivation of both efficient and inefficient equilibria with knife-edge dynamics, which exist for this case.

	$\Delta_h > \Delta_l$	$\Delta_l > \Delta_h$
$\Delta_h > \Delta_g$	efficient, reversed	efficient, standard inefficient, standard efficient, reversed
$\Delta_g > \Delta_h$		efficient, standard inefficient, standard

Figure 3: Existence of equilibria for $r \to 0$.

The equilibrium set can be refined by focusing on, e.g., (i) *undefeated equilibria* with maximal payoffs (primarilty) to buyers and (secondarily) to sellers (Mailath et al., 1993) or (ii) "simple" and "robust" equilibria. The former criterion advocates efficient standard dynamics, which yield the highest payoffs Δ_h to buyers and positive payoffs $\Delta_l - \Delta_h$ to sellers. However, the low information needs speak for efficient reversed dynamics.

Regarding comparative statics, we further observe that buyers' payoffs are increasing in Δ_h (and decreasing in Δ_g) while low quality sellers' payoffs are increasing in Δ_l (and decreasing in Δ_h and Δ_g). This arises because of two forces. The first force is that efficiency considerations combined with flexible screening possibilities allow buyers and sellers to enjoy the entire trade surplus $\frac{\Delta_l+\Delta_h}{2}$. The second novel force is that as frictions vanish optimal screening must keep a buyer indifferent between offering $\min\{x_0, x_l\}$ and offering x_h. As screening intensifies, the payoff of the former approximates V_b and the payoff of the latter approaches Δ_h. Buyers' payoffs will hence turn to $\frac{\Delta_h-\Delta_g}{2}$ under reversed dynamics and to Δ_h under standard dynamics. We are not aware of any counterpart to this result in the literature.

It is also noteworthy that, if there exists a stationary equilibrium, there exists an *efficient* stationary equilibrium. As discussed, efficient screening arises in our model because of two main reasons: (i) frictions of trading are vanishingly small and (ii) information in signals is sufficiently rich (e.g., in Moreno and Wooders (2010) no quality information is observed and in Kaya and Kim (2018) the observed information is coarse). However, a remaining problem that we have is that efficient trading only arises here in a stationary equilibrium. Thereby, unless the market has reached an efficient stationary equilibrium, the properties of the transition path are important for efficiency. This is left for future study. Non-stationary dynamics may also play a key role when no stationary equilibrum exists for $\Delta_g > \Delta_h > \Delta_l$.

4.4 Screening with bounded signal information

To demonstrate the usefulness of considering rich information structures, we next show how our analysis with unboundedly informative signals informs analyses with bounded signal information. To proceed, we thus suppose there exists an upper bound $B < \infty$ on the informativeness of quality signals s, i.e., $\frac{1}{B} \leq \frac{f_h}{f_h}(s) \leq B$.

To transport the idea immediately into our framework, we thus assume that all signals $\frac{f_h}{f_h}(s) < \frac{1}{B}$ and $\frac{f_h}{f_h}(s) > B$ are replaced by, respectively, the (lowest) signal $\underline{s}$ which gives $\frac{f_h}{f_h}(\underline{s}) = \frac{1}{B}$ and the (highest) signal $\overline{s}$ which gives $\frac{f_h}{f_h}(\overline{s}) = B$.[32] To retain the feature that high signals indicate high quality, we assume that $E[u|\overline{s}, q_0] > E[u|q_0] = \bar{U}$.

Our previous analysis permits us to derive limits on the information content of signals that suffices to sustain almost efficient trade with positive trade frictions. In other words, we obtain a new measure for bounded signal information $B < \infty$ needed for "constrained efficient screening" of assets with positive frictions $r > 0$.

Corollary 1 (of Propositions 1 and 2) *For any (small) $r >$ there exists (large) $B < \infty$ such that a stationary equilibrium generates higher welfare than the static Walrasian market if $\Delta_g > \Delta_h$ and $\Delta_l \geq \Delta_h$ and almost equal payoff if $\Delta_h \geq \Delta_g$.*

In the limit $r \to 0$, equilibrium analysis can be conducted similarly as in the previous section. The upper bound on signal informativeness implies that the payoffs of offering x_h cannot exceed

$$E[u|\overline{s}, q_u] - C_h,$$

which gives Δ_h only when market quality is very high $q_c = 1$. Another novelty is that in the limit screening becomes ineffective with bounded signals, i.e., $\nu_\theta(\overline{s}, r) = \frac{r}{1-F_\theta(\overline{s})} \to 0$ as $r \to 0$ for any $\overline{s}$.

Still, if the lemons problem is not severe, the fixed point condition remains

$$\frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\Delta_h + \left(1 - \frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\right)(-\Delta_g) = \tag{11}$$
$$\frac{1}{2+\nu_h(y,r)+\nu_l(y,r)}\Delta_h + \frac{1}{2+\nu_h(y,r)+\nu_l(y,r)}(-\Delta_g),$$

as with unboundedly informative signals. Because $\Delta_h \geq \Delta_g$, we can easily see that a

[32] Because only the upper bound is binding, the lower information bound is redundant.

fixed point exists for low r as the utility of offering x_h on the lhs is $U_l - C_h$ at $y = 0$ and $E[u|\overline{s}, q_0] - C_h$ at $y = \overline{s}$ whereas the utility from x_0 on the rhs is $\bar{U} - C_h$ at $y = 0$ through $y = \overline{s}$. Payoffs thus remain as in Proposition 1 in the limit $r \to 0$.

Remark 1 *If $\Delta_g \leq \Delta_h$, an efficient equilibrium with bounded signals exists for $r \to 0$.*

This contrasts with cases where the lemons problem is severe. The ineffectiveness of screening low quality assets then implies that a stationary limit equilibrium cannot be sustained without mixing.

Remark 2 *If $\Delta_g > \Delta_h$, no pure equilibrium with bounded information exits for $r \to 0$.*

In other words, to make it unattractive for low quality sellers to wait for high prices, a buyer needs to randomize between offering x_l and x_h at $s = \overline{s}$, e.g., in proportions $p_l > 0$ and $p_h > 0$ with $p_l = 1 - p_h$. This mixing is optimal for a buyer at $\overline{s}$ provided

$$\frac{1}{1 + \frac{(1-F_h(\overline{s}))p_h}{1-(1-F_l(\overline{s}))(1-p_h)} \frac{f_l(\overline{s})}{f_h(\overline{s})}}(\overline{s})\,(U_h - C_h) + \left(1 - \frac{1}{1 + \frac{(1-F_h(\overline{s}))p_h}{1-(1-F_l(\overline{s}))(1-p_h)} \frac{f_l(\overline{s})}{f_h(\overline{s})}}\right)(U_l - C_h) =$$
$$\frac{1}{1 + \frac{(1-F_h(\overline{s}))p_h}{1-(1-F_l(\overline{s}))(1-p_h)} \frac{f_l(\overline{s})}{f_h(\overline{s})}} V_b + \left(1 - \frac{1}{1 + \frac{(1-F_h(\overline{s}))p_h}{1-(1-F_l(\overline{s}))(1-p_h)} \frac{f_l(\overline{s})}{f_h(\overline{s})}}\right)(U_l - V_l)\,,$$

Again, sufficient screening of low quality requires $\frac{r}{(1-F_l(\overline{s}))p_h} \geq \frac{\Delta_g}{\Delta_l}$, which implies $p_h \to 0$ as $r \to 0$. Trading at high prices thus becomes very difficult at the limit, increasing the average market quality to the highest possible level, which gives a contradiction

$$\Delta_h = V_b = \frac{\Delta_h}{1 + \frac{r}{(1-F_h(\overline{s}))p_h}}.$$

Because $(r, p_h) \to (0, 0)$, $\frac{r}{(1-F_l(\overline{s}))p_h} \geq \frac{\Delta_g}{\Delta_l}$ is incompatible with $\frac{r}{(1-F_h(\overline{s}))p_h} \to 0$. To reduce average market quality, low quality must trade less often; knife-edge dynamics represent a possibility.

Thereby, if the lemons problem is severe and signals bounded, we need mixing both in high price offers and in low price offers, much like previously in Moreno and Wooders (2010) and Kaya and Kim (2018). Complete analysis lies beyond the scope of this article. However, a fact of life remains that highly informative signals about assets are observed with positive probability, albeit perhaps small, if waiting is costless. Characterizing the equilibrium in these natural circumstances is hence crucial to understanding market performance.

5 Conclusion

The main lessons from our analysis for practical market design are the following.

1. Large enough trade surpluses $\Delta_h > \Delta_g$, for high quality, or $\Delta_l > \Delta_h$, for low quality, are sufficient to guarantee (almost) efficient trade in markets with signals.

2. Information requirements supporting (almost) efficient trade are negligible for vanishing frictions r if $\Delta_h > \Delta_g$ but increase proportional to $\frac{1}{r}\frac{\Delta_g+\Delta_h}{\Delta_l-\Delta_h}$ if $\Delta_l > \Delta_h$.

3. With sufficient information, trading problems thus persist only in markets infested, at the same time, by (i) assets with high value differences (high Δ_g) and (ii) assets with low gains from trade (low Δ_h and Δ_l). Sorting out the assets with negative contribution to market performance, e.g., by a fixed entry cost as in Heinsalu (2020) or by splitting the markets as in Inderst and Müller (2002), can then help to restore efficient trading incentives in the market.

We close by discussing some extensions and alternative modeling frameworks.

Coasian payoffs

Because uninformed buyers are given full bargaining power over informed asset sellers, it is also interesting to study whether payoffs become *Coasian* as frictions disappear, i.e., whether buyers lose all commitment power to low prices and there will be efficient trade in the limit. Fuchs and Skrzypacz (2022) argue that a form of the Coase conjecture often survives even if trading is delayed. According to Fuchs and Skrzypacz (2022), translated to our case a generalized Coase conjecture could also mean that buyers trade at prices equal to (i) the highest seller valuation (i.e., here C_h) or (ii) the marginal buyer utility (i.e., here $E[U|s]$).

Indeed, when dynamics are reversed, we do find that trade only occurs for high prices C_h which both high and low quality sellers can accept. However, when dynamics are standard, a buyer will price at marginal utility for $s = y$ only when indifferent between offering x_l and x_h. In other words, in our model buyers are not always (i) pricing at the highest seller valuation C_h nor (ii) obtaining only the marginal buyer utility V_b. Thus, payoffs are not *Coasian* even when they are efficient.[33]

Here payoffs are non-*Coasian* in the limit under standard dynamics, in short, because signals grant the buyer an additional degree of commitment power, which is absent from models where no information is available to a buyer. Buyers know that, by waiting for a high signal, they can trade high quality with high certainty whereas, if they prefer not to wait, they also have a chance to buy low quality for low prices. Thus, low quality only obtains a payoff of $\Delta_l - \Delta_h > 0$ under standard dynamics whereas buyers obtain the

[33] Yet, a mathematical fact remains that buyers obtain positive payoffs in our model only if there is common knowledge of positive gains from trading high quality, i.e., $V_b \to 0$ as $\Delta_h \to 0$.

payoff of $\Delta_h > 0$ if they trade high quality for x_h and $\Delta_l - (\Delta_l - \Delta_h) > 0$ if they trade low quality for x_l.[34]

Sellers offer prices

The signaling version of our model is studied more closely in Hämäläinen (2015). Focusing on seller-optimal equilibria, this article observes that, if $\Delta_l = \lambda$ is rather high relative to $\Delta_h = 1 - \lambda$, a stationary equilibrium with standard dynamics exists for $\lambda \geq \underline{\lambda}$ but, if $\Delta_h = 1 - \lambda$ is instead high relative to $\Delta_l = \lambda$, a stationary equilibrium with reversed dynamics exists for $\lambda \leq \overline{\lambda}$. In between, for $\lambda \in (\underline{\lambda}, \overline{\lambda})$ both kinds of dynamics can be supported in a stationary equilibrium.

Standard dynamics arise in an equilibrium where sellers are pooling for high signals and separating for low signals. Reversed dynamics arise in an equilibrium where sellers pool for high signals but return to the market for low signals.[35] Seller-optimal prices leave no surplus to buyers, i.e., $V_b = 0$: Pooling prices thus equal $p(s) = E[U|s]$ whereas separating prices are $p_h = U_h$ for high quality and $p_l = U_l$ for low quality. In the seller-optimal case, $p(s)$ and p_l are accepted by buyers with probability one but, to prevent low quality from mimicking high, p_h can only be accepted with probability $\frac{p_l - V_l}{p_h - V_l} < 1$.

Efficiency properties of equilibria are not analyzed for vanishing trade frictions in Hämäläinen (2015). Reasonably, one would think it possible to employ the same cutoffs as in this article, e.g., screen low quality much harder than high quality under a severe lemons problem. A key question then is whether this would allow high quality trade almost certainly for high prices $p(s)$ (or p_h) and low quality trade almost certainly for low prices p_l, implementing therefore an efficient equilibrium where $V_l - C_l \to \Delta_l$ and $V_h - C_h \to \Delta_h$ as $r \to 0$. Lemma 7 suggests this is possible under signaling as well.[36]

Different entry rates

Different entry flows e_h for high quality and $e_l = 1 - e_h$ for low quality, alter the stationary market composition through the following equilibrium condition

$$e_\theta = M_\theta(1 - G_-(x_\theta)).$$

[34] In the efficient equilibrium, the likelihood of the events adjusts so that buyers' payoffs will be given by $\Delta_h/2 + (\Delta_l - (\Delta_l - \Delta_h))/2 = \Delta_h > 0$.

[35] In a so called semi-pooling equilibrium, bridging the pooling and separating cases, low quality sellers mix between offering x_l and x_0 for $s < y$.

[36] The cutoff signal y and the associated screening, $\nu_l(y) \to n_l >> \nu_h(y) \to 0$, should yield $E[U|y] \geq V_b + V_h(\to U_h)$ as $r \to 0$ (high quality sellers offer pooling prices $p(y) = E[U|y]$ for $s = y$) and $U_l \geq V_b + V_l(\to U_l)$ as $r \to 0$ (low quality sellers offer separating prices $p_l = U_l$ for $s < y$); this may require giving at least a small payoff to buyers $V_b \to 0$ as $r \to 0$ to prevent low quality sellers from obtaining more than $U_l - C_l$ for $r \to 0$.

Because buyers' expectations q_u and $q_c(s)$ of sellers assets thus change, the fixed point condition under standard dynamics will transform into

$$\Delta_h - \frac{1-F_h(y)}{1}\frac{f_l(s)}{f_h(s)}\frac{e_l}{e_h}\Delta_g = V_b' + \frac{1-F_h(y)}{1}\frac{f_l(s)}{f_h(s)}\frac{e_l}{e_h}\left(\Delta_l - (V_l' - C_l)\right)$$

where

$$\begin{aligned} V_b' &= \frac{\Delta_h - (1-F_l)\frac{e_l}{e_h}\Delta_g + F_l\frac{e_l}{e_h}(\Delta_l - (V_l' - C_l))}{1+\frac{e_l}{e_h} + r\frac{e_l}{e_h} + \nu_h}, \\ V_l' - C_l &= \frac{1}{1+\nu_l}\left(\Delta_g + \Delta_l\right). \end{aligned}$$

The fixed point condition hence turns into

$$\Delta_h = \frac{\Delta_h + \frac{e_l}{e_h}\Delta_l - \frac{e_l}{e_h}\frac{1}{1+\nu_l}\left(\Delta_g + \Delta_l\right)}{1+\frac{e_l}{e_h}}.$$

for $r \to 0, y \to 1$ and $\nu_h \to 0$ and

$$\Delta_h = \frac{\Delta_h + \frac{e_l}{e_h}\Delta_l}{1+\frac{e_l}{e_h}+\nu_h}.$$

for $r \to 0, y \to 1$ and $\nu_l \to \infty$.

We can thus see that the existence condition and properties of equilibria for standard dynamics are unchanged. The payoffs are in the efficient equilibrium

$$V_b + e_l(V_l - C_l) = \Delta_h + e_l(\Delta_l - \Delta_h) = e_h\Delta_h + e_l\Delta_l$$

and in the inefficient equilibrium $V_l = \Delta_h$ and $V_l - C_l = 0$. An equilibrium with reversed dynamics exists if the following static lemons condition holds

$$\Delta_h \geq \frac{e_l}{e_h}\Delta_g.$$

Ergo, our assumption that different asset qualities enter the market at equal rates is innocuous.

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Appendix

Proof of Lemma 1

We denote by $V_\theta \geq C_\theta$ the continuation value of a seller with quality θ, which gives the opportunity cost of selling an asset of quality θ in the current meeting. Optimally, a seller accepts any price p that is higher or equals V_θ, i.e., $p \geq V_\theta$. Knowing this buyers offer either V_h (to target sellers with $V_\theta \leq V_h$) or V_l (to target sellers with $V_\theta \leq V_l$) or $p_0 < \min_\theta V_\theta$ to pass the meeting without trading. Especially, offering a strictly higher price $p > V_\theta$ to target a seller with quality θ is dominated by lowering the price until it equals the highest continuation value that lies below the offer.

Proof of Lemma 2

Denote by V_b the continuation valuation of a buyer and by $q(s)$ a buyer's belief after seeing signal s, i.e., the probability that the buyer assigns to the random event that the signal comes from a high quality asset. We show in the main text that $q(s)$ is increasing in s. We can therefore conjecture that, if there is a signal y such that

$$V_b = q(y)U_h + (1 - q(y))U_l,$$

then the buyer optimally offers V_h for $s \geq y$ and $\min\{V_l, U_l - V_b\}$ for $s < y$. Otherwise, $y = 0$ or $y = 1$. Note that the maximal price a buyer offers for the asset quality θ is $\max\{U_\theta - V_b, 0\}$ because acquiring the asset now gives U_θ but purchasing another asset later yields V_b. It is assured that $V_h < U_h - V_b$ because $V_h = C_h$ but not certain that $V_l < U_l - V_b$.

If a seller expects to trade for price p with the next buyer, the continuation value of the seller solves the Bellman equation

$$\begin{aligned}
V_\theta(t) &= c_\theta dt + pdt + (1 - dt)(1 - rdt) V_\theta(t + dt) \\
V_\theta(t) &= (c_\theta + p) dt + \left(1 - (1 + r)dt + r(dt)^2\right) V_\theta(t + dt) \\
V_\theta(t + dt) &= \frac{c_\theta + p}{r + 1} + \frac{1}{r + 1} \frac{V_\theta(t + dt) - V_\theta(t)}{dt} + \frac{rdt}{r + 1} V_\theta(t + dt) \\
V_\theta(t) &= \frac{c_\theta + p}{r + 1} + \frac{1}{r + 1} V'_\theta(t) \\
V_\theta(t) &= \frac{rC_\theta + p}{r + 1} + \frac{1}{r + 1} V'_\theta(t)
\end{aligned}$$

as $dt \to 0$. During the interval dt the seller receives a dividend with probability $1dt$ and meets a buyer with probability $1dt$. If the seller does not meet a buyer, time goes on and the seller obtains the continuation value $(1 - rdt)V_\theta(t + dt)$, where $(1 - rdt) \approx e^{-rdt}$ when dt takes a small enough value. This implies that, in a stationary equilibrium, $V_\theta(t) = \frac{rC_\theta + p}{r+1}$ as $V'_\theta(t) = 0$. $V_\theta(t)$ is therefore a weighted average between C_θ and p. The optimality of accepting the price p requires that $p \geq V_\theta(t)$.

This shows that a high quality seller has a higher continuation value than a low quality seller, $V_h > V_l$, because the dividend yield is higher $c_h > c_l$ even in cases where the prices p remain intact.[37]

We next show that the highest price offered by a buyer is $V_h = C_h$. The reasoning

[37] Here prices are higher when a seller has a higher quality asset because buyers offer higher prices for higher signals.

follows Diamond's paradox kind of logic. Suppose instead that the highest price is strictly larger $p' > C_h$. Thus,

$$V_h \leq \frac{rC_h + p'}{r+1} < p'.$$

But then the buyer can lower the offer to $p'' \in (V_h, p')$, which the seller would still accept with certainty. The original price offer $p' > C_h$ is thereby not optimal. The contradiction proves the result. $\square$

Proof of Lemma 3

Lemma 3 follows from Lemmata 1–2 and the following analysis in the text once we note that a buyer strictly prefers offering x_l to x_0 (x_0 to x_l) if $V_b+(V_l-C_l) < \Delta_l$ ($V_b+(V_l-C_l) > \Delta_l$) but remains indifferent between x_l and x_0 if $V_b + (V_l - C_l) = \Delta_l$. Because $x_l = V_l$ and $x_0 = \max\{U_l - V_b, 0\}$, we have that $V_b + (V_l - C_l) = \Delta_l \iff U_l - V_l = V_b \iff U_l - x_l = V_b \iff U_l - x_0 = V_b$.

Buyers' conditional beliefs (3) are obtained directly from (2) by Bayesian updating. We only have to consider the fact that, when buyers offer x_l for $s < y$, high quality trades with probability $1 - F_h(y)$ and low quality with probability 1 but, when buyers offer x_0 for $s < y$, high quality trades with probability $1 - F_h(y)$ and low quality with probability $1 - F_l(y)$ in a meeting.

If buyers instead offer x_0 for $s < z$, x_l for $s \in (z, y)$ and x_l for $s > y$, high quality trades with probability $1 - F_h(y)$ and low quality with probability $1 - F_l(z)$. $\square$

Proof of Lemma 4

Consider a stationary equilibrium with reversed dynamics in a market with a severe lemons problem $\Delta_h < \Delta_g$.

Under reversed dynamics, buyers' conditional beliefs at the cutoff signal $s = y$ are given by $q_c(y) = \left(1 + \frac{1-F_h(y)}{1-F_l(y)} \frac{f_l(y)}{f_h(y)}\right)^{-1}$.

By MLRP, $q_c(y) < 1/2$ such that $q_c(y)(U_h - C_h) + (1 - q_c(y))(U_l - C_h) < 0$ in cases where $\Delta_h < \Delta_g$.

But this implies that a buyer is not willing to make a high price offer $x_h = C_h$ at the cutoff signal $s = y$, which contradicts the assumption that a stationary equilibrium with reversed dynamics can exist for $\Delta_h < \Delta_g$. $\square$

Derivation of value functions

Continuation values, V_l and V_b, are derived by dynamic programming, by defining the value functions (Bellman equations) related to buyers and low quality sellers' optimal stopping problems.[38]

In what follows, $U_b(s \geq y)$ and $U_b(s < y)$ denote the expected flow valuations of a buyer associated with observing a high signal $s \geq y$ and a low signal $s < y$, respectively,

$$\begin{aligned} U_b(s \geq y) :=& q_u \left(1 - F_h(y)\right) \left(U_h - C_h\right) + \\ & (1 - q_u) \left(1 - F_l(y)\right) \left(U_l - C_h\right), \\ U_b(s < y) :=& (1 - q_u) F_l(y) \min\left\{\left(U_l - x_l\right), x_0\right\}. \end{aligned}$$

A buyer meets sellers at a rate equal to unity. The probability of meeting a high quality seller and obtaining a high signal is $q_u \left(1 - F_h(y)\right)$ whereas that of meeting a low quality seller and receiving a high signal is $(1 - q_u)\left(1 - F_l(y)\right)$. If the signal is above c, the buyer offers x_h, which both sellers accept. The probability of observing a low signal when meeting with a high quality seller is $q_u F_h(y)$ and that of detecting a low signal when meeting a low quality seller is $(1 - q_u)F_l(y)$. If the signal is below c, the buyer offers the minimum of x_l (accepted by low quality) and x_0 (rejected by all sellers).

Under standard dynamics, a buyer's value function can be written as follows

$$\begin{aligned} V_b(t) &= dt\left(U_b(s \geq y) + U_b(s < y)\right) + dt q_u F_h(y) V_b(t) + \left(1 - (1 + r)dt\right) V_b(t + dt) \\ (1 + r)V_b(t + dt) - q_u F_h(y) V_b(t) &= U_b(s \geq y) + U_b(s < y) + \frac{V_b(t + dt) - V_b(t)}{dt} \\ V_b(t) &= \frac{U_b(s \geq y) + U_b(s < y)}{1 - q_u F_h(y) + r} + \frac{1}{1 - q_u F_h(y) + r} V_b'(t), \end{aligned} \tag{12}$$

as $dt \to 0$; the second order terms $(dt)^2$ are negligible and can thereby be ignored. Intuitively, a buyer trades under a high signal at rate $U_b(s \geq y)$ and under a low signal at rate $U_b(s < y)$. A buyer continues searching in the market either (i) if the buyer does not meet any seller in the market, which will occur with probability $1 - dt$, or (ii) if the buyer does not trade with a matched seller, happening with probability $q_c F_h(y) dt$. There is no trade in a meeting if the buyer's signal is low but the seller's quality high.

Under reversed dynamics, a buyer's value function can be defined instead as

$$\begin{aligned} V_b(t) &= U_b(s \geq y)dt + \left(1 - \left(1 - q_c F_h(y) - (1 - q_c) F_l(y)\right)dt\right)\left(1 - r dt\right) V_b(t + dt), \\ &= \frac{U_b(s \geq y)}{1 - q_u F_h(y) - (1 - q_u) F_l(y) + r} + \frac{1}{1 - q_u F_h(y) - (1 - q_u) F_l(y) + r} V_b'(t). \end{aligned} \tag{13}$$

as $dt \to 0$; the second order terms $(dt)^2$ are negligible and can thereby be ignored. In this case, there is no trade in a meeting if the signal is low, irrespective of the quality of the seller's asset. As before, a buyer will thus trade under a high signal at rate $U_b(s \geq y)$ but,

[38] We omit here the maximization over the strategy space because we have already described the optimal strategies in Lemmata 1–2 and 3.

when the signal is low, the trade rate is zero. As a result, a buyer continues searching in the market either (i) if there is no meeting with a seller, with probability $1 - dt$, or (ii) if there is no trade in a meeting, with probability $(q_c F_h(y) - (1 - q_c)F_l(y))dt$.

The ordinary differential equations (12) and (13) describe the evolution of V_b under different equilibrium trade dynamics. In a stationary equilibrium, $V_b'(t) = 0$ for all t, because the evolution dynamics of V_b have then reached a steady-state.

In a stationary equilibrium with standard dynamics, we thus obtain that a buyer's continuation value is

$$V_b = \frac{U_b(s \geq y) + U_b(s < y)}{1 - q_u F_h(y) + r},$$

whereas a buyer's continuation value in a stationary equilibrium with reversed dynamics can be expressed as

$$V_b = \frac{U_b(s \geq y)}{1 - q_u F_h(y) - (1 - q_u)F_l(y) + r}.$$

Moving on to sellers, the continuation value of holding a high quality asset is fixed at $V_h = C_h$ whereas the seller's continuation value of keeping a low quality asset is given by

$$\begin{aligned} V_l(t) &= dt c_l + dt\left((1 - F_l(y))\, C_h + F_l(y)V_l(t)\right) + (1 - (1+r)dt)\, V_l(t + dt) \\ (1+r)V_l(t+dt) - F_l(y)V_l(t) &= c_l + (1 - F_l(y))\, C_h + \frac{V_l(t+dt) - V_l(t)}{dt} \\ V_l(t) &= \frac{rC_l + (1 - F_l(y))\, C_h}{1 - F_l(y) + r} + \frac{1}{1 - F_l(y) + r} V_l'(t), \end{aligned} \tag{14}$$

as $dt \to 0$; the second order terms $(dt)^2$ are negligible and can thereby be ignored.

Note that two events may happen to low quality sellers at each time point: (i) the seller's asset may generate a new dividend with payoff c_l, or (ii) the seller may encounter a new potential buyer with signal s. Both events follow a Poisson process with the rate equal to unity. If the buyer's signal is high, with probability $1 - F_l(y)$, the seller obtains $C_h - V_l$ whereas, if the buyer's signal is low, with probability $F_l(y)$, the seller receives V_l, irrespective of which trade dynamics prevail (i.e., with standard trade dynamics, a buyer offers $x_l = V_l$, which the seller accepts, and, with reversed trade dynamics, the buyer offers, $x_0 < V_l$, which the seller rejects). However, if neither event occurs, the seller continues searching in the market, which gives the seller the continuation value, $V_l(t+dt)$. In a stationary equilibrium, V_l is hence given by

$$V_l(t) = \frac{(1 - F_l(y))\, C_h + rC_l}{1 - F_l(y) + r} = \frac{C_h - C_l}{1 + \frac{r}{1 - F_l(y)}} + C_l.$$

The first term captures the value of trading low quality for a high price whereas the second term denotes the valuation of dividends.

Value functions for screening

The screening intensities

$$\nu_h : \nu_h(y,r) = \frac{r}{1-F_h(y)},$$
$$\nu_l : \nu_l(y,r) = \frac{r}{1-F_l(y)},$$

are increasing in r and y.

Rearranging (14) gives

$$\begin{aligned} V_l = \frac{C_h + \nu_l C_l}{1+\nu_l} &= \frac{C_h + \nu_l C_l}{1+\nu_l} = \frac{C_h - C_l + (1+\nu_l) C_l}{1+\nu_l}, \\ &= C_h - \frac{\nu_l}{1+\nu_l}\left(C_h - C_l\right) = C_h - \frac{\nu_l}{1+\nu_l}\left(\Delta_g + \Delta_l\right), \\ &= C_l + \frac{1}{1+\nu_l}\left(C_h - C_l\right) = C_l + \frac{1}{1+\nu_l}\left(\Delta_g + \Delta_l\right), \end{aligned} \tag{15}$$

which shows that, as a function of y, $V_l(y) = V_l(\nu_l(y,r))$ is continuous and decreasing. Note also that $V_l(y)$ attains any value in between $V_l(0) = (C_h + rC_l)/(1+r)$ and $V_l(1) = C_l$ at some unique signal cutoff $y \in (0,1)$.

Assuming standard dynamics, (12) gives

$$\begin{aligned} V_b &= \frac{q_u(1-F_h)\Delta_h - (1-q_u)(1-F_l)\Delta_g + (1-q_u)F_l(U_l - V_l)}{1 - F_h q_u + r} \\ &= \frac{q_u(1-F_h)\Delta_h - (1-q_u)(1-F_l)\Delta_g + (1-q_u)F_l(U_l - V_l)}{q_u(1-F_h) + (1-q_u)(1-F_l) + (1-q_u)F_l + r} \\ &= \frac{\Delta_h - (1-F_l)\Delta_g + F_l(U_l - V_l)}{2 + r\frac{1}{q_u(1-F_h)}} = \frac{\Delta_h - (1-F_l)\Delta_g + F_l(U_l - V_l)}{2 + r\frac{2-F_h}{1-F_h}} \\ &= \frac{\Delta_h - (1-F_l)\Delta_g + F_l(U_l - V_l)}{2 + r\left(1 + \frac{1}{1-F_h}\right)} = \frac{\Delta_h - (1-F_l)\Delta_g + F_l(U_l - V_l)}{2 + r + \nu_h} \end{aligned} \tag{16}$$

if $V_b + (V_l - C_l) < \Delta_l$ and $V_b \geq 0$, and

$$\begin{aligned} V_b &= \frac{q_u(1-F_h)\Delta_h - (1-q_u)(1-F_l)\Delta_g + (1-q_u)F_l V_b}{1 - F_h q_u + r} \\ &= \frac{\Delta_h - (1-F_l)\Delta_g + F_l V_b}{2 + r + \nu_h} = \frac{\Delta_h - (1-F_l)\Delta_g}{2 - F_l + r + \nu_h} \end{aligned} \tag{17}$$

if $V_b + (V_l - C_l) \geq \Delta_l$ and $V_b \geq 0$.[39]

Above, we have thus expressed average quality q_u in terms of y and F_l, F_h. To derive

[39] To make sure the later given fixed point correspondences FP and incentive condition correspondences IC are everywhere continuous, we need both payoffs in our fixed point analysis for standard dynamics, although only the former ones are consistent with the assumed dynamics.

the first lines, we have used the fact that, assuming standard dynamics,

$$
\begin{aligned}
q_u(1-F_h) &= \frac{1-F_h}{2-F_h}, \\
(1-q_u)(1-F_l) &= (1-F_l)\frac{1-F_h}{2-F_h}, \\
(1-q_u)F_l &= F_l\frac{1-F_h}{2-F_h}.
\end{aligned}
$$

Assuming reversed dynamics, (13) gives

$$
\begin{aligned}
V_b &= \frac{q_u(1-F_h)\Delta_h-(1-q_u)(1-F_l)\Delta_g}{1-q_uF_h(y)-(1-q_u)F_l+r} \\
&= \frac{q_u(1-F_h)\Delta_h-(1-q_u)(1-F_l)\Delta_g}{q_u(1-F_h)+(1-q_u)(1-F_l)+r} \\
&= \frac{\Delta_h-\Delta_g}{2+r\frac{1}{q_u(1-F_h)}} = \frac{\Delta_h-\Delta_g}{2+r\frac{2-F_h-F_l}{(1-F_l)(1-F_h)}} \\
&= \frac{\Delta_h-\Delta_g}{2+r\left(\frac{1}{1-F_h}+\frac{1}{1-F_h}\right)} = \frac{\Delta_h-\Delta_g}{2+\nu_h+\nu_l}
\end{aligned} \tag{18}
$$

for $V_b+(V_l-C_l)>\Delta_l$ and $V_b\geq 0$.

To obtain the first lines, we have used the fact that, assuming reversed dynamics,

$$
\begin{aligned}
q_u(1-F_h) &= (1-F_h)\frac{1-F_l}{2-F_h-F_l}, \\
(1-q_u)(1-F_l) &= (1-F_l)\frac{1-F_h}{2-F_h-F_l}.
\end{aligned}
$$

Independent of which trade dynamics prevail, we can thus see that $V_b(y)$ is continuous for all $y\in[0,1]$, and first increasing in y and later decreasing in y. With standard dynamics, $V_b(0)=\max\left\{0,\frac{\Delta_h-\Delta_g}{2+\nu_h}\right\}$ and, with reversed dynamics, $V_b(0)=\max\left\{0,\frac{\Delta_h-\Delta_g}{2+\nu_h+\nu_l}\right\}$. In both cases, $V_b(y)\to 0$ as $y\to 1$.

Proof of Lemma 5

Suppose trade takes place with probability one. By Lemmata 1–2 and 3 this implies that $y=0$. As a result, buyers offer C_h, which all sellers accept, for all signals s.

By the continuity of f_l and f_h, the buyers' conditional beliefs (3) are continuous and, according to Assumption 1, $q_c(0)=0$. The continuity of beliefs q_c in signals s entails that for any number $\epsilon>0$ there exists a number $\delta(\epsilon)>0$ such that $q_c(s)<\epsilon$ for all $s<\delta$.

Accordingly, if we choose the number $\epsilon=\frac{C_h-U_l}{U_h-U_l}=\frac{\Delta_g}{\Delta_h+\Delta_g}>0$, then for all signals $s<\delta(\epsilon)$, beliefs $q_c(s)$ are so low that a buyer's expected valuation for offering a high price is negative,

$$E(u|s) - C_h < \epsilon U_h + (1-\epsilon)U_l - C_h = \epsilon\Delta_h - (1-\epsilon)\Delta_g < 0,$$

which shows that a buyer strictly prefers offering $\min\{x_0, x_l\}$ to offering C_h for $s < \delta(\epsilon)$. This contradicts the assumption that trade takes place with probability one for all signals. $\square$

Proof of Lemma 6

Note first that for any $y < 1$

$$V_l(r) = C_l + \frac{1}{1+\nu_l(r)}(C_h - C_l) \to C_h,$$

as $r \to 0$, which means that

$$V_l + V_b > V_l > U_l$$

for low values of r because $C_h > U_l$. Thus, $y \to 1$ as $r \to 0$ is a necessary condition for the existence of a limit equilibrium under standard trade dynamics. The general proof below is somewhat more complex, though.

According to (1), the cutoff is the signal y solving the following fixed point condition,

$$U_h - C_h + \frac{1-G_-(x_h)}{1-G_-(x_l)}\frac{f_l(y)}{f_h(y)}(U_l - C_h) = V_b + \frac{1-G_-(x_h)}{1-G_-(x_l)}\frac{f_l(y)}{f_h(y)}\max\{U_l - x_l, V_b\}. \quad (19)$$

This condition is obtained from (1) by rewriting the equation in terms of $q_c(y)$ and suppressing the denominators $1 + \frac{1-G_-(x_h)}{1-G_-(x_l)}\frac{f_l(y)}{f_h(y)}$.

As a preliminary observation, note that for the lowest signal values $s \in (0,1)$, the lhs of (19) is smaller than the rhs of (19). The lhs is negative for low enough s by Assumption 1. The rhs is non-negative because V_b is non-negative by definition. Instead, for the highest signal values $s \in (0,1)$, the lhs of (19) is larger than the rhs of (19) because the lhs is positive for low enough s whereas the rhs approaches zero by Assumption 1 for $V_b \to 0$ as $y \to 1$. By the continuity of the condition (19) a fixed point y will thus exist.

The smallest fixed point corresponds to a cutoff signal at which the sides of (19) are larger than zero. To see why satisfying (19) as $0 = 0$ is impossible, consider signals $y < 1$ for which the lhs of (19) is equal to zero. This entails both (i) zero buyers' payoff for offering x_h at $s = y$ and (ii) positive buyers' payoff for offering x_h for $s > y$. As a result, we can see from (12) and (13) that $V_b > 0$ because $U(s < y) > V_b$ (irrespective of whether $V_b \geq U_l - x_l$ for which $U(s < y) = V_b$ or $V_b < U_l - x_l$ for which $U(s < y) > V_b$).

Lemma 8 shows later in more detail that, there is both a (higher) signal $y < 1$ which satisfies (19) as

$$U_h - C_h + \frac{1 - G_h(x_h-)}{1 - G_l(x_l-)} \frac{f_l(y)}{f_h(y)} (U_l - C_h) = V_b + \frac{1 - G_-(x_h)}{1 - G_-(x_l)} \frac{f_l(y)}{f_h(y)} (U_l - x_l),$$

and a (lower) signal $y > 0$ which satisfies (19) as

$$U_h - C_h + \frac{1 - G_h(x_h-)}{1 - G_l(x_l-)} \frac{f_l(y)}{f_h(y)} (U_l - C_h) = \left(1 + \frac{1 - G_h(x_h-)}{1 - G_l(x_l-)} \frac{f_l(y)}{f_h(y)}\right) V_b.$$

For standard dynamics, the latter condition results in[40]

$$\frac{\Delta_h - (1 - F_h)\frac{f_l}{f_h}\Delta_g}{1 + (1 - F_h)\frac{f_l}{f_h}} = \frac{\Delta_h - (1 - F_l)\Delta_g}{2 + \nu_h + r - F_l} = V_b, \tag{20}$$

whereas, for reversed dynamics, the same condition implies

$$\frac{\Delta_h - \frac{1-F_h}{1-F_l}\frac{f_l}{f_h}\Delta_g}{1 + \frac{1-F_h}{1-F_l}\frac{f_l}{f_h}} = \frac{\Delta_h - \Delta_g}{2 + \nu_h + \nu_l} = V_b. \tag{21}$$

We consider these cases one by one next. For (weakly) standard dynamics, observe that (20) can be written as

$$\alpha\Delta_h - (1 - \alpha)\Delta_g = \alpha'\Delta_h - \frac{1 - F_l}{1 - F_l + r + \nu_h}(1 - \alpha')\Delta_g,$$

where

$$\begin{aligned}
\alpha &= \frac{1}{1 + (1 - F_h(y))\frac{f_l(y)}{f_h(y)}}, \\
\alpha' &= \frac{1}{2 + \nu_h + (r - F_l(y))}.
\end{aligned}$$

For any $y < 1$, $\frac{1-F_l}{1-F_l+r+\nu_h} \to 1$ as $r \to 0$. To satisfy (20), we thus need either $y \to 1$ as $r \to 0$ or $\alpha \to \alpha'$ as $r \to 0$ (or both).

Setting $\alpha = \alpha'$ results in

$$\begin{aligned}
(1 - F_h(y))\frac{f_l(y)}{f_h(y)} &= r + \nu_h + (1 - F_l(y)) \\
(1 - F_h(y))\frac{f_l(y)}{f_h(y)} - (1 - F_l(y)) &= r\frac{2 - F_h}{1 - F_h} \\
\frac{1}{2 - F_h}\left[(1 - F_h(y))^2\frac{f_l(y)}{f_h(y)} - (1 - F_h(y))(1 - F_l(y))\right] &= r,
\end{aligned} \tag{22}$$

[40]We suppress the arguments of $f_\theta(y)$'s, $F_\theta(y)$'s and $\nu_\theta(y, r)$'s to abbreviate the expressions.

where $\frac{1}{2-F_h} > 1/2$ whereas the function inside the square brackets is strictly positive by MLRP for all $y < 1$. By L'Hopital's rule, $(1 - F_l(y))/(1 - F_h(y)) \to f_l(y)/f_h(y)$ as $y \to 1$, which shows that the inside of the brackets approaches zero as y approaches one (but generally not otherwise). Thus, $\alpha \to \alpha'$ as $r \to 0$ cannot hold unless $y \to 1$ as $r \to 0$.

For (weakly) reversed dynamics, notice that (21) can be written as

$$\beta\Delta_h - (1-\beta)\,\Delta_g = \beta'\Delta_h - \frac{1}{1+\nu_h+\nu_l}\,(1-\beta')\,\Delta_g,$$

where

$$\beta = \frac{1}{1 + \frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}},$$
$$\beta' = \frac{1}{2+\nu_h+\nu_l}.$$

For any $y < 1$, $\frac{1}{1+\nu_h+\nu_l} \to 1$ as $r \to 0$. To satisfy (21), we thus need either $y \to 1$ as $r \to 0$ or $\beta \to \beta'$ as $r \to 0$ (or both).

Setting $\beta = \beta'$ results in

$$\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)} = 1 + \frac{r}{1-F_h(y)} + \frac{r}{1-F_l(y)}$$
$$\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)} - 1 = r\left(\frac{1}{1-F_h(y)} + \frac{1}{1-F_l(y)}\right)$$
$$\frac{1-F_h(y)-F_l(y)-F_h(y)F_l(y)}{2-F_h(y)-F_l(y)}\left[\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)} - 1\right] = r \tag{23}$$

Note that $\frac{1-F_h(y)-F_l(y)-F_h(y)F_l(y)}{2-F_h(y)-F_l(y)} > 0$ whereas the function inside the square brackets is strictly positive by MLRP for all $y < 1$.

By L'Hopital's rule, $(1 - F_l(y))/(1 - F_h(y)) \to f_l(y)/f_h(y)$ as $y \to 1$, which shows that the function in the brackets approaches zero as y approaches one (but generally not otherwise). Thus, $\beta \to \beta'$ as $r \to 0$ cannot hold unless $y \to 1$ as $r \to 0$. $\square$

Proof of Lemma 7

We start by rewriting $\nu_l(r, s)$ as

$$\underbrace{\frac{r}{1-F_l(s)}}_{\nu_l(s,r)} = \underbrace{\frac{r}{1-F_h(s)}}_{\nu_h(s,r)}\frac{1-F_h(s)}{1-F_l(s)},$$

and study the limit as $s \to 1$. Because $F_\theta(s) \to 1$ as $s \to 1$, we can use L'Hopital's rule to have

$$\frac{1-F_h(s)}{1-F_l(s)} \to \frac{f_h(s)}{f_l(s)} \to \infty,$$

as $s \to 1$. This equals saying that for any $M > 1$ there exists a signal $s_l < 1$ such that

$$\nu_h(s_l, r)M^2 < \nu_l(s_l, r) \tag{24}$$

for any r and for $s > s_l$. We then turn to

$$\nu_\theta(s_0, r) = \frac{r}{1 - F_\theta(s_0)},$$

which clearly approach zero as $r \to 0$ and infinity as $r \to \infty$. For any $M > 1$ and for $s_l < 1$ we can thus find $r_0 \in (0, \infty)$ and $s_0 \in (0, s_l)$ such that

$$\nu_h(s_l, r_0) = \frac{r_0}{1 - F_h(s_l)} = 1/M, \tag{25}$$

$$\nu_l(s_0, r_0) = \frac{r_0}{1 - F_l(s_0)} = 1/M. \tag{26}$$

The result follows from (24), (25) and (26). $\square$

Lemma 8 *For any large enough $M > 1$, $0 < y_0(r) < y_l(r) < 1$ for all $r = 1/M$.*

1. *Under standard dynamics, $y(r) \geq y_l(r)$ where $y_l^1(r^1), y_l^2(r^2), ... \to 1$ as $r^1, r^2, ... \to 0$ along a sequence for which $\nu_l(y_l^1(r^1)), \nu_l(y_l^2(r^2)), ... \to n_l \geq \Delta_g/\Delta_l$.*

2. *Under reversed dynamics, $y(r) = y_0(r)$ where $y_0^1(r^1), y_0^2(r^2), ... \to 1$ as $r^1, r^2, ... \to 0$ along a sequence for which $\nu_l(y_0^1(r^1)), \nu_l(y_0^2(r^2)), ... \to n_0 = 0$.*

Proof of Lemma 8

To search for the lowest y_0, we continue the analysis from (22) and (23), which gives

$$\frac{1}{2 - F_h}\left[(1 - F_h(y))\frac{f_l(y)}{f_h(y)} - (1 - F_l(y))\right] =: \nu_h', \tag{27}$$

$$\frac{1 - F_h(y) - F_l(y) - F_h(y)F_l(y)}{2 - F_h(y) - F_l(y)}\left[\frac{1}{1 - F_l(y)}\frac{f_l(y)}{f_h(y)} - \frac{1}{1 - F_h(y)}\right] =: \nu_h'.. \tag{28}$$

We consider signal cutoffs y' for which buyers would be indifferent between offering x_0 and x_h at $s = y'$.

Under (weakly) standard dynamics, we can employ (27) and express $\nu_l' := \nu_l(y')$ as

$$\nu_l' = \frac{1 - F_h(y')}{1 - F_l(y')}\nu_h'$$

$$\nu_l' = \frac{1 - F_h(y')}{1 - F_l(y')}\frac{1}{2 - F_h}\left[(1 - F_h(y))\frac{f_l(y)}{f_h(y)} - (1 - F_l(y))\right]$$

$$\nu_l' = \frac{1 - F_h(y')}{2 - F_h(y')}\left[\frac{1 - F_h(y)}{1 - F_l(y')}\frac{f_l(y)}{f_h(y)} - 1\right] \to 0, \text{ as } y \to 1.$$

Under (weakly) reversed dynamics, we can apply (28) and rewrite $\nu_l' := \nu_l(y')$ as

$$\nu_l' = \frac{1 - F_h(y')}{1 - F_l(y')}\nu_h'$$
$$\nu_l' = \frac{1 - F_h(y')}{1 - F_l(y')} \frac{1 - F_h(y) - F_l(y) - F_h(y)F_l(y)}{2 - F_h(y) - F_l(y)} \left[\frac{1}{1 - F_l(y)} \frac{f_l(y)}{f_h(y)} - \frac{1}{1 - F_h(y)}\right]$$
$$\nu_l' = \frac{1 - F_h(y)}{(1 - F_h(y)) + (1 - F_l(y))} \left[\frac{1 - F_h(y')}{1 - F_l(y)} \frac{f_l(y)}{f_h(y)} - 1\right] \to 0, \text{ as } y \to 1.$$

This shows that, irrespective of which trade dynamics prevail, $y' \to 1$ as $r \to 0$ along a sequence for which $\nu_l(y')$ stays close to zero.

Turning to incentive conditions, note that $(V_l - C_l) \leq \Delta_l$ is a necessary condition for $V_b + (V_l - C_l) \leq \Delta_l$.

$$\begin{aligned}
(V_l - C_l) &\leq \Delta_l \\
\frac{1}{1 + \nu_l}(\Delta_g + \Delta_l) &\leq \Delta_l \\
\Delta_g + \Delta_l &\leq (1 + \nu_l)\Delta_l \\
\Delta_g &\leq \nu_l \Delta_l \\
\frac{\Delta_g}{\Delta_l} &\leq \nu_l
\end{aligned} \tag{29}$$

Next, consider y'' defined by (29) as

$$\nu_l'' := \nu_l(y'') = \frac{r}{1 - F_l(y'')} = \frac{\Delta_g}{\Delta_l}.$$

We can see that $y'' \to 1$ as $r \to 0$ along a sequence for which $\nu_l(y'')$ remains bounded away from zero.[41]

Comparing ν_l' to ν_l'' we thus find that $y_0(r) < y_l(r)$ for low r because $y_0 \to y'$ as $r \to 0$ and $y_l \geq y''$.

To complete the proof, we also need to show that $0 < y_0(r)$ and that $y_l(r) < 1$. This is easy. First, consider the incentives for offering x_0 as opposed to x_l, i.e., the roots of

$$IC_{0l} : IC_{0l}(y) = V_b(y) + (V_l(y) - C_l) - \Delta_l.$$

We have shown above that V_l is decreasing in y and $(V_l(0) - C_l) = \Delta_g + \Delta_l$, $(V_l(1) - C_l) = 0$, and $V_b + (V_l - C_l) \to 0$ as $y \to 1$, which shows by continuity that $y_l \in (0, 1)$.

[41] The bound is not tight. Indeed, it is straightforward to show that $\nu_l(y_l) \to n_l = \frac{\Delta_g + \Delta_h}{\Delta_l - \Delta_h} > \frac{\Delta_g}{\Delta_l}$ as $r \to 0$.

Second, consider the fixed point condition that defines the cutoff y,

$$\Delta_h + (1 - F_l(y))\frac{f_l(y)}{f_h(y)}(-\Delta_g) = V_b + (1 - F_l(y))\frac{f_l(y)}{f_h(y)} \max\{U_l - V_l, V_b\} \geq 0,$$

with standard dynamics and

$$\Delta_h + \frac{1 - F_h(y)}{1 - F_l(y)} \frac{f_l(y)}{f_h(y)}(-\Delta_g) = V_b + \frac{1 - F_h(y)}{1 - F_l(y)} \frac{f_l(y)}{f_h(y)} \max\{U_l - V_l, V_b\} \geq 0,$$

with reversed dynamics.

Clearly, because $\frac{f_l(y)}{f_h(y)} \to \infty$ as $y \to 0$, the lhs is (strictly) negative and the rhs is (weakly) positive in any sufficiently small neighborhood of $y = 0$. Similarly, as $\frac{f_l(y)}{f_h(y)} \to 0$ as $y \to 1$, and $V_b(y) \to 0, V_l(y) \to C_l$ as $y \to 1$, the lhs is positive and the rhs is negative in a neighborhood of $y = 1$. Thus, the result $y_0 \in (0, 1)$ obtains by continuity. $\square$

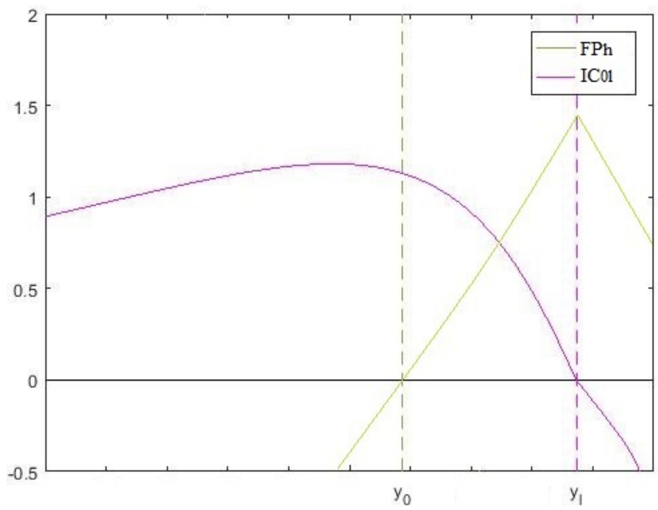


Figure 4: Illustration of Lemma 8.

Figure 4 illustrates the effect of different tentative cutoffs y on incentives to offer different prices x_0, x_l and x_h. The green line depicts a function $y \mapsto FP_h(y)$ (for "fixed point condition") which is positive for cutoffs for which a buyer prefers offering x_h over $\min\{x_0, x_l\}$. $FP_h(y)$ will thus cross the s-axis at y_0. The pink line defines a function $y \mapsto IC_{0l}(y)$ (for "incentive condition") which is positive for cutoffs for which a buyer prefers offering x_0 over x_l for $s < y$. $IC_{0l}(y)$ therefore crosses the s-axis at y_l. Both of these lines are generally non-monotone.

For IC_{0l}, this is because the effect of screening on a buyer's payoff is first positive and later negative whereas the effect on a low quality seller's payoff is negative. $IC_{0l} = V_b + V_l - \Delta_l$ can hence be first increasing (as V_b increases faster than V_l decreases) and thereafter decreasing (when both V_b and V_l are decreasing). Indeed, we find that a *single-crossing property* must hold for incentives of offering x_0 over x_l. Thereby, the pink IC_{0l}

curve crosses the s-axis once from above and divides the cutoffs into (i) lower ones $y < y_l$ for which dynamics would be reversed and (ii) higher ones $y > y_l$ for which dynamics would be standard.

Instead, the green FP_h curve is increasing for low screening, $y < y_l$, crossing the s-axis therefore at $y_0 < y_l$ already for relatively low screening. As detailed in Proposition 1, this low root of FP_h may for suitable parameters correspond to an equilibrium with reversed dynamics. However, as the benefit of trading low quality for a low price x_l begins to affect the payoff of a buyer V_b, FP_h generally decreases for $y \in (y_l, s_h)$ and ultimately increases for $y \in [s_h, 1]$, possibly thus crossing the s-axis for a second and a third time. These higher roots of FP_h would then represent equilibria with standard dynamics, outlined in Proposition 2.

We proceed by proving Proposition 2 first and then move to Propositions 1 and 3.

Proof of Proposition 2

Step I. Screening over different cutoff sequences $(y(r^i))_{r^i \to 0}$

By Lemma 7, for any $M > 1$ there exist $s_0 < s_l < s_h$ and $r < 1/M$ such that

$$\nu_l(s_0) = \nu_h(s_l) = 1/M < M \leq \nu_l(s_l) = \nu_h(s_h).$$

By Lemma 8, for any $M > \Delta_l/\Delta_g$ and $r < 1/M$ there exist y_0 and y_l such that

$$\nu_h(y_0) < \nu_l(y_0) < \nu_h(y_l) \leq 1/M < \Delta_g/\Delta_l \leq \nu_l(y_l),$$

where the cutoffs are defined by Lemma 8 in such a way that, if $y = y_0$, a buyer is indifferent between x_0 and x_h and, if $y = y_l$, a buyer is indifferent between x_0 and x_l.

For any sequence $r^1, r^2, \ldots \to 0$ we thus obtain five related cutoff sequences $(s_0(r^i), s_l(r^i), s_h(r^i), y_0(r^i), y_l(r^i))_{i=1,2,\ldots}$ with different associated screening intensities.

Because $s_0 \to 1$ and $y_0 \to 1$ as $r \to 0$, we also know that

$$\begin{aligned} \frac{f_l}{f_h}(y(r)) &< 1/N_f \qquad && \text{for all } r < 1/M, y = y_0, y_l, s_0, s_l, s_h, \\ 1 - F_\theta(y(r)) &< 1/N_F \qquad && \text{for all } r < 1/M, y = y_0, y_l, s_0, s_l, s_h, \end{aligned}$$

where $N_f \to \infty$ and $N_F \to \infty$ as $M \to \infty$.

Step II. Existence of fixed point sequences $(y(r^i))_{r^i \to 0}$

According to (19), y satisfies the following fixed point condition under standard trade dynamics

$$FP_h : FP_h = \frac{\Delta_h - (1 - F_h)\frac{f_l}{f_h}\Delta_g}{1 + (1 - F_h)\frac{f_l}{f_h}} - \frac{V_b + (1 - F_h)\frac{f_l}{f_h}\max\{U_l - V_l, V_b\}}{1 + (1 - F_h)\frac{f_l}{f_h}} = 0, \tag{30}$$

where V_b is defined by (16) for $V_b + V_l \leq U_l$ and by (17) for $V_b + V_l > U_l$ and V_l is defined by (15). Under these assumptions, FP_h is continuous in y and in r.

We proceed by proving that (30) is satisfied at some $y^1(r) \in (y_l(r), s_l(r))$ and at some $y^2(r) \in (y^1(r), s_h(r))$ for all low enough (fixed) values of r. As FP_h is continuous in y, it suffices to show that

$$FP_h(y,r) > 0, \qquad \text{for all } y \in (y_0(r), y_l(r)), \tag{31}$$

$$FP_h(y,r) < 0, \qquad \text{at } y = s_l(r), \tag{32}$$

$$FP_h(y,r) > 0, \qquad \text{at } y = s_h(r). \tag{33}$$

Case 1. To show that (31) holds, we consider (20) satisfied by y_0

$$\alpha\Delta_h - (1-\alpha)\,\Delta_g - \alpha'\Delta_h + \frac{1-F_l}{1-F_l+r+\nu_h}\,(1-\alpha')\,\Delta_g = 0, \tag{34}$$

where

$$\alpha(y) = \frac{1}{1+(1-F_h(y))\frac{f_l(y)}{f_h(y)}},$$
$$\alpha'(y) = \frac{1}{1+(1-F_l(y))+\nu_h(y,r)+r}.$$

Lemma 6 shows that $y_0(r) \to 1$ as $r \to 0$. Lemma 8 proves that $\alpha(y_0) \to \alpha'(y_0)$ as $r \to 0$. Further, the terms multiplied by $(1-\alpha)$ or $(1-\alpha')$ become negligible for low r because $f_l/f_h(y) \to 0$, $1 - F_l(y(r)) \to 0$, and $\nu_h(y(r)) \to 0$ as $r \to 0$ for all $y \in (y_0, y_l)$. To sign the lhs of (34) for low values of r, we can hence focus on

$$\alpha - \alpha',$$

or, equivalently, on the difference between the numerators

$$(1+(1-F_h)\frac{f_l}{f_h}) - (1+(1-F_l)+r+\nu_h).$$

Differentiating this expression with respect to y results in

$$-f_h\frac{f_l}{f_h} + (1-F_h)\frac{\partial}{\partial y}\frac{f_l}{f_h} + f_l - \frac{\partial}{\partial y}\nu_h = (1-F_h)\underbrace{\frac{\partial}{\partial y}\frac{f_l}{f_h}}_{<0} - \underbrace{\frac{\partial}{\partial y}\nu_h}_{>0} < 0,$$

which shows that $\alpha - \alpha'$ is increasing in y for $y \in (y_0, y_l)$.

To show that (32) and (33) also hold true, we proceed by proving that

$$\Delta_h - (1 - F_h)\frac{f_l}{f_h}\Delta_g < V_b + (1 - F_h)\frac{f_l}{f_h}\max\{U_l - V_l, V_b\} \text{ at } y = s_l \tag{35}$$

$$\Delta_h - (1 - F_h)\frac{f_l}{f_h}\Delta_g > V_b + (1 - F_h)\frac{f_l}{f_h}\max\{U_l - V_l, V_b\} \text{ at } y = s_h. \tag{36}$$

Case 2. We can see from above that (35) is satisfied providing that

$$V_b(s_l) = \frac{\Delta_h - (1 - F_l(s_l))\Delta_g + F_l(s_0)(U_l - V_l(s_l))}{2 + r + \nu_h(s_l)} > \Delta_h.$$

Given the assumptions in Step I, $V_b(s_l)$ is clearly larger than

$$\underline{V}_b(s_l(M)) = \frac{\Delta_h - 1/N_F\Delta_g + (1 - 1/N_F)(\Delta_l - \frac{\Delta_g + \Delta_l}{1+M})}{2 + 1/M + 1/M}.$$

Taking the limit as $M \to \infty$, we thus observe as required that

$$\underline{V}_b(s_l(M)) \to \frac{\Delta_h + \Delta_l}{2} > \Delta_h$$

and

$$\underline{V}_b(s_l(M)) + (\underline{V}_l(s_l(M)) - C_l) \to \frac{\Delta_h + \Delta_l}{2} < \Delta_l.$$

Case 3. Additionally, we can see that (36) is satisfied at $y = s_h$ for any high enough values of M because the assumptions made in Step I imply here that

$$\Delta_h - 1/(M^2)\Delta_g > \frac{\Delta_h - (1 - F_l(s_h))\Delta_g + F_l(s_h)\Delta_l}{2 + M + M} + 1/(M^2)(\Delta_l - \frac{\Delta_g + \Delta_l}{1 + M})$$

$$\Delta_h > \underbrace{\frac{\Delta_h - (1 - F_l(s_h))\Delta_g + F_l(s_h)\Delta_l}{2 + M + M}}_{\to 0, \text{ as } M \to \infty} + \underbrace{\frac{1}{M(1 + M)}(\Delta_l + \Delta_g)}_{\to 0, \text{ as } M \to \infty}.$$

Figure 5 shows the graph of $FP_h(y)$ for $r = 0.05$ (Figure 5a) and $r = 0.005$ (Figure 5a) illustrating how decreasing r affects the position of fixed points y^1 and y^2 ('red dots') with respect to y_0 ('green line') and y_l ('pink line'). Especially, the gap between y_0 and $y_l > y_0$ remains whereas the gap between y_l and $y^1 > y_l$ vanishes as $r \to 0$.

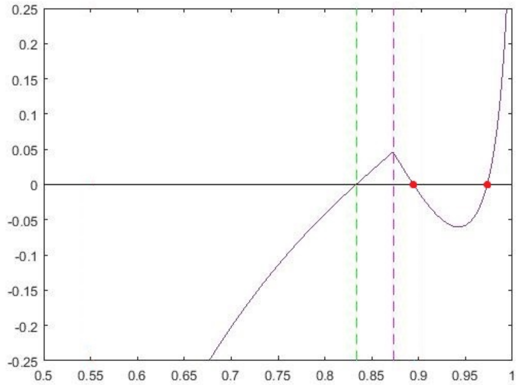
Details of equilibria with $(\Delta_h, \Delta_g, \Delta_l) = (0.5, 1.5, 1)$ and $r = 0.05$ (as in Figure 5a):

1st equilibrium at $y^1 \approx 0.894$: $V_b(0.894) \approx 0.450$ and $V_l(0.894) \approx 0.466$ such that $V_b + 0.5(V_l - C_l) \approx 0.683 < 0.5(\Delta_h + \Delta_l) = 0.75$,

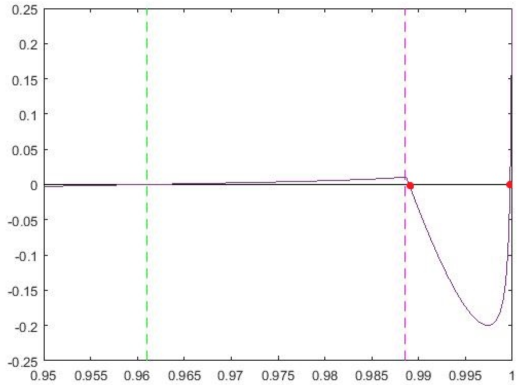
2nd equilibrium at $y^2 \approx 0.974$: $V_b(0.974) \approx 0.498$ and $V_l(0.974) \approx 0.034$ such that $V_b + 0.5(V_l - C_l) \approx 0.515 < 0.5(\Delta_h + \Delta_l) = 0.75$.

Step III. Price offers at y^1 and y^2

One might still wonder whether the solutions to (30) that we have identified correspond



(a) $r = 0.05$



(b) $r = 0.0005$

Figure 5: FP_h for $(\Delta_h, \Delta_g, \Delta_l) = (0.5, 1.5, 1)$.

to equilibria with standard dynamics or whether the preference for offering x_l over x_0 (as with standard dynamics) might change again to a preference for offering x_l over x_0 (as with reversed dynamics). However, if that was the case, (35) and (36) imply a buyer is indifferent between offering x_h and x_0 at y^1 and at y^2. This means that (34) holds at $y^1 \in (y_l, s_l)$ and at $y^2 \in (y^1, s_h)$.

But then our assumption made in Step I will imply that $\nu_h(y^1) < 1/M$ and $\nu_h(y^1) < 1/M$. As we can see following the proof of Lemma 7, $\nu_h(y^1) < 1/M$ and $\nu_h(y^1) < 1/M$ implies first $\alpha \to \alpha'$ as $M \to \infty$ which implies $\nu_l(y^1) \to 0$ and $\nu_l(y^2) \to 0$ as $M \to \infty$. This contradicts our assumptions in Step I. As a result, we can conclude that the cutoffs y^1 and y^2 must be such that a buyer prefers to offer x_l over x_0 at each of them as required by standard trade dynamics.

Step IV. Limit payoffs at y^1 and y^2

It remains to calculate the payoffs in the limit equilibria where $y = y^1 \in (y_l, s_h)$ and $y = y^2 \in (y^1, s_h)$ satisfy (30). These limit payoffs depend on $\nu_l(y, r)$ and $\nu_h(y, r)$, which assume different values for $(y, r) = (y^1(r), r)$ and $(y, r) = (y^2(r), r)$ for low r, although both $y^1(r) \to 1$ and $y^2(r) \to 1$ as $r \to 0$.

Case 1. The payoffs at $y^1(r)$ for $r \to 0$.

We have shown that $\nu_h(y^1) < 1/M$ but $\nu_l(y^1) \in (\frac{\Delta_g}{\Delta_l}, M)$. Applying (16) and (30), the equation defining y^1 thus becomes for high values of M, approximately,

$$\Delta_h - \frac{1}{N_f N_F}\Delta_g = \frac{\Delta_h - 1/N_F\Delta_g + (1 - 1/N_F)(\Delta_l - \frac{\Delta_g+\Delta_l}{1+\nu_l(y^1)})}{2 + 2/M} + \frac{1}{N_f N_F}\left(\Delta_l - \frac{\Delta_g + \Delta_l}{1 + \nu_l(y^1)}\right).$$

We are interested in the limiting payoffs as $M \to \infty$, which gives us

$$\begin{aligned}
\Delta_h =& \frac{\Delta_h + \Delta_l - \frac{\Delta_g+\Delta_l}{1+\nu_l(y^1)}}{2} \\
\Longrightarrow& \nu_l(y^1) = \frac{\Delta_g + \Delta_h}{\Delta_l - \Delta_h} > 0 \\
\Longrightarrow& V_l - C_l = \frac{\Delta_l + \Delta_g}{1 + \nu_l(y^1)} = \Delta_l - \Delta_h \\
\Longrightarrow& V_b = \frac{\Delta_h + \Delta_l - (\Delta_l - \Delta_h)}{2} = \Delta_h \\
\Longrightarrow& V_b + V_l - C_l = \Delta_h + \Delta_l - \Delta_h = \Delta_l \\
\Longrightarrow& W = V_b + \frac{1}{2}(V_l - C_l) = \Delta_h + \frac{1}{2}(\Delta_l - \Delta_h) = \frac{\Delta_h + \Delta_l}{2}.
\end{aligned}$$

Two results are notable. First, the limiting equilibrium payoffs will approach from below the payoffs at which a buyer is indifferent between offering x_0 and x_l for $s < y$. Second, the limiting equilibrium payoffs are efficient, equaling the payoffs from immediate trading. Moreover, the limiting payoffs are also higher than the static *Walrasian* payoffs $\Delta_l/2$.

Case 2. The payoffs at $y^2(r)$ for $r \to 0$.

We first make a guess that $y^2 \in (s_l, s_h)$ such that $\nu_l(y^2) > M$ but $\nu_h(y^2) \in (1/M, M)$.[42] The equation defining y^2 thus becomes for high values of M, approximately,

$$\Delta_h - \frac{1}{N_f N_F}\Delta_g = \frac{\Delta_h - 1/N_F\Delta_g + (1 - 1/N_F)(\Delta_l - \frac{\Delta_g+\Delta_l}{1+M})}{2 + 1/M + \nu_h(y^2)} + \frac{1}{N_f N_F}\left(\Delta_l - \frac{\Delta_g + \Delta_l}{1 + M}\right)$$

[42] The other possibility would be $\nu_l(y^2) > M$ and $\nu_h(y^2) > M$, which would result in lower limit payoffs.

We concentrate on the limiting payoffs as $M \to \infty$, which here gives us

$$\begin{aligned}
\Delta_h &= \frac{\Delta_h + \Delta_l}{2 + \nu_h(y^2)} \\
\Longrightarrow \nu_h(y^1) &= \frac{\Delta_l - \Delta_h}{\Delta_h} > 0 \\
\Longrightarrow V_l - C_l &= 0 \\
\Longrightarrow V_b &= \frac{\Delta_h + \Delta_l}{1 + \frac{\Delta_l}{\Delta_h}} = \Delta_h \\
\Longrightarrow V_b + V_l - C_l &= \Delta_h < \Delta_l \\
\Longrightarrow W = V_b + \frac{1}{2}(V_l - C_l) &= \Delta_h < \frac{\Delta_h + \Delta_l}{2}.
\end{aligned}$$

Now, the limiting payoffs are inefficient, below the payoffs from immediate trading. However, the limiting equilibrium payoffs may still exceed the static *Walrasian* payoffs $\Delta_l/2$.

r	y^1	V_b	V_l	y^2	V_b	V_l
0.01	0.953	0.488	0.468	0.995	0.526	0.009
0.001	0.985	0.487	0.510	0.999	0.599	0.003
0.0001	0.995	0.501	0.494	1.000	0.529	0.000

Table 1: Comparison of equilibria with $(\Delta_h, \Delta_g, \Delta_l) = (0.5, 1.5, 1)$.

Table 1 compares equilibrium payoffs for different $r \leq 0.01$.

Proof of Proposition 1

Under reversed trade dynamics, the cutoff signal y satisfies the following fixed point condition and incentive condition

$$\begin{aligned}
FP_h : FP_h &= \frac{\Delta_h - (1 - F_h)\frac{f_l}{f_h}\Delta_g}{1 + (1 - F_h)\frac{f_l}{f_h}} - V_b = 0, \\
IC_{0l} : IC_{0l}(y) &= V_b(y) + (V_l(y) - C_l) - \Delta_l \geq 0.
\end{aligned}$$

In this case, the existence of equilibrium follows directly from Lemma 8, which proves that there exists y_0 at which a buyer is indifferent between x_h and x_0 and prefers offering x_0 over x_l.

Equilibrium uniqueness is given by the proof of Proposition 2 (Step II, Case 1), which shows that the fixed point y of F_{0h} is unique because F_{0h} is increasing when $IC_{0l} \geq 0$ is satisfied.

We can derive payoff limits as before. We know from Lemma 8 that $y_0 \to 1$ along a path such that $\nu_h < \nu_l < 1/M$, where M is a large value:

$$\begin{aligned}
&\Longrightarrow V_l - C_l = \frac{\Delta_l + \Delta_g}{1 + 1/M} \to \Delta_l + \Delta_g \\
&\Longrightarrow V_b = \frac{\Delta_h - \Delta_g}{2 + 1/M + 1/M} \to \frac{\Delta_h - \Delta_g}{2} \\
&\Longrightarrow W = V_b + \frac{1}{2}\left(V_l - C_l\right) = \frac{\Delta_h - \Delta_g}{2} + \frac{\Delta_l + \Delta_g}{2} = \frac{\Delta_h + \Delta_l}{2}.
\end{aligned}$$

The limiting payoffs are thereby efficient and equal the static *Walrasian* payoffs.

Proof of Proposition 3

By Lemma 4, we know that an equilibrium cannot feature reversed dynamics if the static lemons problem is severe. Because we assume in this case that $\Delta_g > \Delta_h$, an equilibrium must thus feature standard dynamics. Under standard trade dynamics, the cutoff signal y satisfies the following fixed point condition and incentive condition

$$FP_h : FP_h = \Delta_h - \frac{1}{N_f N_F}\Delta_g - V_b - \frac{1}{N_f N_F}\max\left\{U_l - V_l, V_b\right\} = 0, \tag{37}$$

$$IC_{0l} : IC_{0l}(y) = V_b(y) + (V_l(y) - C_l) - \Delta_l \leq 0. \tag{38}$$

However, satisfying both conditions at the same time is impossible for sufficiently low r if $\Delta_h > \Delta_l$.

To see why, note that, by Lemma 8, satisfying the incentive condition $IC_{0l} \leq 0$ for low enough r requires high enough $y > y_l(r) > y_0(r)$, where $y_0(r) \to 1$ as $r \to 0$.

As a result, applying the notation in the proof of Proposition 2 (Step I), the fixed point condition $FP_h = 0$ for $y > y_0$ and $r < 1/M$ can be approximated by

$$\Delta_h - \underbrace{\left(\frac{1}{N_f N_F}\right)}_{\to 0}\Delta_g - V_b - \underbrace{\left(\frac{1}{N_f N_F}\right)}_{\to 0}\max\left\{U_l - V_l, V_b\right\} = 0,$$

where $1/N_f \to 0$, $1/N_F \to 0$ as $M \to \infty$.

We also show in the proof of Proposition 2 (Step II Case 1.) that $FP_h > 0$ for $y \in (y_0, y_l)$ and $r < 1/M$. By the continuity of $FP_h(y)$, it is thus easy to see that satisfying $FP_h(y) = 0$ for $y \in (y_0, y_l)$ and $r < 1/M$ is impossible without assuming that $V_b \geq \Delta_h$, which would violate $IC_{0l}(y) \leq 0$.

This also covers the case of weak incentives $IC_{0l} = 0$. However, Proposition 3 may mislead some readers into thinking that the above shows that we cannot obtain an equilibrium where buyers offer two prices x_h and x_l but does not preclude the existence of an equilibrium with three price offers x_h, x_l and x_0.

To convince all readers, we thus show that offering three prices would result in a contradiction as y_0 and y_l require different screening intensities $\nu_l(y,r)$ for low r, as shown in Lemma 8. So, let us try to construct an equilibrium where $y = y_0 = y_l$ such that a buyer would be willing to propose x_0 or x_l for $s < y_0 = y_l$ (i.e., x_0 for $s \in (0,z)$ and x_l for $s \in (z,y)$) and x_h for $s > y_0 = y_l$.

The probability of trading for low quality is given by $1 - F_l(z)$, which we also use in the following notation $\nu_0 = \frac{r}{1-F_l(z)} < \frac{r}{1-F_h(y)} = \nu_h(y)$. We can thus express the valuation of buyers as

$$V_b = \frac{q_u(y,z)(1-F_h(y))\Delta_h - (1-q_u(y,z))(1-F_l(y))\Delta_g}{1 - q_u(y,z)F_h(y) - (1-q_u(y,z))F_l(y) + r}$$
$$\frac{\Delta_h - \frac{\nu_0(z)}{\nu_l(y)}\Delta_g}{1 + \frac{\nu_0(z)}{\nu_l(y)} + r\frac{1}{q_u(y,z)(1-F_h(y))}}$$
$$\frac{\Delta_h - \frac{\nu_0(z)}{\nu_l(y)}\Delta_g}{1 + \frac{\nu_0(z)}{\nu_l(y)} + r\left(\frac{1-F_l(z)}{(1-F_l(z))(1-F_h(y))} + \frac{1-F_h(y)}{(1-F_l(z))(1-F_h(y))}\right)}$$
$$\frac{\Delta_h - \frac{\nu_0(z)}{\nu_l(y)}\Delta_g}{1 + \frac{\nu_0(z)}{\nu_l(y)} + \nu_0(z) + \nu_h(y)} \tag{39}$$

where

$$q_u(y,z) = \frac{1}{1 + \frac{1-F_h(y)}{1-F_l(z)}}.$$

By definition, y_l satisfies the incentive condition $IC_{0l}(y) = 0$, which can be written as

$$V_b = \frac{\Delta_l - \frac{1}{\nu_l(y)}\Delta_g}{1 + \frac{1}{\nu_l(y)}},$$

whereas y_0 satisfies the fixed point condition $FP_h(y) = 0$, which can be expressed as

$$V_b = \frac{\Delta_h - \frac{\nu_0(z)}{\nu_l(y)}m(y)\Delta_g}{1 + \frac{\nu_0(z)}{\nu_l(y)}m(y)}.$$

Above,

$$m(y) = \frac{f_l(y)}{f_h(y)}\frac{1-F_h(y)}{1-F_l(y)}.$$

Note that $m(y)$ is larger than unity by MLRP but approaches one as $y \to 1$. As shown by Lemma 8, satisfying the incentive condition requires that $y \to 1$. Thus, there exists no cutoff $y = y_0 = y_l$ that satisfies all the conditions for low r. $\square$

Documentation for Figures 1, 4 and 5

Figures 1, 4 and 5 are plotted using the function forms as follows:

$$
\begin{aligned}
& f_h(s) = 2s, F_h(s) = s^2, f_l(s) = 2 - 2s, F_l(s) = 2s - s^2, & \\
& \nu_h(y,r) = \frac{r}{1 - F_h(y)} & \text{'blue line'} \\
& \nu_l(y,r) = \frac{r}{1 - F_l(y)} & \text{'red line'} \\
& y = y_0(r) & \text{'green line'} \\
& y = y_l(r) & \text{'pink line'}
\end{aligned}
$$

Proof of Corollary 1

Propositions 1 and 2 demonstrate that any (sufficiently large) information bound $B < \infty$ is associated with $y = \overline{s} < 1$ and $r > 0$ on the path $(y, r) \to (1, 0)$ to the efficient limit equilibrium.

If $\Delta_g > \Delta_h$ and $\Delta_l \geq \Delta_h$, the payoffs in the static Walrasian market equal $\Delta_l/2$ and those in the stationary equilibrium with small frictions

$$
\begin{aligned}
V_b &= \frac{\Delta_h - (1 - F_l(y))\Delta_g + F_l(y)(U_l - V_l)}{2 + r + \nu_h(y,r)} + \\
(V_l - C_l)/2 &= \frac{\Delta_g + \Delta_l}{2 + 2\nu_l(y,r)}.
\end{aligned}
$$

Instead, if $\Delta_g \leq \Delta_h$, the payoffs in the static Walrasian market equal $\Delta_h/2 + \Delta_l/2$ and those in the stationary equilibrium with small frictions

$$
\begin{aligned}
V_b = V_b(y) &= \frac{\Delta_h - \Delta_g}{2 + \nu_h(y,r) + \nu_l(y,r)} + \\
(V_l - C_l)/2 &= \frac{\Delta_g + \Delta_l}{2 + 2\nu_l(y,r)}. \quad \square
\end{aligned}
$$

Knife-edge dynamics

An equilibrium with knife-edge dynamics for vanishing frictions $r \to 0$ is given by y, $z < y$, and (V_b, V_l) satisfying the following system

$$\begin{aligned}
& q_c(s) = \frac{1}{1 + \frac{1-F_h(y)}{1-F_l(z)} \frac{f_l(s)}{f_h(s)}}, \text{ for } s \in [0,1] \\
& q_c(y)\,(U_h - C_h) + (1 - q_c(y))\,(U_l - C_h) = V_b, && (40) \\
& V_b = \frac{\Delta_h}{1 + \nu_h(y,r)}, \\
& V_l = C_l + \frac{\Delta_g + \Delta_l}{1 + \nu_l(y,r)}, \\
& V_b + (V_l - C_l) = \Delta_l. && (41)
\end{aligned}$$

Note that we can set $y \to 1$ because it otherwise becomes impossible to satisfy Eq. (41). Eq. (41) further implies that $\frac{\Delta_h}{1+\nu_h(y,r)} + \frac{\Delta_g+\Delta_l}{1+\nu_l(y,r)} = \Delta_l$. Joining Eqs. (40) and (41), we thus obtain that

$$q_c(y)\Delta_h + (1 - q_c(y))\,(-\Delta_g) = \frac{\Delta_h}{1 + \nu_h(y,r)} = \Delta_l - \frac{\Delta_g + \Delta_l}{1 + \nu_l(y,r)}. \tag{42}$$

As $(y,r) \to (1,0)$, market quality depends on the evolution of $z(y,r)$ as frictions disappear. In principle, $z(y,r)$ could assume any values between 0 and $y \to 1$. Depending on $z(y,r)$, buyers' beliefs

$$q_c(y) = \frac{1}{1 + \frac{1-F_h(y)}{1-F_l(z)} \frac{f_l(s)}{f_h(s)}}$$

thus span all the values from 1/2 (attained by letting $z(y,r) \to y$ as $(y,r) \to (1,0)$) to 1 (attained by letting $z(y,r) \to 0$ as $(y,r) \to (1,0)$). This allows some leeway in equilibrium construction because any triplet (q_c, ν_h, ν_l) satisfying Eq. (42) for $1/2 \le q_c \le 1$ and $0 \le \nu_h \le \nu_l \le \infty$ defines a stationary limit equilibrium for $(y,r) \to (1,0)$.

$$\begin{aligned}
\frac{\Delta_h}{1 + \nu_h} &= q_c\Delta_h + (1 - q_c)(-\Delta_g) \\
\frac{\Delta_h}{1 + \nu_h} &= \Delta_l - \frac{\Delta_g + \Delta_l}{1 + \nu_l}.
\end{aligned}$$

For example, by letting $\nu_l \to \infty$, we immediately find an equilibrium with knife-edge dynamics given by

$$\nu_h \to \frac{\Delta_h}{\Delta_l} - 1, \text{ and } q_c \to \frac{\Delta_l + \Delta_g}{\Delta_h + \Delta_g}, \text{ as } (y,r) \to (1,0),$$

for cases $\Delta_g > \Delta_h > \Delta_l$ where Proposition 3 shows that no equilibrium with standard or reversed dynamics exists.

This equilibrium is inefficient as $V_b + (V_l - C_l)/2 = \Delta_l < \frac{\Delta_l + \Delta_h}{2}$. To derive an efficient equilibrium, we thus require that

$$V_b = \frac{\Delta_l + \Delta_h}{2} - \frac{\Delta_g + \Delta_l}{(1+\nu_l)2}$$

$$\frac{\Delta_l + \Delta_h}{2} - \frac{\Delta_g + \Delta_l}{(1+\nu_l)2} = \Delta_l - \frac{\Delta_g + \Delta_l}{1+\nu_l},$$

which gives $\frac{1}{1+\nu_l}$ as a solution to the second-order equation

$$-\left(\Delta_g + \Delta_l\right)\left(\frac{1}{1+\nu_l}\right)^2 + \left(\Delta_g + \Delta_l\right)\frac{1}{1+\nu_l} + \frac{\Delta_h - \Delta_l}{2} = 0.$$

Presuming $\Delta_h > \Delta_l$, the equation has a positive solution

$$\frac{1}{1+\nu_l} = \frac{-\left(\Delta_g + \Delta_l\right) + \sqrt{\left(\Delta_g + \Delta_l\right)^2 + 2\left(\Delta_g + \Delta_l\right)\left(\Delta_h - \Delta_l\right)}}{2\left(\Delta_g + \Delta_l\right)} = \frac{-1 + \sqrt{1 + 2\frac{\Delta_h - \Delta_l}{\Delta_g + \Delta_l}}}{2},$$

Inserting this solution into Eqs. (42) gives

$$\begin{aligned}
\nu_l &= \frac{\sqrt{1 + 2\frac{\Delta_h - \Delta_l}{\Delta_g + \Delta_l}} + 3}{\sqrt{1 + 2\frac{\Delta_h - \Delta_l}{\Delta_g + \Delta_l}} - 1} \\
\nu_h &= \frac{\Delta_h - \Delta_l + \frac{\Delta_g + \Delta_l}{2}\left(\sqrt{1 + 2\frac{\Delta_h - \Delta_l}{\Delta_g + \Delta_l}} - 1\right)}{\Delta_l - \frac{\Delta_g + \Delta_l}{2}\left(\sqrt{1 + 2\frac{\Delta_h - \Delta_l}{\Delta_g + \Delta_l}} - 1\right)} \\
q_c &= \frac{\Delta_h}{\Delta_h + \Delta_g}\frac{1}{1+\nu_h} + \frac{\Delta_g}{\Delta_h + \Delta_g} \in (1/2, 1)
\end{aligned}$$

The existence of efficient knife-edge dynamics requires that $\nu_h < \nu_h$ and $q_c \in (1/2, 1)$. The latter condition is clearly satisfied for all $\nu_h \geq 0$ if $\Delta_g > \Delta_h$.

It is also easy to confirm that the former one is satisfied, e.g., if $\Delta_g = 3 > \Delta_h = 2 > \Delta_l = 1$ for which $\nu_l \approx 19.2 > \nu_h \approx 2.6$ and $q_c \approx 0.71 \in (1/2, 1)$.

More general analysis demonstrates that $\nu_h < \nu_l$ is equivalent to

$$\frac{y+4}{y} > \frac{\Delta_h - \Delta_l + \frac{\Delta_g + \Delta_l}{2}y}{\Delta_l - \frac{\Delta_g + \Delta_l}{2}y} \iff -\left(\Delta_g + \Delta_l\right)y^2 - \left(2\Delta_g + \Delta_h\right)y + 4\Delta_l > 0$$

where $y = \left(\sqrt{1 + 2\frac{\Delta_h - \Delta_l}{\Delta_g + \Delta_l}} - 1\right)$. The lhs of the inequality represents a downward sloping parabola, with both a negative root and a positive root, which transforms our conditions for $\nu_l < \nu_h$ into

$$-\sqrt{\left(\frac{\Delta_g + \Delta_h/2}{\Delta_g + \Delta_l}\right)^2 + 4\Delta_l} - \frac{\Delta_g + \Delta_h/2}{\Delta_g + \Delta_l} < \sqrt{1 + 2\frac{\Delta_h - \Delta_l}{\Delta_g + \Delta_l}} - 1 < \sqrt{\left(\frac{\Delta_g + \Delta_h/2}{\Delta_g + \Delta_l}\right)^2 + 4\Delta_l} - \frac{\Delta_g + \Delta_h/2}{\Delta_g + \Delta_l}.$$