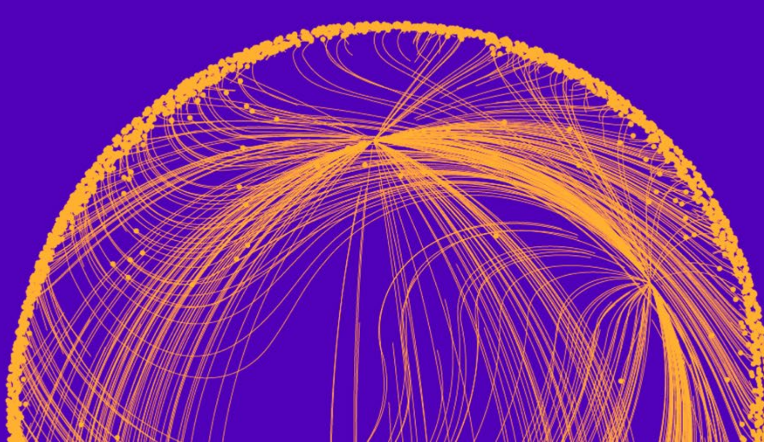
Helsinki GSE

HELSINKI GSE DISCUSSION PAPERS 56 · 2026

Licensing resource exploitation with endogenous and privately known reserves

Jarmo Jääskeläinen
Pauli Lappi



HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

HANKEN

Aalto University

Helsinki GSE Discussion Papers

Helsinki GSE Discussion Papers 56 · 2026

Jarmo Jääskeläinen and Pauli Lappi:
Licensing resource exploitation with endogenous and privately known reserves

ISBN 978-952-7543-55-9 (PDF)
ISSN 2954-1492

Helsinki GSE Discussion Papers:
https://www.helsinkigse.fi/discussion-papers

Helsinki Graduate School of Economics
PO BOX 21210
FI-00076 AALTO
FINLAND

Helsinki, March 2026

Licensing resource exploitation with endogenous and privately known reserves

Jarmo Jääskeläinen* and Pauli Lappi†

Abstract

We design optimal mechanism to collect revenues from nonrenewable resource exploitation, when the resource must be explored and discovered and when there is asymmetric information. The exploration effort may result in a discovery that is revealed only to the firm, and the effort is not observed by the owner. After discovery, the firm may pay development costs including set-up costs to build extraction capacity. To overcome the discontinuity caused by the set-up costs in this adverse selection problem, we use a two-step procedure to solve the model and show that the optimal license contract involves a discontinuity at the reserve cut-off level. We show that compared to the case without any pricing of the resource exploitation, the optimal mechanism requires richer discoveries to yield a reserve, induces less exploration, and results in lower capacity and extraction. We also argue that often applied royalties and resource rent taxes distort exploration and extraction even when deductions for exploration expenses are allowed.

Keywords: discontinuous adverse selection; exploration; hidden effort; information acquisition; limited liability; nonrenewable resources; reserves; resource tax.

JEL codes: D82; D86; H21; Q30.

"*A mine is a hole in the ground owned by a liar.*" – Mark Twain[1]

1 Introduction

Both economic growth and actions to mitigate climate change, including renewable energy investments and electrification of transport, require large quantities of raw materials from nonrenewable sources. Current reserve levels of many critical metals such as lithium, cobalt, copper and rare earth metals are only able to supply a fraction of the growing demand, which is spurring exploration for new resource deposits. At the same time, the resource rights owners often wish to obtain revenues from the resource extraction on their turf. This desire comes with fundamental incentive problems because the exploration effort is often unobservable to the owner and the results of such actions, i.e., discovered deposits and their sizes, remain private information. How then should the owner design resource pricing mechanisms that it offers to a resource exploration and extraction firm, when the firm takes unobservable exploration actions, and – to paraphrase Mark Twain's words with some benefit of the doubt – may have incentives to lie about the discovered stock?

*University of Helsinki; jarmo.jaaskelainen@helsinki.fi.

†University of Helsinki; pauli.lappi@helsinki.fi. Corresponding author.

[1] The quote is often attributed to Mark Twain, e.g., see Ernest Scheyder (2024), The War Below.

Acknowledgements: We are thankful to Lassi Ahlvik and the participants at EAERE 2025 and NAERE 2025 for useful comments. This paper has received funding from Research Council of Finland, grant number 348718.

In this paper we design an optimal contract offered by the resource rights owner to the firm under two information asymmetries involving unobservable actions by the firm to gather private information over the resource stock. First, our main focus is on private information over the resource stock, but contrary to Osmundsen (1998) and Martimort *et al.* (2018), we explicitly model exploration incentives and the discovery process that generates the stock, as this aspect of nonrenewable resource exploitation is currently a pressing one.[2] Second, in our model the exploration effort by the firm is unobservable to the owner. Specifically, the resource firm is first offered a contract consisting of extraction capacity and a license fee, and after accepting the contract it explores the area for the resource and possibly discovers or learns the size of the stock in secret. Finally, the firm builds capacity and extracts the resource based on the contract and pays the fee. We jointly model exploration (information acquisition), private information over the resource stock and costly capacity build-up with set-up costs (discontinuous adverse selection), and analyze which discoveries yield reserves and which do not.

We develop a two-step procedure to solve the model. At Step 1, we focus on a model without the set-up costs, which is an information acquisition model with unobservable effort and private information. Here we apply a first-order approach, where we replace the incentive compatibility constraint with its derivative and check ex-post that the contract is incentive compatible. At Step 2, we use the optimal contract from the first step to construct the solution to the discontinuous adverse selection problem with the set-up costs. The optimal contract is obtained as a restriction of the contract from Step 1.

Our main results show how optimal pricing of nonrenewable resources under these information problems affects the discovery of new reserves (extensive margin) and the extraction of discovered reserves (intensive margin). First, the optimal mechanism requires the discovered resource to be greater than without such a mechanism to yield a new reserve for extraction. Second, the exploration effort is lower than without the mechanism. These two results mean that, under the mechanism, fewer discoveries are made and the discoveries qualify as reserves less often than when the resource is not priced. Third, when the discovery does yield a reserve, we have the typical screening result that the optimal mechanism induces the firm to build a smaller extraction capacity than without the licensing mechanism in order to reduce information rents left to the firm. We also argue that the unobservability of the effort brings no extra rent for the firm.

The previous literature on nonrenewable resources and asymmetric information consists mainly of models, where the private information is over initial resource stock (Osmundsen, 1998; Martimort *et al.*, 2018), extraction cost (Gaudet *et al.*, 1995; Osmundsen, 1995; Hung *et al.*, 2006; Ing, 2020), both extraction cost and exploration cost (Castonguay and Lasserre, 2016) or extraction site reclamation cost (Lappi, 2020). Even though private information over costs is important for contract design, we focus only on the initial resource stock as the source of the information problem. The firm acquires this private information through its exploration efforts. In our model, as in practice, the resource owner need not to be able to observe the exploration effort that the firm exerts. Hence the framework also contains an unobservable action by the

[2] There exists a considerable literature on the exploration and extraction of nonrenewable resources (e.g. Arrow and Chang, 1982; Quyen, 1988, 1991; Cairns, 1990), but with little weight on asymmetric information.

agent (the firm), and has both hidden action and private information. Such models are often challenging, but have been analyzed in various contexts by, e.g., Laffont and Tirole (1986), Baron and Besanko (1987), Roger (2013), Gottlieb and Moreira (2022) and Castro-Pires *et al.* (2024). The only such nonrenewable resource model that we know of is found in Herrnstadt *et al.* (2024), who analyze a revenue maximizing contract, when well productivity is private information and the effort to boost production (water use) is unobservable. Here the exploration effort is unobservable, and it is exerted before the firm can exploit its information advantage over the resulting initial stock.

Many papers have analyzed information acquisition and contracts, but not in the resource economics context even though exploration is information acquisition. Crémer *et al.* (1998) apply the Baron-Myerson model to costly information acquisition by the agent of its production cost. The agent may acquire information before it accepts the contract at a constant cost. Effort is either exerted or not, but in our model the exploration effort is continuous and the firm may acquire information about the stock after accepting the contract: high exploration unit cost implies no exploration, but low value that exploration is conducted and the resource stock may be discovered. In our model the acquisition is more extreme as compared to e.g. Szalay (2009) as the stock is either discovered or not. This and our assumption that exploration effort and the size of the discovery are independent considerably simplify the analysis, but allow us to connect information asymmetries with both the extensive and intensive margins even though the agent's payoff is not linear in type as in e.g. Szalay (2009). In addition, these mean that the owner is able to obtain the same expected revenue whether exploration is observable or not. Muratov (2023) models the bargaining between an entrepreneur and a (venture capital) investor in a setting, where the entrepreneur may generate valuable information of its product but also divert the funds that have been invested. In his model information generation is costless and the results are observable, while in ours exploration is costly and the results remain firm's secret.[3]

In addition to information acquisition, we allow the contract to be discontinuous in type (the resource stock). This is necessary due to the set-up costs, which make the resource rights owner's objective discontinuous. Therefore our model is a discontinuous adverse selection model, see e.g. Crémer *et al.* (1998), Hellwig (2010) and Martimort and Stole (2022). We argue that the optimal capacity jumps upwards at the reserve cut-off.

Finally, we also contribute to the discussion on neutral resource taxation. The standard recommendation to collect revenues using resource licensing fees or taxes is to apply rent taxes (Daniel *et al.*, 2010), which can be designed to collect maximal revenues without affecting the resource firm's decision to explore and extract.[4] There are multiple reasons why the actual practice deviates from this neutral tax recommendation in favor of fees and taxes that distort decisions, and it is well-known that private information is a major

[3]In addition to contracts, information acquisition has been analyzed in many other contexts, including recently in bargaining (Chatterjee *et al.*, 2025), product differentiation (Biglaiser *et al.*, 2025) and delegation (Ball and Gao, 2024; Krähmer and Kováč, 2016).

[4]Neutral, or non-distorting taxation, has been studied by many (e.g. Boadway and Bruce (1984); Fane (1987); Bond and Devereux (1995)). Recent theoretical literature on resource taxation has focused on explaining tax cycles (Jaakkola *et al.*, 2019) and on mechanism design and private information (e.g. Martimort *et al.*, 2018; Ing, 2020).

obstacle for neutral tax design. We highlight this by showing that both royalties and rent taxes always distort firms' decisions on how to explore and extract.[5]

In our model the firm obtains private information after exploration and the feasibility constraints that the contract must satisfy include an incentive compatibility constraint and a limited liability constraint. The limited liability constraint states that the extraction profit net of the license fee must be non-negative and this effectively constraints the firm's maximal losses to the exploration expenses when no discovery is made. Importantly, without any pricing the maximal losses equal the exploration expenses too, and in practise the pricing is often targeted at the resource profit obtained from extraction.[6] Apart from information asymmetry, our model focuses on the uncertainty over discoveries and the generation of new reserves as exploration activities are in practice very uncertain and quite seldom produce resource discoveries of sufficient sizes for extraction. Therefore our model includes – as any model on exploration and extraction should – endogenous reserves and allows for the possibility that a discovery may not yield a reserve.

2 Model

2.1 Exploration and extraction

Suppose the resource owner (e.g. the land owner or the government) has resource rights over some land area but does not possess the exploration and extraction technology. To benefit from the resource, it offers a contract to a risk-neutral firm, and this contract gives the firm the right to explore the area for resources and to extract the discovery in exchange for a license fee or a resource tax. The problem for the owner is that it cannot observe either the exploration effort or the resource stock discovered. Therefore it designs a mechanism consisting of a stock-dependent extraction capacity and a resource license fee (or tax) to maximize the expected revenue. This design must take into account that the firm should have incentives to report the true discovered resource stock and to build the extraction capacity after discovery. Figure 1 illustrates the timeline of the model, where the firm learns the stock size based on its exploration effort such as exploratory drilling.[7]

To analyze the firm's incentives to explore and lie about the discovery, we use a similar exploration and discovery model as Quyen (1988) and Quyen (1991), and define the expected exploration profit, when the firm exerts exploration effort y, as

$$D(y)\,\mathbb{E}[\pi(q;x) - F\mathbb{1}_{\{x:q(x)>0\}}] - zy. \tag{1}$$

Here the discovery probability satisfies properties $D'(y) > 0$, $D''(y) < 0$, $D(0) = 0$ and $D(y) \in (0, 1)$ for all

[5]Both royalties and rent taxes are used in practice, but in various forms. For example, in Western Australia the royalties are based either on the quantity produced or on the production value (Department of Mines, Petroleum and Exploration. Western Australia, 2025), and in Canada the taxes are on profit (Natural Resources Canada, 2024). Deductions are often allowed, e.g. in Canada the exploration expenses are deductible.

[6]A lump-sum payment collected before exploration may result in greater losses, if nothing is discovered: losses equal the payment and the exploration expenses.

[7]Note that the owner makes the contract dependent on the discovered stock that is unknown to both parties when the contract is signed. This differs from the standard adverse selection setting as the firm does not know its type when signing the contract, but learns it later based on its actions, as in the information acquisition literature.

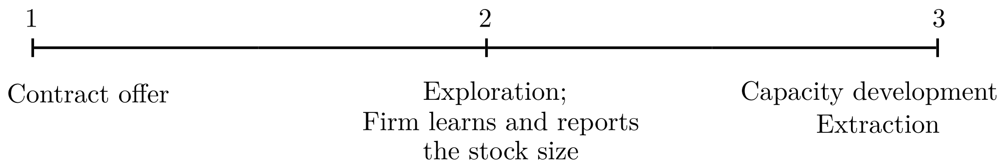


Figure 1: The time-line of the model. The firm is offered a contract consisting of a stock-dependent extraction capacity and a fee at stage 1, and after that, at stage 2, it chooses exploration effort. If it makes a discovery, it reports the discovered resource stock, and at stage 3 builds the extraction capacity and extracts the resource.

$y > 0$. This probability is assumed to be independent of the resource stock size in the ground. The quantity $\mathbb{E}\,\pi(q;x)$, where the expectation is taken over the unknown resource stock x, is the expected resource profit from capacity development q and extraction at this capacity, $F \geq 0$ is the fixed set-up cost related to capacity development (paid only if the capacity is positive), and the parameter z is the exploration unit cost. The continuous density of the unknown x is $g > 0$ and it is defined on $[0, \bar{x}]$ (with the cumulative distribution G), where $\bar{x}$ is some finite upper limit for the stock in this area. The density g is public knowledge, i.e., the owner and the firm know it, and it is based on the known geological properties of the area (and possibly on prior commonly known exploration results).

The resource profit π related to development and extraction is assumed to have a unique maximum with respect to capacity $q \geq 0$, and $\pi_{qq} < 0$ for all q, $\pi(0;x) = 0$ and $\pi(q;0) = 0$. Hence the (strictly concave) profit is zero, when the extraction capacity is zero or when the resource stock is zero. The resource is extracted by assumption at the maximal rate using the capacity developed.[8] Furthermore, we make the following assumption on how the resource profit depends on the discovered resource stock:

Assumption 1. *The resource profit from capacity development and extraction, $\pi(q;x)$, has the following properties (in addition to a unique maximum with respect to $q \geq 0$, $\pi_{qq} < 0$, $\pi(0;x) = 0$ and $\pi(q;0) = 0$):*

$$\pi_x > 0, \quad \pi_{qx} > 0, \quad \pi_{qqx} \geq 0, \quad \pi_{qxx} \leq 0 \quad \textit{for all} \;\; q > 0 \; \textit{and} \; x \in [0, \bar{x}].$$

Both the profit and the marginal profit are strictly increasing in the stock.[9]

2.2 Feasible contracts

When no mechanism is applied to collect revenue, the firm is free to claim bankruptcy for example when no discovery was made and limit its losses to exploration expenses. We impose the same structure for the maximal losses and focus on contracts, where the license fee T is collected from the resource profit π as is often the case in practice.

[8] See Appendix A.1 for an example of an extraction model with optimal extraction at the capacity. In capacity constrained nonrenewable resource models it is common that the extraction is at the capacity at least for some initial time interval (see e.g. Campbell (1980) and Cairns (2001)).

[9] Properties in this assumption hold at least in a simple capacity constrained nonrenewable resource model (see Appendix A.1), where the given stock is extracted at the maximal installed capacity, except possibly for the assumptions related to π_{qqx} and π_{qxx}. These partials have no clear economic interpretations, but are standard (Fudenberg and Tirole, 1991).

Remark 1. Note that any lump-sum payment collected before exploration would imply greater losses in case of a small discovery. Indeed, building capacity when the discovery is small implies by Equation (1) that the resource profit does not cover the set-up cost. Therefore the firm has no incentives to build capacity for a small find and the losses equal the exploration expenses plus the lump-sum payment.[10]

We define the contract as a pair of extraction capacity and license fee (or tax) for each discovered resource stock, namely as a pair (q, T), where q and T are functions defined on $[0, \bar{x}]$.

A contract is feasible if it satisfies the three constraints described next. For the first constraint, note that the firm can always claim bankruptcy after the exploration and discovery and obtain zero resource profit by doing so (the loss $-zy$ being sunk at this point). Hence the contract must satisfy the following limited liability type constraint:

$$\pi(q(x); x) - T(x) \geq F\mathbb{1}_{\{x:q(x)>0\}} \quad \text{for all} \quad x \in [0, \bar{x}]. \tag{LL}$$

In other words, the contract that the firm has accepted must give it non-negative profit after the discovery has occurred and limit the firm's maximal losses to exploration expenses.

Second, the discovery reveals the size of the stock only to the firm and not to the owner and therefore the contract must provide incentives for reporting the true discovery. This is captured by the following incentive compatibility constraint:

$$\pi(q(x); x) - F\mathbb{1}_{\{x:q(x)>0\}} - T(x) \geq \pi(q(\hat{x}); x) - F\mathbb{1}_{\{\hat{x}:q(\hat{x})>0\}} - T(\hat{x}) \quad \text{for all} \quad (x, \hat{x}) \in [0, \bar{x}]^2. \tag{IC}$$

The final constraint is that the unobservable exploration effort chosen by the firm maximizes the expected exploration profit, that is,

$$y = \arg\max_{\{\hat{y}\}} \{D(\hat{y})\, \mathbb{E}[\pi(q(x); x) - T(x) - F\mathbb{1}_{\{x:q(x)>0\}}] - z\hat{y}\}. \tag{EC}$$

A contract is said to be feasible if it satisfies constraints (IC), (LL), and (EC).

3 Results

3.1 Symmetric information without pricing

To characterize the optimal exploration effort and the optimal capacity without pricing, we define the reserve cut-off as the largest discovery for which the optimal capacity is zero and denote it with r^{F} for $F > 0$. Given the stock x, we denote the optimal capacity that solves problem $\max_{q \geq 0}\{\pi(q; x) - F\mathbb{1}_{\{x:q(x)>0\}}\}$ by $q^{\mathrm{F}}(x)$. Also, we let y^{F} be the optimal exploration effort for $F > 0$, i.e., the effort $y \geq 0$ that maximizes (1). The corresponding notation for the case $F = 0$ is r^0, q^0 and y^0. The optimal effort and capacity are characterized next.

[10] In addition, in practice the resource fees or taxes are often collected from the resource profit instead of collecting them before exploration through the selling of the rights.

Proposition 1. *Suppose that the resource extraction is not priced.*

1. *When the set-up costs are zero ($F = 0$), the optimal extraction capacity is given by*

$$q^0(x) = \begin{cases} 0, & x \le r^0, \\ \text{the solution to } \pi_q(q; x) = 0, & x > r^0, \end{cases}$$

where r^0 is the reserve cut-off. The capacity q^0 is continuous, piecewise C^1 and increasing.

The optimal exploration effort is given by

$$y^0 = \begin{cases} 0, & D'(0)\,\mathbb{E}\,\pi(q^0(x), x) \le z, \\ \text{the solution to } D'(y)\,\mathbb{E}\,\pi(q^0(x), x) - z = 0, & D'(0)\,\mathbb{E}\,\pi(q^0(x), x) > z. \end{cases} \tag{2}$$

2. *When the set-up costs are positive ($F > 0$), the optimal extraction capacity is given by*

$$q^{\mathrm{F}}(x) = \begin{cases} 0, & x \le r^{\mathrm{F}}, \\ q^0(x), & x > r^{\mathrm{F}}, \end{cases}$$

where the reserve cut-off $r^{\mathrm{F}} > r^0$ is the solution to $\pi(q^0(x); x) = F$, if it is smaller than $\bar{x}$. The capacity q^{F} is piecewise C^1, jumps upwards at r^{F} and is increasing.

The optimal exploration effort is given by

$$y^{\mathrm{F}} = \begin{cases} 0, & D'(0)\,\mathbb{E}\,\Pi^{\mathrm{F}} \le z, \\ \text{the solution to } D'(y)\,\mathbb{E}\,\Pi^{\mathrm{F}} - z = 0, & D'(0)\,\mathbb{E}\,\Pi^{\mathrm{F}} > z, \end{cases} \tag{3}$$

where $\Pi^{\mathrm{F}} := \pi(q^{\mathrm{F}}(x), x) - F\mathbb{1}_{\{x: q^{\mathrm{F}}(x) > 0\}}$.

Proof. Appendix A.2. □

The discoverable resource stock sizes are therefore divided into two parts. In the first, the stock is so small that the firm has no incentive to build the capacity at all. But there always exists the second part, where the capacity is built and the marginal resource profit equals zero as long as the set-up cost is not too large. When the set-up costs are zero, the optimal capacity is continuous in the stock. With positive set-up costs, the firm must be able to break even and therefore the optimal capacity jumps upwards at the reserve cut-off. The optimal capacity with the set-up costs equals the the optimal capacity without these costs, when it is positive. Intuitively, the reserve cut-off is larger, when the set-up cost is higher.

For the optimal effort, the firm exerts exploration effort and equalizes the expected marginal exploration revenue with the unit exploration cost, when the expected resource profit from the optimal capacity is large enough, namely, when the marginal increase in the expected exploration revenue at zero effort is greater than the unit cost of exploration. It is worth noting that the firm may make a loss $-zy^{\mathrm{F}}$ if it does not discover a deposit rich enough to build any capacity.

3.2 Asymmetric information: non-neutrality of royalties and resource rent taxes

It is well-known, see, e.g., Boadway and Keen (2015) and Gaudet and Lasserre (2015), that under complete information a government can design a resource tax on resource rents such that the choices of the firm are not distorted from the allocation presented in Proposition 1. Interestingly, the standard tools in resource taxation such as royalties based on the extraction or the extraction revenue and the resource rent tax with or without deductions for exploration expenses cannot be designed to deliver the allocation $(q^{\mathrm{F}}(x), y^{\mathrm{F}})$, i.e., the capacity and the effort without taxation. That is, royalties and resource rent taxes are not neutral under asymmetric information, which is a result well-known from the previous literature without exploration (e.g. Osmundsen (1998); Martimort *et al.* (2018)).

Royalties and rent taxes remain non-neutral also with exploration as shown next. For that we define a resource rent tax as any fee collected as a fraction from the resource profit that may depend on the stock size and on the exploration effort. Under the resource rent tax, the firm's after-tax profit is $[\pi(q(x); x) - F\mathbb{1}_{\{x:q(x)>0\}}][1 - T(x, y)]$. A royalty is defined as any fee collected in an additive-manner from the resource profit that may depend on the stock size and on the exploration effort. With the royalty, the firm's after-tax profit is given by $\pi(q(x); x) - F\mathbb{1}_{\{x:q(x)>0\}} - T(x, y)$. Thus in both cases, the instrument may depend on the stock size and the exploration effort, which means for example that deductions for exploration expenses are allowed.[11]

Proposition 2. *Assume that the tax function T is piecewise C^1. Then the allocations $(q^0(x), y^0)$ (for $F = 0$) and $(q^{\mathrm{F}}(x), y^{\mathrm{F}})$ (for $F > 0$) are not feasible, when*

1. *T is a royalty, or*
2. *T is a resource rent tax.*

Proof. See Appendix A.3. □

In other words, both of these schemes are non-neutral: no matter how royalties and rent taxes are designed, they will not deliver the same allocation as without them. The tax design, i.e., the form of the tax scheme, can be as complicated as wanted and it can be made contingent on any observable (prices, extraction cost, capacity, etc.) and unobservable characteristics (stock size and exploration effort). Any such design attempt fails, because the scheme cannot induce the correct allocation without breaking at least one of the feasibility constraints.

[11] For example, exploration expenses are deductible in Canada (Natural Resources Canada, 2024). In our model this means that the after-tax profit could take the form $\pi(q(x); x) - F\mathbb{1}_{\{x:q(x)>0\}} - [\pi(q(x); x) - F\mathbb{1}_{\{x:q(x)>0\}} - zy]\alpha$, where α is the tax rate.

3.3 Asymmetric information: optimal licensing

Therefore to design a feasible contract one must accept distortions in the allocation and leave some rent to the firm.[12] A feasible contract that maximizes the expected revenues of the resource owner is obtained as the solution to

$$\max_{\{q(x),T(x)\}} \int_0^{\bar{x}} D(y)T(x)g(x)\ \mathrm{d}x \quad \text{subject to (IC), (LL), and (EC).} \tag{F}$$

Note that there is no need for an ex-ante participation constraint, because the firm can guarantee itself zero payoff by not exerting any exploration effort. What is also notable is that the firm may make a loss after it has accepted the contract, if the resource profit is not sufficient to cover the exploration expenses that have already been paid.

We analyze the discontinuous adverse selection model in Problem (F) using the following two steps:

1. We study a problem without the set-up cost F, i.e. a model, where we set $F = 0$:
   1.1 in that problem, we first replace the incentive compatibility constraint with its necessary condition, and
   1.2 show then that the solution to this problem is a solution to problem with $F = 0$;
2. We use the solution from step 1 to construct the solution in the discontinuous adverse selection model (F).

Step 1

This step involves a relatively standard problem except that the contract is not a priori restricted to be continuous. The problem without set-up costs is

$$\begin{aligned} &\max_{\{q(x),T(x)\}} \int_0^{\bar{x}} D(y)T(x)g(x)\ \mathrm{d}x \\ &\text{subject to } \pi(q(x);x) - T(x) \geq \pi(q(\hat{x});x) - T(\hat{x}), \\ &\qquad \pi(q(x);x) - T(x) \geq 0, \\ &\qquad y = \arg\max_{\{\hat{y}\}}\{D(\hat{y})\,\mathbb{E}[\pi(q(x);x) - T(x)] - z\hat{y}\}. \end{aligned} \tag{P}$$

The rent left to the firm is defined here as $U(x) := \pi(q(x);x) - T(x)$. We first characterize the solution to a simplified problem, where we replace the incentive compatibility constraint of Problem (P) with its necessary condition. The simplified problem is

$$\begin{aligned} &\max_{\{q(x)\}} \int_0^{\bar{x}} D(y)[\pi(q(x);x) - U(x)]g(x)\ \mathrm{d}x \\ &\text{subject to} \quad U'(x) = \pi_x(q(x);x), \quad U(x) \geq 0, \\ &\qquad y = \arg\max_{\{\hat{y}\}}\{D(\hat{y})\,\mathbb{E}\,U - z\hat{y}\}, \end{aligned} \tag{S}$$

[12] We assume that the owner only wishes to obtain resource revenues and gives zero weight to other objectives including externalities, consumer surplus and alternative land-use options.

where we substituted $\pi(q(x);x) - U(x)$ for $T(x)$ in the objective of Problem (P) and used the rent equation in the limited liability constraint and in the exploration effort maximization constraint. The equation $U'(x) = \pi_x(q(x);x)$ holds at almost every point. We show afterwards that a solution to Problem (S) is a solution to Problem (P). In particular, we check that the obtained candidate solution is incentive compatible (Proposition 6).

Similarly to the case without a resource license fee (or tax), we denote the optimal capacity with the license fee by q_{t}^0 and define the reserve cut-off with the license fee as the largest discovery for which the capacity is zero and denote it with r_{t}^0.

We make the following standard assumption:

Assumption 2. *The density of the unknown resource stock satisfies the inequality* $\mathrm{d}/\mathrm{d}x[(1-G(x))/g(x)] \leq 0$ *for all* $x \in [r_{\mathrm{t}}^0, \bar{x}]$.

That is, the typical monotone hazard rate assumption holds for all resource stock sizes larger than the reserve cut-off r_{t}^0. Hence for "small" resource stocks, the monotone hazard rate assumption may not hold. This reflects the idea that most areas contain small amounts of resources, which means that the stock sizes may be sufficiently packed around zero to break the monotone hazard rate assumption for low resource stock sizes. However, the density is sufficiently regular for larger stocks.

We derive the optimal capacity and the optimal license fee related to Problem (S).

Proposition 3. *Consider Problem (S). The optimal extraction capacity under unobservable exploration effort and hidden discovery is continuous, piecewise C^1 and increasing. It is given by*

$$q_{\mathrm{t}}^0(x) = \begin{cases} 0, & x \leq r_{\mathrm{t}}^0, \\ \text{the solution to } \pi_q(q;x) - \frac{1-G(x)}{g(x)}\pi_{qx}(q;x) = 0, & x > r_{\mathrm{t}}^0, \end{cases} \tag{4}$$

when $r_{\mathrm{t}}^0 < \bar{x}$.

Proof. See Appendix A.4. □

This proposition says that, compared to the case without license fee, the optimal interior capacity is distorted downwards for each resource stock except for the largest realization $\bar{x}$. This distortion essentially makes it less lucrative for the firm to claim that the stock size is lower than it really is. This capacity is the first part of the contract.

Proposition 4. *Consider Problem (S). The optimal license fee is continuous, piecewise C^1 and increasing in the resource discovery. It is given by the formula*

$$T^0(x) = \begin{cases} 0, & x \leq r_{\mathrm{t}}^0, \\ \pi(q_{\mathrm{t}}^0(x);x) - \int_{r_{\mathrm{t}}^0}^{x} \pi_x(q_{\mathrm{t}}^0(s);s)\ \mathrm{d}s, & x > r_{\mathrm{t}}^0. \end{cases} \tag{5}$$

Proof. See Appendix A.5. □

Hence overly poor discoveries are excluded from the license contract. But for sufficiently large discoveries the optimal fee takes away all resource profit from the cut-off reserve type and the fee is heavier for all types greater than the cut-off. Still, the profit left to the firm is positive for rich deposits, which is used to incentivize truthful revelation of discoveries.

Before completing the analysis of Problems (S) and (P) by showing that the obtained contract $(q_{\mathrm{t}}^0(x), T^0(x))$ is incentive compatible, we analyze the optimal exploration effort that the firm exerts given the contract. While the optimal exploration effort is independent of the realization of the resource stock, it is not independent of the contract, because the contract affects the marginal expected revenue from effort, which is $D'(y)\,\mathbb{E}[\pi(q_{\mathrm{t}}^0(x); x) - T^0(x)]$.

Proposition 5. *Consider Problem (S). The optimal exploration effort under unobservable exploration effort and hidden discovery is given by*

$$y_{\mathrm{t}}^0 = \begin{cases} 0, & D'(0)\,\mathbb{E}\,U \le z, \\ \text{the solution to } D'(y)\,\mathbb{E}\,U - z = 0, & D'(0)\,\mathbb{E}\,U > z, \end{cases} \tag{6}$$

where the expected rent left for the firm is

$$\mathbb{E}\,U = \int_{r_{\mathrm{t}}^0}^{\bar{x}} \pi_x(q_{\mathrm{t}}^0(s); s)(1 - G(s))\ \mathrm{d}s.$$

Proof. The proof is similar to the proof of Proposition 1 on the optimal effort and is omitted. For the rent, note that we have $U(x) = 0$ for all $x \le r_{\mathrm{t}}^0$. The expectation of the rent becomes

$$\mathbb{E}\,U = \int_{r_{\mathrm{t}}^0}^{\bar{x}} U(x)g(x)\ \mathrm{d}x = U(\bar{x}) - \int_{r_{\mathrm{t}}^0}^{\bar{x}} U'(x)G(x)\ \mathrm{d}x = \int_{r_{\mathrm{t}}^0}^{\bar{x}} \pi_x(q_{\mathrm{t}}^0(s); s)(1 - G(s))\ \mathrm{d}s,$$

where integration by parts was used in the second equality. □

The equation characterizing the optimal interior effort matches the expected marginal revenue from effort with its unit cost, but as in the case without license fee the optimal effort may be zero. This occurs when the expected rent left to the firm is too small compared to the marginal discovery probability at zero effort and unit exploration cost.

Finally, we check that the contract $(q_{\mathrm{t}}^0(x), T^0(x))$ given by Propositions 3 and 4 is incentive compatible (in the model with $F = 0$), which means that this contract is the optimal one for Problem (P). The reason for doing this is that in the propositions we replaced the incentive compatibility constraint with its necessary condition.

Proposition 6. *The contract $(q_{\mathrm{t}}^0(x), T^0(x))$ is incentive compatible and a solution to Problem (P).*

Proof. The proof is standard and omitted (see e.g. Fudenberg and Tirole (1991)), because we have shown in Proposition 3 and in Proposition 4 that the contract is continuous for every x and because $q_{\mathrm{t}}^{0\prime}(x) \ge 0$ for all points of differentiability. □

Step 2

In the previous proofs we argued that the optimal contract is continuous when the set-up costs are zero. With the set-up cost, it can be expected that the optimal contract lacks this property. To obtain the solution to the model in Problem (F), we use the optimal contract with zero set-up costs from step 1 to construct the solution. The inclusion of positive set-up costs implies that the optimal capacity jumps upwards at the reserve cut-off, which is larger than r_t^0, as shown next. We denote the reserve cut-off with set-up costs by r_t^F and the rent by $\hat{U}(x) := \pi(q(x); x) - T(x) - F\mathbb{1}_{\{x:q(x)>0\}} \geq 0$.

Proposition 7. *Consider Problem (F), where the development of extraction capacity has a set-up cost $F > 0$, which is borne only when capacity is positive. Then,*

1. *the optimal capacity is given by*
$$q_\mathrm{t}^\mathrm{F}(x) = \begin{cases} 0, & x \leq r_\mathrm{t}^\mathrm{F}, \\ q_\mathrm{t}^0(x), & x > r_\mathrm{t}^\mathrm{F}, \end{cases} \tag{7}$$
where the reserve cut-off $r_\mathrm{t}^\mathrm{F} > r_\mathrm{t}^0$ is the solution to
$$\pi(q_\mathrm{t}^0(x); x) - T^0(x) = F,$$
if it is smaller than $\bar{x}$. The optimal capacity is increasing and jumps upwards at r_t^F and is C^1 elsewhere. Otherwise there is no reserve cut-off and $q_\mathrm{t}^\mathrm{F} \equiv 0$,

2. *the optimal license fee jumps upwards at r_t^F and is C^1 elsewhere and increasing. The fee is given by*
$$T^\mathrm{F}(x) = \begin{cases} 0, & x \leq r_\mathrm{t}^\mathrm{F}, \\ \pi(q_\mathrm{t}^0(x); x) - F - \int_{r_\mathrm{t}^\mathrm{F}}^x \pi_x(q_\mathrm{t}^0(s); s)\ \mathrm{d}s, & x > r_\mathrm{t}^\mathrm{F}. \end{cases} \tag{8}$$
If there is no cut-off, then $T^\mathrm{F} \equiv 0$, and

3. *the optimal exploration effort is given by*
$$y_\mathrm{t}^\mathrm{F} = \begin{cases} 0, & D'(0)\,\mathbb{E}\,\hat{U} \leq z, \\ \text{the solution to } D'(y)\,\mathbb{E}\,\hat{U} - z = 0, & D'(0)\,\mathbb{E}\,\hat{U} > z, \end{cases}$$
where the expected rent left to the firm is
$$\mathbb{E}\,\hat{U} = \int_{r_\mathrm{t}^\mathrm{F}}^{\bar{x}} \pi_x(q_\mathrm{t}^0(s); s)(1 - G(s))\ \mathrm{d}s.$$

Proof. See Appendix A.6 □

With positive set-up costs, the optimal reserve cut-off level increases and therefore fewer discoveries result in new reserves. The contract includes a capacity that jumps upwards at the optimal cut-off level, which enables the firm to obtain resource profit to cover the set-up costs. The jump is from zero capacity to the capacity level from the model without these costs. License fee is collected only from those discoveries that are greater than the cut-off, and it equals the fee T^0 deducted by the set-up costs.[13]

[13] The same allocation can be achieved with an indirect fee. Let x^F denote the inverse of q_t^F, when $x \geq r_\mathrm{t}^\mathrm{F}$. One can verify

3.4 Comparisons

The optimal contract is given by (4) and (5) (if $F = 0$) or by (7) and (8) (if $F > 0$). These contracts have apparent differences to the case without resource pricing and affect how much exploration effort is exerted, which discoveries become reserves, and how large a capacity is installed. These comparisons are presented in the following result:

Proposition 8. *When the exploration effort is unobservable and the discovery is hidden,*

1. *the reserve cut-off is larger with resource pricing than without pricing, that is,*

$$r_{\mathrm{t}}^0 > r^0 \quad \text{for} \;\; F = 0 \quad \text{and} \quad r_{\mathrm{t}}^{\mathrm{F}} > r^{\mathrm{F}} \quad \text{for} \;\; F > 0,$$

2. *the exploration effort is smaller with resource pricing than without pricing, that is,*

$$y_{\mathrm{t}}^0 < y^0 \quad \text{for} \;\; F = 0 \quad \text{and} \quad y_{\mathrm{t}}^{\mathrm{F}} < y^{\mathrm{F}} \quad \text{for} \;\; F > 0.$$

Proof. See Appendix A.7. □

This result means that resource discoveries for capacity buildup must be richer with resource pricing than without it, when there is information asymmetry between the owner and the firm. Combining this with Propositions 1 and 3 shows that the optimal capacity is larger without pricing than with it, except when capacity is not built even without pricing or for the largest discovery $\bar{x}$ (as the capacities are equal at $\bar{x}$, i.e., there is no distortion at the top).

Figure 2 illustrates these optimal capacities and also the license fees and firms' profits, when the resource is not priced and when it is. Notably, the set-up costs require greater discoveries to yield a reserve and lower the fee level by the costs at each positive reserve level (see the middle panel). The fee jumps upwards at the reserve cut-off $r_{\mathrm{t}}^{\mathrm{F}}$ and there are discoveries larger than r_{t}^0, which do not qualify as reserves with set-up costs causing a further loss to the (expected) resource revenues that can be collected. The panel on the right illustrates how the resource profit left to the firm decreases as the resource is priced. The optimal exploration effort is smaller with the set-up costs compared to the effort without them, because the expected rent is lower.

that the indirect fee $P(q) := \pi(q; x(q)) - \int_{r_{\mathrm{t}}^{\mathrm{F}}}^{x(q)} \pi_x(q_{\mathrm{t}}^{\mathrm{F}}(s); s)\, \mathrm{d}s$, where

$$x(q) = \begin{cases} r_{\mathrm{t}}^{\mathrm{F}}, & q = 0, \\ x^{\mathrm{F}}(q) & q > 0, \end{cases}$$

incentivizes the firm to choose $q_{\mathrm{t}}^{\mathrm{F}}$ for any discovery x.

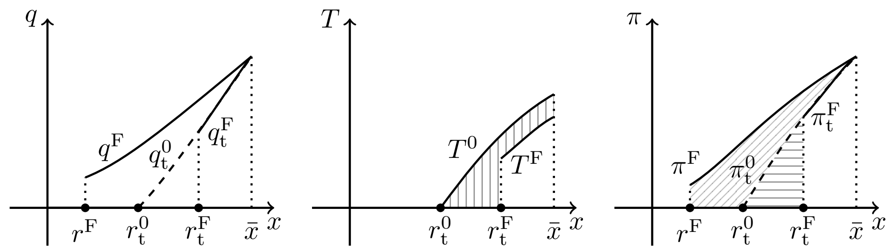


Figure 2: Illustration of optimal capacities, license fees and firms' profits. The figure on the left shows the optimal capacities without pricing and with set-up costs $q^{\rm F}$, with pricing and no set-up cost $q_{\rm t}^0$ and with pricing and set-up costs $q_{\rm t}^{\rm F}$. They are zero until the reserve cut-offs $r^{\rm F}$, $r_{\rm t}^0$ and $r_{\rm t}^{\rm F}$, respectively, and positive afterwards. The optimal license fees are shown in the middle for $F = 0$ and $F > 0$. The shaded area is the resource license revenue that is not collected (without weighting the revenues with the density g), when there are set-up costs. The figure on the right shows the respective resource profits using similar notation as for the capacities and cut-offs. The shaded areas are the (unweighted) lost resource profits not collected by anyone due to pricing.

4 Conclusions

We argue that there is little hope in designing neutral resource taxation using royalties and rent taxes, when the resource firm exerts hidden exploration effort to discover the resource and obtains by doing so private information over the stock size. Any attempt to use these tools will distort extraction and exploration choices of the firm. To counter the information advantage of the firm, the owner needs to leave information rents to it and also accept distortions in extraction, exploration and reserve discovery.

The optimal contract model shows that the endogeneity of reserves, asymmetric information, and payment for resource exploitation have two channels through which a smaller amount of new reserves are born. First, licensing of resource exploitation drives a wedge between the profit without licensing fee and with fee, and this makes building the extraction capacity less profitable. Thus richer deposits are required to build capacity with the licensing mechanism. Second, the mechanism also cuts firm's expected profits, which results in smaller incentives to exert exploration effort. This has the effect that fewer discoveries are made. These results thus show how the need to collect revenues from resource exploitation under asymmetric information over discoveries reduces both new reserves and extraction. The optimal mechanism developed in this paper effectively results in resource conservation.

There are multiple open questions for future research. For example, the model could be extended to allow the probability of discovery to depend on the amount of resource in the ground, which would require a different plan of attack to disentangle the effects of private information over the stock and the unobservability of the exploration effort. However, in our opinion the following two are the most interesting ones. First, how should one modify the mechanism when private information is multidimensional, e.g., when the cost is also private information? Second, what is the optimal mechanism to collect maximal revenues under the same information problems as in the current model when the owner has multiple land areas open for exploration and extraction?

A Appendix

A.1 Resource extraction model satisfying Assumption 1

Let x be the discovered resource deposit size and let the net resource price be a constant $p - c$. Define the discounted revenue from a resource deposit of size x with

$$R(q;x) := \begin{cases} 0, & q = 0, \\ \int_0^{x/q} (p-c) q e^{-\rho t} \, \mathrm{d}t, & q > 0, \end{cases}$$

where q is the extraction capacity (and the constant extraction rate), x/q is the operation interval and ρ is the interest rate. An example of an extraction model with constant optimal extraction rate that equals the capacity is

$$\max_{\{z(t),\mathcal{T}\}} \int_0^{\mathcal{T}} (p-c) z(t) e^{-\rho t} \, \mathrm{d}t$$
$$\text{s.t.} \quad z(t) \in [0,q], \quad \mathcal{T} \geq 0, \quad \dot{s}(t) = -z(t), \quad s(0) = x, \quad s(\mathcal{T}) \geq 0,$$

where z is the rate of extraction, $\mathcal{T}$ is the shut-down date and s is the remaining resource stock. Constant extraction rate $z(t) \equiv q$ and $\mathcal{T} = x/q$ solve this problem. The corresponding value function is R.

Building extraction capacity is costly. The capacity cost is given by $C(q;x)$, where C is assumed to satisfy $C(0;x) = 0$, $C_q > 0$, $C_{qq} \geq 0$, $C_x < 0$ and $C_{qx} < 0$ for all $q \geq 0$ and $x \in [0, \bar{x}]$. The resource profit π is defined as the difference between the discounted revenue and the capacity cost,

$$\pi(q;x) := R(q;x) - C(q;x).$$

The properties of π in Assumption 1 follow from the definition of R and the assumptions on C by direct calculation (except $\pi_{qqx} \geq 0$ and $\pi_{qxx} \leq 0$ for which one needs to assume more on C).

A.2 Proof of Proposition 1

Part 1: $F = 0$. We first show that $r^0 = \sup\{x : \arg\max_{q \geq 0} \pi(q;x) = 0\}$, and $q^0(x) = 0$ for all $x \leq r^0$ and $q^0(x) > 0$ for all $x > r^0$.

Denote $x^{\mathrm{m}} = \sup\{x : \arg\max_{q \geq 0} \pi(q;x) = 0\}$. By Assumption 1, given x, the unique maximum point q exists for $\pi(q;x)$ and

$$\arg\max_{q \geq 0} \pi(q;x) = \begin{cases} 0, & \\ q & \text{such that } \pi_q(q;x) = 0. \end{cases}$$

Now, $\pi_q(q, x^{\mathrm{m}}) \leq 0$ for all $q > 0$. For stocks $x < x^{\mathrm{m}}$, we have that $\pi_q(q,x) < 0$ (for all $q > 0$), since $\pi_{qx} > 0$. Thus $q^0(x) = 0$ for $x < x^{\mathrm{m}}$.

By the definition of x^{m}, $q^0(x) > 0$ for stocks $x > x^{\mathrm{m}}$.

We show next that q^0 is piecewise C^1. By the implicit function theorem, q^0 is C^1, when $x > r^0$ as a solution to $\pi_q(q;x) = 0$. For $x < r^0$, $q^0(x) = 0$. Thus q^0 is piecewise C^1. In addition, it is either constant or strictly increasing ($q^{0\prime} = -\pi_{qx}/\pi_{qq} > 0$ at each point of differentiability $x > r^0$).

Capacity q^0 is continuous by Berge's Maximum Theorem, because problem $\max_{q\geq 0} \pi(q;x)$ has a unique solution, π is continuous in q and x and $\pi_{qq} < 0$ for all $q > 0$.

For the optimal effort in Equation (2), note first that (1) with $F = 0$ is strictly concave, and has a unique maximum either at $y = 0$ or at $y > 0$. If $D'(0)\,\mathbb{E}[\pi(q^0(x), x)] - z \leq 0$, the expected exploration profit is strictly decreasing for all y, and $y^0 = 0$ is optimal. And if $D'(0)\,\mathbb{E}[\pi(q^0(x), x)] - z > 0$, the optimal effort is positive.

Part 2: $F > 0$. With positive set-up costs, the profit must be at least as great as this cost, and therefore the reserve cut-off r^{F} is the solution to $\pi(q(x); x) = F$, where q is a solution to $\pi_q = 0$ (or in other words $q = q^0$ of the above case $F = 0$). The optimal capacity jumps at r^{F} and is given by the first-order condition for $x > r^{\mathrm{F}}$. As q^0 is C^1 for $x > r^{\mathrm{F}}$ (see Part 1 above), q^{F} is piecewise C^1.

The optimal effort in Equation (3) is obtained using similar arguments as in Part 1 except that $F > 0$.

□

A.3 Proof of Proposition 2

We assume on the contrary that a neutral tax scheme can be designed (i.e., the scheme incentivizes the allocation $(q^{\mathrm{F}}(x), y^{\mathrm{F}})$ or allocation $(q^0(x), y^0)$). Let x and $\hat{x}$ be the true and the reported discovery, respectively. When the tax depends on the unobservable exploration effort, we denote the actual effort with y and the reported effort with $\hat{y}$.

Let $F > 0$. The proof for the other case with $F = 0$ is similar and is omitted.

1. (Royalty) Note first, that the tax scheme can have jumps in x only at r^{F}, because other jumps would imply that the scheme is not incentive compatible, i.e., would not satisfy

$$\pi(q(x); x) - F\mathbb{1}_{\{x:q(x)>0\}} - T(x, y) \geq \pi(q(\hat{x}); x) - F\mathbb{1}_{\{\hat{x}:q(\hat{x})>0\}} - T(\hat{x}, \hat{y}) \tag{A.1}$$

for all $(x, \hat{x}) \in [0, \bar{x}]^2,\ y > 0,\ \hat{y} > 0$, as a firm with stock close to the jump point other than r^{F} would have incentive to misreport.

Case a: T depends only on x. Suppose that T is not constant on $[0, \bar{x}]$. Then there either exists an $x \in (0, \bar{x})$ such that $T'(x) \neq 0$ or T is piecewise constant with a jump at r^{F}.

Consider the first case and let $x > r^{\mathrm{F}}$. Then the firm with resource stock x chooses the contract that gives it the largest after-tax profit, i.e., it solves $\max_{\hat{x}\in[0,\bar{x}]}\{\pi(q^{\mathrm{F}}(\hat{x}); x) - F\mathbb{1}_{\{\hat{x}:q^{\mathrm{F}}(\hat{x})>0\}} - T(\hat{x})\}$. The value $\hat{x} = x$, for which $T'(x) \neq 0$, does not maximize the firm's after-tax profit, because the derivative of the after-tax profit $\pi_q(q^{\mathrm{F}}(\hat{x}); x)q^{\mathrm{F}\prime}(\hat{x}) - T'(\hat{x})$ evaluated at x is $-T'(x) \neq 0$. This means that the royalty is not incentive compatible and (A.1) breaks.

Consider the second case and let T piecewise constant with a jump at r^{F}. If $T > 0$ for $x \leq r^{\mathrm{F}}$, then $q(x) = q^{\mathrm{F}}(x)$ implies that for $x = 0$ the left-side of the constraint (LL) becomes $\pi(0, 0) - T = -T < 0$, which breaks (LL). Thus $T = 0$ for $x \leq r^{\mathrm{F}}$ and $T > 0$ for $x > r^{\mathrm{F}}$. However, type $x = r^{\mathrm{F}} + \epsilon$, where $\epsilon > 0$ is small, obtains profit equal to $\pi(q^{\mathrm{F}}(r^{\mathrm{F}} + \epsilon); r^{\mathrm{F}} + \epsilon) - F > 0$, but $\pi(q^{\mathrm{F}}(r^{\mathrm{F}} + \epsilon); r^{\mathrm{F}} + \epsilon) - F - T < 0$ when ϵ is small enough (since π and q^{F} are continuous). Thus the constraint (LL) breaks.

Case b: T is constant. Suppose that the royalty is a constant, $T > 0$. Such a tax is incentive compatible, but constraint (LL) is again broken for $x = 0$ as above.

Case c: T depends on y. Suppose that the royalty depends on y. Note that royalty $T(x, y)$ with $T_x(x, y) \neq 0$ for some x breaks the constraint (A.1) as we showed above in Case a. Also, a piecewise constant tax with a jump at r^{F} breaks the constraint (LL) for each fixed y.

Suppose then that with our undistorted allocation (i.e., $q = q^{\mathrm{F}}$, $y = y^{\mathrm{F}}$) we have $T(y^{\mathrm{F}}) > 0$. Again, as in Case b, the constraint (LL), i.e., $\pi(q^{\mathrm{F}}(x); x) - F\mathbb{1}_{\{x:q^{\mathrm{F}}(x)>0\}} - T(y^{\mathrm{F}}) \geq 0$ for all $x \in [0, \bar{x}]$, breaks for $x = 0$: $\pi(q^{\mathrm{F}}(0); 0) - T(y^{\mathrm{F}}) = -T(y^{\mathrm{F}}) < 0$.

2. (Rent tax) This tax is paid as a fraction of the resource profit meaning that $T \in (0, 1]$ (zero is excluded as it does not collect any revenue). Note that as firm's true resource stock is not observed, the rent tax is based on the profit evaluated at the reported resource stock $\hat{x}$. Firm's after-tax resource profit with report $\hat{x}$ is $\pi(q(x); x) - F\mathbb{1}_{\{x:q(x)>0\}} - \pi(q(\hat{x}); \hat{x})T$. In general, when the tax depends on both x and y, the incentive compatibility constraint becomes

$$\pi(q(x); x) - F\mathbb{1}_{\{x:q(x)>0\}} - \pi(q(x); x)T(x, y) \geq \pi(q(\hat{x}); x) - F\mathbb{1}_{\{\hat{x}:q(\hat{x})>0\}} - \pi(q(\hat{x}); \hat{x})T(\hat{x}, \hat{y}) \tag{A.2}$$

for all $(x, \hat{x}) \in [0, \bar{x}]^2$ and $y > 0$, $\hat{y} > 0$.

Case a: T is constant. Suppose that the tax is a constant. Let $x > r^{\mathrm{F}}$ and evaluate (A.2) at $q(x) = q^{\mathrm{F}}(x)$, and note that the derivative of the firm's after-tax profit on the right-side of (A.2) with respect to the reported resource stock $\hat{x}$ is

$$\pi_q(q^{\mathrm{F}}(\hat{x}); x)q^{\mathrm{F}\prime}(\hat{x}) - \pi_q(q^{\mathrm{F}}(\hat{x}); \hat{x})q^{\mathrm{F}\prime}(\hat{x})T - \pi_x(q^{\mathrm{F}}(\hat{x}); \hat{x})T$$

This equals $-\pi_x(q^{\mathrm{F}}(x); x)T < 0$ at $\hat{x} = x$, and therefore the incentive compatibility constraint breaks.

Case b: T depends only on y. Suppose that the tax depends on the effort y. The same arguments as above in Case a show that the firm has incentives to lie.

Case c: T depends on x and y. Let $x > r^{\mathrm{F}}$. Similarly to Case a., the incentive compatibility (A.2) evaluated at $q(x) = q^{\mathrm{F}}(x)$, $y = y^{\mathrm{F}}$, implies that equation

$$-\pi_x(q^{\mathrm{F}}(x); x)T(x, y^{\mathrm{F}}) - \pi(q^{\mathrm{F}}(x); x)T_x(x, y^{\mathrm{F}}) = 0$$

must hold at intervals where functions are C^1. The solution to this differential equation is

$$T(x, y^{\mathrm{F}}) = \frac{c(y^{\mathrm{F}})}{\pi(q^{\mathrm{F}}(x); x)},$$

where $c(y^{\mathrm{F}}) > 0$. The limited liability constraint $\pi(q^{\mathrm{F}}(x); x) - F\mathbb{1}_{\{x:q^{\mathrm{F}}(x)>0\}} - \pi(q^{\mathrm{F}}(x); x)T(x, y^{\mathrm{F}}) \geq 0$, breaks for x close to r^{F}, because $\pi(q^0(r^{\mathrm{F}}); r^{\mathrm{F}}) = F$ and it is continuous in some interval $[r^{\mathrm{F}}, \tilde{x}]$. Thus the left-side of the limited liability constraint becomes $\pi(q^{\mathrm{F}}(x); x) - F - c(y^{\mathrm{F}}) < 0$ for x close to r^{F}, since $q^{\mathrm{F}} = q^0$ in $(r^{\mathrm{F}}, \bar{x}]$. □

A.4 Proof of Proposition 3

We can omit the exploration effort maximization constraint and $D(y)$ in the objective, because the expected exploration profit depends on the expected rent $\mathbb{E}\,U$ as the firm does not yet know the stock possibly to be revealed by discovery, and therefore the optimal exploration effort by the firm is independent of x.

The rent is zero at $x = 0$ and it can be expressed, by Milgrom and Segal (2002, Theorem 2), as

$$U(x) = \int_0^x \pi_x(q(s); s)g(s)\ \mathrm{d}x. \tag{A.3}$$

After inserting the rent into the objective of Problem (S) and integrating by parts, we pointwise maximize function, for fixed x,

$$q \mapsto \pi(q; x) - \frac{1 - G(x)}{g(x)}\pi_x(q; x)$$

over the interval $[0, q^{\max}]$, where $q^{\max} = \arg\max_{q\geq 0} \pi(q; \bar{x})$. Denote the value of this function with $\psi(q; x)$. Note that $q^{\max}$ is the largest of the maximizers of π over the interval $[0, \bar{x}]$ and that it exists (by assumption).

Function ψ is strictly concave in q and therefore there exists a unique maximizer on the interval $[0, q^{\max}]$ for each x. Denote it with q_{t}^0 and note that it is continuous by Berge's Maximum Theorem, because ψ is continuous in q and x and strictly concave in q.

Thus, pointwise maximization shows that the solution to Problem (S) satisfies

$$\pi_q - \frac{1 - G}{g}\pi_{qx} \leq 0, \quad q \geq 0, \quad q\left(\pi_q - \frac{1 - G}{g}\pi_{qx}\right) = 0,$$

which gives Equation (4), since one can show that it holds for $r_{\mathrm{t}}^0 = \sup\{x : q_{\mathrm{t}}^0(x) = 0\}$ that $r_{\mathrm{t}}^0 = \sup\{x : \arg\max_{q\in\{[0,q^{\max}]\}} \psi(q; x) = 0\}$ and $q_{\mathrm{t}}^0(x) = 0$ for all $x \leq r_{\mathrm{t}}^0$ and $q_{\mathrm{t}}^0(x) > 0$ for all $x > r_{\mathrm{t}}^0$ using similar arguments as in the proof of Proposition 1.

To show that q_{t}^0 is increasing, note that for $x > r_t^0$

$$q_{\mathrm{t}}^{0\prime}(x) = \frac{\left(\frac{1-G}{g}\right)' \pi_{qx} - \pi_{qx} + \left(\frac{1-G}{g}\right)\pi_{qxx}}{\pi_{qq} - \left(\frac{1-G}{g}\right)\pi_{qqx}} > 0,$$

for all points of differentiability by Assumptions 1 and 2, and therefore the optimal capacity is either constant or strictly increasing. □

A.5 Proof of Proposition 4

We have shown in Proposition 3 that the optimal capacity is $q = q_{\mathrm{t}}^0$ and the rent is given by the integral representation (A.3). Thus the optimal fee is

$$T^0(x) = \pi(q_t^0(x); x) - U(x) = \pi(q_t^0(x); x) - \int_0^x \pi_x(q_t^0(s); s)g(s)\ \mathrm{d}x.$$

We have $T^0(x) = 0$ for $x \leq r_{\mathrm{t}}^0$, since $\pi_x(0; x) = 0$ and we showed in Proposition 3 that $q_{\mathrm{t}}^0 = 0$, when $x \leq r_{\mathrm{t}}^0$. The previous claim that the partial derivate is 0 follows as the differential quotients are identically zero, since $\pi(0; x) = 0$ by Assumption 1.

Furthermore, the above reasoning implies that, for $x > r_{\mathrm{t}}^0$,

$$T^0(x) = \pi(q_t^0(x); x) - U(x) = \pi(q_t^0(x); x) - \int_{r_{\mathrm{t}}^0}^{x} \pi_x(q_t^0(s); s) g(s)\ \mathrm{d}x.$$

Hence, we have shown the formula (5) for the fee T^0. T^0 is continuous, as π is continuous by assumption, the rent is continuous and we have proved in Proposition 3, that q_{t}^0 is continuous.

Differentiating the formula with respect to x gives, for $x > r_t^0$, $T^{0\prime}(x) = \pi_q(q_{\mathrm{t}}^0(x); x) q_{\mathrm{t}}^{0\prime}(x) > 0$ at each point of differentiability of q_{t}^0. $T^{0\prime}(x)$ is positive as both terms of the product are positive. The first term is positive, by the definition of q_{t}^0 in (4) and the assumption $\pi_{qx} > 0$, and the positivity of the second term is shown in the proof of Proposition 3.

Therefore the optimal fee is either constant or strictly increasing. Moreover, T^0 is piecewise C^1, since q_{t}^0 is piecewise C^1 by Proposition 3. □

A.6 Proof of Proposition 7

Part 1: Assume that F is not too large, i.e., that the optimal contract offers positive capacity and fee for some stock sizes. Similarly to the proof of Proposition 3, without loss of generality, we omit exploration. The problem is then to

$$\max_{\{q(x),T(x)\}} \int_0^{\bar{x}} T(x) g(x)\ \mathrm{d}x,$$

subject to (IC) and the limited liability constraint for the resource rent $\hat{U}(x) = \pi(q(x); x) - T(x) - F\mathbb{1}_{\{x:q(x)>0\}} \geq 0$.

We proceed in the following three steps to construct the solution:

1. We form a simplified problem using two implications of the incentive compatibility constraint: a.) a derivative condition for the rent, $\hat{U}'(x) = \pi_x(q(x); x) > 0$ almost everywhere, and b.) $q \equiv 0$ and $T \equiv 0$, when x is smaller than the zero x° of the rent $\hat{U}$;

2. We show that (q_{t}^0, T^0) is a solution for the simplified problem for $x > r_{\mathrm{t}}^{\mathrm{F}}$, where $r_{\mathrm{t}}^{\mathrm{F}}$ is the zero of $\hat{U}$ for the contract choice $(q, T) = (q_{\mathrm{t}}^0, T^0)$;

3. We conclude the proof by showing the solution to the simplified problem is actually incentive compatible (IC).

1. a.) First, constraint (IC) implies $\hat{U}'(x) = \pi_x(q(x); x) > 0$ almost everywhere. This replaces (IC) in the above optimization problem.

b.) Second, we can focus on a solution with zero rent at some x: We solve T from the rent equation and plug it to the objective integral, and suppose on the contrary that the pair $X^+ = (q^+(x), \hat{U}^+(x))$, with $\hat{U}^+(x) > 0$ for all x, solves the problem. However, the pair $X := (q^+(x), \hat{U}^+(x) - \min_{s\in[0,\bar{x}]} \hat{U}^+(s))$ is feasible, because X^+ is feasible, and gives a greater value for the objective integral and satisfies $\hat{U}(x) = 0$ for some x.

Let (q, T) be any contract and x° be such that $\hat{U}(x^\circ) = \pi(q(x^\circ); x^\circ) - F\mathbb{1}_{\{x^\circ:q(x^\circ)>0\}} - T(x^\circ) = 0$.

For the second implication of (IC), we show that (IC) implies $q \equiv 0$ on $[0, x^\circ)$. Assume that $q > 0$ somewhere on $[0, x^\circ)$. Take $\tilde{x} < x^\circ$ with the property that $q(\tilde{x}) > 0$. Then the feasibility of (q, T) implies that $\pi(q(\tilde{x});\tilde{x}) - T(\tilde{x}) - F \geq 0$. This, $\pi_x > 0$ and $\tilde{x} < x^\circ$ imply then $\pi(q(\tilde{x}); x^\circ) - T(\tilde{x}) - F > 0$, which contradicts incentive compatibility (as a firm with x° obtains zero by telling the truth). Thus $q \equiv 0$ on $[0, x^\circ)$, which implies by feasibility that $T \equiv 0$ on $[0, x^\circ)$.

2. Taking $q = q^0_{\mathrm{t}}$ and $T = T^0$, we see that there is $x^\circ > r^0_{\mathrm{t}}$ such that $\hat{U}(x^\circ) = 0$, since $\pi(q^0_{\mathrm{t}}(r^0_{\mathrm{t}}); r^0_{\mathrm{t}}) - T^0(r^0_{\mathrm{t}}) - F = -F$ and $\pi(q^0_{\mathrm{t}}(x); x) - T^0(x)$ is increasing in x and F is not too large as assumed at the beginning. We denote this x° by $r^{\mathrm{F}}_{\mathrm{t}}$.

Now,

$$\begin{aligned}\int_0^{\bar{x}} T(x)g(x)\,\mathrm{d}x &= \int_0^{\bar{x}} \left(\pi(q(x);x) - \hat{U}(x) - F\mathbb{1}_{\{x:q(x)>0\}}\right) g(x)\,\mathrm{d}x, \\ &= \int_{r^{\mathrm{F}}_{\mathrm{t}}}^{\bar{x}} \left(\pi(q(x);x) - \hat{U}(x) - F\right) g(x)\,\mathrm{d}x,\end{aligned}$$

since the integrand is zero on $[0, r^{\mathrm{F}}_{\mathrm{t}}]$ by the step 1b of this proof, and thus

$$\int_0^{\bar{x}} T(x)g(x)\,\mathrm{d}x = \int_{r^{\mathrm{F}}_{\mathrm{t}}}^{\bar{x}} \left(\pi(q(x);x) - \hat{U}(x)\right) g(x)\,\mathrm{d}x - \int_{r^{\mathrm{F}}_{\mathrm{t}}}^{\bar{x}} F\ g(x)\,\mathrm{d}x.$$

The last integral is a constant and hence, similarly as in the proofs of Proposition 3 and Proposition 4, we see that the pair

$$(q(x), T(x)) = \begin{cases}(0,0), & x \leq r^{\mathrm{F}}_{\mathrm{t}}, \\ (q^0_{\mathrm{t}}(x), T^0(x) - F), & x > r^{\mathrm{F}}_{\mathrm{t}},\end{cases}$$

is a solution to the maximization problem that satisfies the constraints $\hat{U}'(x) = \pi_x(q(x);x)$ and $\hat{U}(x) \geq 0$. In addition, $q'(x) \geq 0$ when it exists. This pair gives $q^{\mathrm{F}}_{\mathrm{t}}$ and T^{F}.

If there is no zero that is smaller than $\bar{x}$ (i.e. F is too large), the optimal contract is the zero contract $(q, T) \equiv (0, 0)$: Assume on the contrary that there would be a non-zero, better contract (q, T); especially $q > 0$ for big enough x. Then the rent $\hat{U}(x) = \pi(q(x); x) - T(x) - F$ must have a zero that is smaller than $\bar{x}$ (as we showed at the step 1). We have shown above that after this zero the contract $(q^0_{\mathrm{t}}, T^0 - F)$ is the optimal one. This is a contradiction with the assumption that there is no zero that is smaller than $\bar{x}$ for the rent $\hat{U}(x) = \pi(q^0_{\mathrm{t}}(x); x) - T^0(x) - F$.

3. We are left to show that $(q^{\mathrm{F}}_{\mathrm{t}}, T^{\mathrm{F}})$ is actually incentive compatible. We denote the true discovery by x and the reported discovery by $\hat{x}$. For the incentive compatibility (IC), we need to show

$$\pi(q^{\mathrm{F}}(x);x) - F\mathbb{1}_{\{x:q^{\mathrm{F}}(x)>0\}} - T^{\mathrm{F}}(x) \geq \pi(q^{\mathrm{F}}(\hat{x});x) - F\mathbb{1}_{\{\hat{x}:q^{\mathrm{F}}(\hat{x})>0\}} - T^{\mathrm{F}}(\hat{x}),$$

for all $(x, \hat{x}) \in [0, \bar{x}]^2$.

We consider all the possible cases, that is, a. $x \leq r^{\mathrm{F}}_{\mathrm{t}}$ and $\hat{x} \leq r^{\mathrm{F}}_{\mathrm{t}}$, b. $x > r^{\mathrm{F}}_{\mathrm{t}}$ and $\hat{x} > r^{\mathrm{F}}_{\mathrm{t}}$, c. $x > r^{\mathrm{F}}_{\mathrm{t}}$ and $\hat{x} \leq r^{\mathrm{F}}_{\mathrm{t}}$, d. $x \leq r^{\mathrm{F}}_{\mathrm{t}}$ and $\hat{x} > r^{\mathrm{F}}_{\mathrm{t}}$.

Case a. Constraint (IC) holds, since both sides of (IC) are 0.

Case b. By Propositions 3 and 4, $q^{\mathrm{F}}_{\mathrm{t}}$ is C^1 and $q^{\mathrm{F}\prime}_{\mathrm{t}}(x) \geq 0$, when $x > r^{\mathrm{F}}_{\mathrm{t}}$. The (IC) follows similarly as in Proposition 6, since $\hat{x} > r^{\mathrm{F}}_{\mathrm{t}}$.

Case c. By definition of $q^{\mathrm{F}}_{\mathrm{t}}$, T^{F} and π, $q^{\mathrm{F}}_{\mathrm{t}}(\hat{x}) = 0$, $T^{\mathrm{F}}(\hat{x}) = 0$ and $\pi(q^{\mathrm{F}}_{\mathrm{t}}(\hat{x}); x) - T^{\mathrm{F}}(\hat{x}) = 0$. Thus (IC) holds as $\pi(q^{\mathrm{F}}_{\mathrm{t}}(x); x) - F - T^{\mathrm{F}}(x) \geq 0$ for $x > r^{\mathrm{F}}_{\mathrm{t}}$, by definitions of π, T^{F} and $r^{\mathrm{F}}_{\mathrm{t}}$.

Case d. As $\hat{x} > r_{\mathrm{t}}^{\mathrm{F}}$,

$$\begin{aligned}
\pi(q_{\mathrm{t}}^{\mathrm{F}}(\hat{x}); x) - F - T^{\mathrm{F}}(\hat{x}) &= \pi(q_{\mathrm{t}}^{\mathrm{F}}(\hat{x}); x) - F - \left(\pi(q_{\mathrm{t}}^{\mathrm{F}}(\hat{x}); \hat{x}) - F - \int_{r_{\mathrm{t}}^{\mathrm{F}}}^{\hat{x}} \pi_x(q_{\mathrm{t}}^{\mathrm{F}}(s); s) \, \mathrm{d}s \right) \\
&= - \int_{x}^{\hat{x}} \pi_x(q_{\mathrm{t}}^{\mathrm{F}}(\hat{x}); s) \, \mathrm{d}s + \int_{r_{\mathrm{t}}^{\mathrm{F}}}^{\hat{x}} \pi_x(q_{\mathrm{t}}^{\mathrm{F}}(s); s) \, \mathrm{d}s \\
&\leq - \int_{r_{\mathrm{t}}^{\mathrm{F}}}^{\hat{x}} \pi_x(q_{\mathrm{t}}^{\mathrm{F}}(\hat{x}); s) \, \mathrm{d}s + \int_{r_{\mathrm{t}}^{\mathrm{F}}}^{\hat{x}} \pi_x(q_{\mathrm{t}}^{\mathrm{F}}(\hat{x}); s) \, \mathrm{d}s = 0 \\
&= \pi(q_{\mathrm{t}}^{\mathrm{F}}(x); x) - F \mathbb{1}_{\{x : q_{\mathrm{t}}^{\mathrm{F}}(x) > 0\}} - T^{\mathrm{F}}(x).
\end{aligned}$$

The last inequality follows, since $\pi_x > 0$ and π_x is increasing in the q-variable, by Assumption 1, and $q_{\mathrm{t}}^{\mathrm{F}}$ is increasing. The last equality holds as $q_{\mathrm{t}}^{\mathrm{F}}(x) = 0$ and $T^{\mathrm{F}}(x) = 0$ for $x \leq r_{\mathrm{t}}^{\mathrm{F}}$. Thus (IC) holds in this case, too.

Part 2: Shown in Part 1.

Part 3: The proof is similar to the proofs of Propositions 1 and 5 and is omitted. □

A.7 Proof of Proposition 8

Part 1, reserve cut-off: Consider first the case, where the set-up costs are zero, i.e. $F = 0$. We need to show that $r_{\mathrm{t}}^0 > r^0$. Suppose on the contrary that $r_{\mathrm{t}}^0 \leq r^0$. Assumption $\pi_{qx}(q; x) > 0$ and the result $r^0 = \sup\{x : \arg\max_{q \geq 0} \pi(q; x) = 0\}$ (see Proposition 1 Part 1) imply $\pi_q(q; r_{\mathrm{t}}^0) \leq \pi_q(q; r^0) \leq 0$. Our assumption $\pi_{qq} < 0$ implies $\pi_q(q; r_{\mathrm{t}}^0) < 0$ for $q > 0$. Then

$$\pi_q(q; r_{\mathrm{t}}^0) - \frac{1 - G(r_{\mathrm{t}}^0)}{g(r_{\mathrm{t}}^0)} \pi_{qx}(q; r_{\mathrm{t}}^0) < 0 \tag{A.4}$$

by $\pi_{qx}(q; x) > 0$. Hence by the continuity with respect to x, there exists a small neighborhood of r_{t}^0 where inequality (A.4) holds. But then $q = 0$ is optimal on that neighborhood, which contradicts the definition of r_{t}^0 being the least upper bound of the stock level at which optimal q is zero. Hence $r_{\mathrm{t}}^0 > r^0$ in the case with $F = 0$.

Consider then the case, where $F > 0$. To show that $r^{\mathrm{F}} < r_{\mathrm{t}}^{\mathrm{F}}$, we suppose on the contrary that $r^{\mathrm{F}} \geq r_{\mathrm{t}}^{\mathrm{F}}$. Then we have

$$q_{\mathrm{t}}^0(r_{\mathrm{t}}^{\mathrm{F}}) \leq q_{\mathrm{t}}^0(r^{\mathrm{F}}),$$

where the inequality follows from the monotonicity of q_{t}^0 (see the proof of Proposition 3). Furthermore, recall from Proposition 1 that it holds for the reserve cut-off without pricing that $\pi(q^0(r^{\mathrm{F}}); r^{\mathrm{F}}) = F$, where $q^0(r^{\mathrm{F}})$ solves $\pi_q(q^0; r^{\mathrm{F}}) = 0$, and from Proposition 7 that $r_{\mathrm{t}}^{\mathrm{F}} > r_{\mathrm{t}}^0$. Because $r^{\mathrm{F}} \geq r_{\mathrm{t}}^{\mathrm{F}}$ by assumption, we have

$$\pi_q(q_{\mathrm{t}}^0(r^{\mathrm{F}}); r^{\mathrm{F}}) - \frac{1 - G(r^{\mathrm{F}})}{g(r^{\mathrm{F}})} \pi_{qx}(q_{\mathrm{t}}^0(r^{\mathrm{F}}); r^{\mathrm{F}}) = 0.$$

Therefore, when $r^{\mathrm{F}} \geq r_{\mathrm{t}}^{\mathrm{F}}$, we have

$$q_{\mathrm{t}}^0(r^{\mathrm{F}}) < q^0(r^{\mathrm{F}}),$$

because $\pi_q(q^0_{\mathrm{t}}(r^{\mathrm{F}}); r^{\mathrm{F}}) > 0$. Then

$$T^0(r^{\mathrm{F}}_{\mathrm{t}}) + F = \pi(q^0_{\mathrm{t}}(r^{\mathrm{F}}_{\mathrm{t}}); r^{\mathrm{F}}_{\mathrm{t}}) \leq \pi(q^0_{\mathrm{t}}(r^{\mathrm{F}}_{\mathrm{t}}); r^{\mathrm{F}}) < \pi(q^0(r^{\mathrm{F}}); r^{\mathrm{F}}) = F,$$

which is a contradiction as $T^0(r^{\mathrm{F}}_{\mathrm{t}}) > 0$. Hence $r^{\mathrm{F}} < r^{\mathrm{F}}_{\mathrm{t}}$.

Part 2, optimal effort: As with the capacity, consider first the case $F = 0$. Note that $\mathbb{E}\, T^0 > 0$ and recall that the rent is $U(x) = \pi(q^0_{\mathrm{t}}(x); x) - T^0(x)$. Then

$$\begin{aligned}
\mathbb{E}\,\pi(q^0(x); x) - \mathbb{E}\, U &= \mathbb{E}\,\pi(q^0(x); x) - \mathbb{E}\,\pi(q^0_{\mathrm{t}}(x); x) + \mathbb{E}\, T^0 > \mathbb{E}\,\pi(q^0(x); x) - \mathbb{E}\,\pi(q^0_{\mathrm{t}}(x); x) \\
&= \int_{r^0}^{\bar{x}} \pi(q^0(x); x) g(x)\,\mathrm{d}x - \int_{r^0_{\mathrm{t}}}^{\bar{x}} \pi(q^0_{\mathrm{t}}(x); x) g(x)\,\mathrm{d}x \\
&> \int_{r^0_{\mathrm{t}}}^{\bar{x}} \pi(q^0(x); x) g(x)\,\mathrm{d}x - \int_{r^0_{\mathrm{t}}}^{\bar{x}} \pi(q^0_{\mathrm{t}}(x); x) g(x)\,\mathrm{d}x \geq 0,
\end{aligned}$$

where the last inequality follows from $q^0(x) \geq q^0_{\mathrm{t}}(x)$ for all x. This and the optimality conditions for (interior) exploration effort in (2) and (6) give

$$D'(y^0) = \frac{z}{\mathbb{E}\,\pi(q^0(x); x)} < \frac{z}{\mathbb{E}\, U} = D'(y^0_{\mathrm{t}}).$$

This and $D'' < 0$ imply $y^0_{\mathrm{t}} < y^0$.

Consider then the other case, where $F > 0$. We show that $y^{\mathrm{F}} > y^{\mathrm{F}}_{\mathrm{t}}$. Recall that the rent is now $\hat{U}(x) = \pi(q^{\mathrm{F}}_{\mathrm{t}}(x); x) - T^{\mathrm{F}}(x) - F\mathbb{1}_{\{x: q^{\mathrm{F}}_{\mathrm{t}}(x) > 0\}}$. We have

$$\begin{aligned}
&\mathbb{E}[\pi(q^{\mathrm{F}}(x); x) - F\mathbb{1}_{\{x: q^{\mathrm{F}}(x) > 0\}}] - \mathbb{E}\,\hat{U} \\
&\quad = \mathbb{E}[\pi(q^{\mathrm{F}}(x); x) - F\mathbb{1}_{\{x: q^{\mathrm{F}}(x) > 0\}}] - \mathbb{E}[\pi(q^{\mathrm{F}}_{\mathrm{t}}(x); x) - F\mathbb{1}_{\{x: q^{\mathrm{F}}_{\mathrm{t}}(x) > 0\}}] + \mathbb{E}\, T^{\mathrm{F}} \\
&\quad = \int_{r^{\mathrm{F}}}^{r^{\mathrm{F}}_{\mathrm{t}}} (\pi(q^{\mathrm{F}}(x); x) - F) g(x)\,\mathrm{d}x + \int_{r^{\mathrm{F}}_{\mathrm{t}}}^{\bar{x}} (\pi(q^{\mathrm{F}}(x); x) - \pi(q^{\mathrm{F}}_{\mathrm{t}}(x); x)) g(x)\,\mathrm{d}x + \mathbb{E}\, T^{\mathrm{F}} > 0,
\end{aligned}$$

where the last inequality follows, since $\pi(q^{\mathrm{F}}(x); x) - F \geq 0$ for $x \geq r^{\mathrm{F}}$, $q^{\mathrm{F}}(x) \geq q^{\mathrm{F}}_{\mathrm{t}}(x)$ for all x and $\mathbb{E}\, T^{\mathrm{F}} > 0$. As in the case with $F = 0$ above, by the optimality conditions for (interior) exploration effort, we have

$$D'(y^{\mathrm{F}}) = \frac{z}{\mathbb{E}[\pi(q^{\mathrm{F}}(x); x) - F\mathbb{1}_{\{x: q^{\mathrm{F}}(x) > 0\}}]} < \frac{z}{\mathbb{E}\,\hat{U}} = D'(y^{\mathrm{F}}_{\mathrm{t}})$$

and we have shown $y^{\mathrm{F}} > y^{\mathrm{F}}_{\mathrm{t}}$, as $D'' < 0$. □

References

Arrow, K. and Chang, S. (1982) Optimal pricing, use, and exploration of uncertain natural resource stocks, *Journal of Environmental Economics and Management*, **9**, 1–10.

Ball, I. and Gao, X. (2024) Benefiting from bias: Delegating to encourage information acquisition, *Journal of Economic Theory*, **217**, 105816.

Baron, D. and Besanko, D. (1987) Monitoring, moral hazard, asymmetric information, and risk sharing in procurement contracting, *RAND Journal of Economics*, **18**, 509–532.

Biglaiser, G., Gu, J. and Li, F. (2025) Information acquisition and product differentiation perception, *American Economic Journal: Microeconomics*, **17**, 1–34.

Boadway, R. and Bruce, N. (1984) A general proposition on the design of a neutral business tax, *Journal of Public Economics*, **24**, 231–239.

Boadway, R. and Keen, M. (2015) Rent taxes and royalties in designing fiscal regimes for nonrenewable resources, in *Handbook on the Economics of Natural Resources* (Eds.) R. Halvorsen and D. F. Layton, Edward Elgar Publishing, chap. 5, pp. 97–139.

Bond, S. and Devereux, M. (1995) On the design of a neutral business tax under uncertainty, *Journal of Public Economics*, **58**, 57–71.

Cairns, R. (1990) The economics of exploration for non-renewable resources, *Journal of Economic Surveys*, **4**, 361–395.

Cairns, R. D. (2001) Capacity choice and the theory of the mine, *Environmental and Resource Economics*, **18**, 129–148.

Campbell, H. F. (1980) The effect of capital intensity on the optimal rate of extraction of a mineral deposit, *Canadian Journal of Economics*, **13**, 349–356.

Castonguay, F. and Lasserre, P. (2016) Resource agency relationship with privately known exploration and extraction costs, CIRANO discussion paper.

Castro-Pires, H., Chade, H. and Swinkels, J. (2024) Disentangling moral hazard and adverse selection, *American Economic Review*, **114**, 1–37.

Chatterjee, K., Dong, M. and Hoshino, T. (2025) Bargaining and information acquisition, *American Economic Journal: Microeconomics*, **17**, 75–102.

Crémer, J., Khalil, F. and Rochet, J.-C. (1998) Contracts and productive information gathering, *Games and Economic Behavior*, **25**, 174–193.

Daniel, P., Keen, M. and McPherson, C. (2010) *The Taxation of Petroleum and Minerals: Principles, Problems and Practice*, Routledge, 1. edn.

Department of Mines, Petroleum and Exploration. Western Australia (2025) Mineral royalties, https://www.wa.gov.au/organisation/department-of-mines-petroleum-and-exploration/mineral-royalties, retrieved 28.8.2025.

Fane, G. (1987) Neutral taxation under uncertainty, *Journal of Public Economics*, **33**, 95–105.

Fudenberg, D. and Tirole, J. (1991) *Game Theory*, MIT Press, 1. edn.

Gaudet, G. and Lasserre, P. (2015) The taxation of nonrenewable natural resources, in *Handbook on the Economics of Natural Resources* (Eds.) R. Halvorsen and D. F. Layton, Edward Elgar Publishing, chap. 4, pp. 66–96.

Gaudet, G., Lasserre, P. and Van Long, N. (1995) Optimal resource royalties with unknown and temporally independent extraction cost structures, *International Economic Review*, **36**, 715–749.

Gottlieb, D. and Moreira, H. (2022) Simple contracts with adverse selection and moral hazard, *Theoretical Economics*, **17**, 1357–1401.

Hellwig, M. F. (2010) Incentive problems with unidimensional hidden characteristics: A unified approach, *Econometrica*, **78**, 1201–1237.

Herrnstadt, E., Kellogg, R. and Lewis, E. (2024) Drilling deadlines and oil and gas development, *Econometrica*, **92**, 29–60.

Hung, N., Poudou, J. and Thomas, L. (2006) Optimal resource extraction contract with adverse selection, *Resources Policy*, **31**, 78–85.

Ing, J. (2020) Adverse selection, commitment and exhaustible resource taxation, *Resource and Energy Economics*, **61**.

Jaakkola, N., Spiro, D. and Benthem, A. (2019) Finders, keepers?, *Journal of Public Economics*, **169**, 17–33.

Krähmer, D. and Kováč, E. (2016) Optimal sequential delegation, *Journal of Economic Theory*, **163**, 849–888.

Laffont, J. and Tirole, J. (1986) Using cost observation to regulate firms, *Journal of Political Economy*, **94**, 614–641.

Lappi, P. (2020) On optimal extraction under asymmetric information over reclamation costs, *Journal of Economic Dynamics and Control*, **119**.

Martimort, D., Pouyet, J. and Ricci, F. (2018) Extracting information or resource? The Hotelling rule revisited under asymmetric information, *RAND Journal of Economics*, **49**, 311–347.

Martimort, D. and Stole, L. A. (2022) Participation constraints in discontinuous adverse selection models, *Theoretical Economics*, **17**, 1145–1181.

Milgrom, P. and Segal, I. (2002) Envelope theorems for arbitrary choice sets, *Econometrica*, **70**, 583–601.

Muratov, O. (2023) Entrepreneur–investor information design, *International Economic Review*, **64**, 1431–1467.

Natural Resources Canada (2024) Mining taxation, https://natural-resources.canada.ca/minerals-mining/mining-policy-taxation-industry/mining-taxation/taxes-levies-level-taxation, retrieved 28.8.2025.

Osmundsen, P. (1995) Taxation of petroleum companies possessing private information, *Resource and Energy Economics*, **17**, 357–377.

Osmundsen, P. (1998) Dynamic taxation of non-renewable natural resources under asymmetric information about reserves, *Canadian Journal of Economics*, **31**, 933–951.

Quyen, N. (1988) The optimal depletion and exploration of a nonrenewable resource, *Econometrica*, **56**, 1467–1471.

Quyen, N. (1991) Exhaustible resources: A theory of exploration, *Review of Economic Studies*, **58**, 777–789.

Roger, G. (2013) Optimal contract under moral hazard with soft information, *American Economic Journal: Microeconomics*, **5**, 55–80.

Szalay, D. (2009) Contracts with endogenous information, *Games and Economic Behavior*, **65**, 586–625.