Helsinki GSE

HELSINKI GSE DISCUSSION PAPERS 49 · 2025

Entry and exit

Klaus Kultti
Eemil Ylä-Nikkilä

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

HANKEN

Aalto University

Helsinki GSE Discussion Papers

Helsinki GSE Discussion Papers 49 · 2025

Klaus Kultti and Eemil Ylä-Nikkilä
Entry and exit

ISBN 978-952-7543-48-1 (PDF)
ISSN 2954-1492

Helsinki GSE Discussion Papers:
https://www.helsinkigse.fi/discussion-papers

Helsinki Graduate School of Economics
PO BOX 21210
FI-00076 AALTO
FINLAND

Helsinki, November 2025

Entry and exit

Klaus Kultti* Eemil Ylä-Nikkilä†

November 5, 2025

Abstract

We turn the zero-profit condition that is typically used to determine the number of firms into a game of entry/exit. We assume that identical firms compete in Cournot fashion, and when market conditions change, the equilibrium number of firms is determined in an entry/exit game. Focusing on symmetric equilibrium, the game may take several periods, and we determine the expected waiting time to reach the new equilibrium, as well as the economic value created during the adjustment period. The model is highly parametrised to allow for numerical results.

Keywords: Entry/Exit dynamics, Mixed strategies.

JEL: C72, C73, D43, L13.

1. INTRODUCTION

Entry and exit of firms is recognised as an important feature of well-functioning markets. Entry of new firms, and exit of declining firms, is behind the dynamism of so-called free markets. This is recognised in economic research, where the search words "entry/exit" produce more than a million hits. In oligopolistic markets, the literature tends to model the issues of entry and exit as games where some players may have private information about costs or some other payoff-relevant matters. Things are different when there is perfect competition or some type of competitive setting. Then it is simply assumed that, in the case of entry, from an unmodelled pool of firms, a number of them enters such that they make non-negative profits, and one more entrant would make negative profits. In particular, the entry and exit decisions are not modelled as strategic ones.

But this is not satisfactory, as the potential entry and exit dynamics are assumed away. There is a very easy cure for this, as it is essentially a standard modelling

*University of Helsinki, Helsinki Graduate School of Economics. Email: klaus.kultti@helsinki.fi. Address: P.O. Box 17 (Arkadiankatu 7), FIN-00014 University of Helsinki, Finland.

†University of Helsinki and Aalto University. Email: yeemil@ad.helsinki.fi.

We are grateful to Topi Hokkanen for constructive comments.

exercise to make this a game. Assuming that in a particular sector there is an equilibrium number of firms, and assuming that market conditions worsen, an exit game among these firms ensues. If the firms are identical, the symmetric equilibrium of the exit game is in mixed strategies. Once the first round is played, there may be any number of firms between zero and the original number in the market. If the number is not the equilibrium number, the firms play the entry/exit game repeatedly until there is the equilibrium number of firms in the market. This creates dynamics from which one can calculate the expected time to reach the equilibrium, as well as the time spent in various non-equilibrium states.

If the conditions in the market improve, an entry game ensues. This is otherwise similar to the exit game above, but now the number of players is potentially much larger, as there are presumably quite a few firms/individuals who are willing to grab economic opportunities. Our aim is to analyse these issues, in particular the asymmetry of the game emanating from the worsening of market conditions and the game emanating from the improvement of market conditions. We do this in a highly parametrised environment where all the firms are identical and the active ones compete in the standard Cournot fashion. As the equilibrium is in mixed strategies, and we need to determine the numerical values of equilibrium probabilities to build the matrices that depict the transitions between different states, we resort to numerical methods.

The main contribution of the article is two-fold. First, we replace the standard zero-profit condition that determines the number of firms by a game that determines the number of firms and produces, as a side product, dynamics of market entry and exit. Secondly, we highlight the difference between entry and exit games. In the latter game, the participants are the firms in the market, while in the former game, there are most likely many more participants, as also firms/individuals all over the economy are ready to reap the potential rewards. This asymmetry may be empirically interesting as to the dynamics.

1.1 RELATED LITERATURE

The most common way to address entry and exit is simply to assume that the zero-profit condition determines the number of firms in the market. When something more interesting is done, the problem is formulated as a timing or coordination problem, or a dynamic stochastic model of an industry is postulated with the aim of replicating industry dynamics. An early example of the first class of models is Diamond (1971), which focuses on price adjustment but also addresses the problem of entry towards the end. Diamond observes that new firms should enter the market until they make a normal return. He does not engage in formal modelling.

Dynamic and strategic frameworks for firm turnover include, e.g., Ghemawat and Nalebuff (1985) who examine the timing of exit in deterministically declining industries. They show that it is not necessarily the weakest firms that exit first when anticipating rivals' strategic responses. Fudenberg and Tirole (1985, 1986) extend this logic to stochastic and continuous-time settings. Their war-of-attrition and pre-emption models exhibit mixed strategies as to the timing of entry, investment, or exit. In our setting, the stochastic transition dynamics are generated solely by the equilibrium being in mixed strategies; everything else in the model is deterministic and common knowledge.

The contribution closest to ours is Vettas (2000). He analyses dynamic entry and exit under imperfect coordination, where symmetric firms choose mixed strategies over market participation. The resulting equilibrium balances entry and exit incentives through a single participation probability that equalises expected payoffs across states, i.e., the number of firms in the market. Vettas shows the existence of equilibrium and analytically characterises its properties in a continuous-time framework, whereas we calculate explicit entry and exit probabilities that are used to construct a Markovian transition matrix. This allows the analysis to move beyond equilibrium existence and examine adjustment dynamics, including the time it takes to converge, how welfare evolves, and how entry and exit differ in terms of speed and consumer effects. In this sense, we translate Vettas's equilibrium logic into a computational form that yields quantitative results.

There is more recent literature focusing on exit from the markets. The driving force may be stochastic profitability or uncertain demand, from which firms only receive partial information. These are typically timing games where the firms learn, or rather conduct Bayesian updating, about market condition; the equilibria are often in mixed strategies. An example is Murto (2004).

Parallel to above theoretical developments, the literature on dynamic industry models aims at replicating the real world dynamics of particular industries. Ericson and Pakes (1995) introduce the framework of Markov-perfect industry dynamics, where firms make entry, exit and investment decisions in stochastic environments. Their model captures the essentials of dynamic oligopoly and has served as the foundation for empirical implementations. Doraszelski and Satterthwaite (2010) extend this framework to computable Markov-perfect equilibria, focusing on numerical methods for solving high-dimensional dynamic games. Our work can be seen as an analytically simplified counterpart: a fully symmetric, tractable Markov system concentrating exclusively on entry and exit transitions, without any stochastic elements or investment dimensions.

Finally, there are many empirical studies of market dynamics; here are two from cement industry. Ryan (2012) estimate a dynamic entry/exit model for the U.S. cement industry, showing how environmental regulation affects cost thresholds and market structure. Collard-Wexler (2013) study the ready-mix concrete industry and find persistent cycles of entry and exit, often described as "market churn", consistent with stochastic fluctuations around equilibrium participation levels. These findings support interpreting market structure as an evolving stochastic process converging to equilibrium states, a perspective we formalize explicitly in our Markov framework.

2. SET-UP

Assume that there is a market where firms compete in Cournot fashion. Marginal costs are zero but each period there is a fixed cost c_k where k indicates the equilibrium number of firms when there is free entry and exit. Assume further that there are $n > k$ firms in the economy, and that those firms not active in the market under study are in some other markets where they earn zero profits, or are inactive but ready to enter the market under study if it becomes profitable.

We need explicit functional forms as the main results are numerical calculations. The demand is given by $q = 1 - p$, and with i firms each gets revenue $\frac{1}{(i+1)^2}$. The costs are assumed to be just periodic constants of being in the market which is, of course, a strong simplification. However, since the primary focus is on how changes in profitability affect market participation, this abstraction is sufficient for our purposes. In a more complicated setting, the changes in costs could be related to changes in taxation, production costs or demand.

We study situations where the fixed cost increases such that the equilibrium number of firms decreases to $h < k$. This situation is compared to the one where the cost then decreases to its original level. These changes are not symmetric: in the first case, the increase in the cost induces an exit game with the k firms as players. In the second case, the decrease in the cost induces an entry game with n firms as players; we mainly study the situation where n is large compared to k, or where the entry game has many more players than the exit game.

We focus on a symmetric equilibrium for the firms that participate in the game at a particular stage; it is in mixed strategies where the actions are entry/no-entry or exit/no-exit. There are several possibilities to model both the exit and entry game. Under mixed strategies, it may happen that the number of firms that enter the market is less than the equilibrium number. Then the entrants make a positive profit in that period. Next period, it could be that only the remaining firms participate in the entry game, or it could be that all the firms participate. As our aim is to focus on the difference between relatively few players in the exit game and relatively numerous players in the entry game, we choose the latter option. In other words, we assume that in each period before the equilibrium is reached, all the firms play the game. This also contributes towards symmetry and simplicity. We also ignore discounting, even though it may take several periods before the equilibrium is reached.

To get an idea about how things work, assume that to start with the equilibrium number of firms in the market is seven, and the total number of firms in the economy is $n = 30$. Then the cost increases such that the new equilibrium supports four firms. Each of the seven firms then flips a properly weighted coin to decide whether to exit or not. Any outcome between zero and seven firms in the market is possible. As long as the outcome is something other than four entrants, all seven firms play the exit game again. If the cost then decreases such that instead of four firms seven firms can be supported in the market, an entry game with thirty firms ensues. If the outcome is anything but seven entrants, all thirty firms play the same game next period, and this continues until the outcome is seven entrants.

The dynamics of the games following an increase or decrease in cost can be modelled by a Markov-chain. To construct the transition matrix, we first have to determine the equilibrium entry and exit probabilities for individual players. From these, we can then determine the transition probabilities.

2.1 INCREASE IN THE COST

To establish notation, notice that one of the actions is staying out of the market, and the other one is entering or staying in the market. We denote the probability of being in the market by σ_i in both entry and exit games. The subscript always denotes the number of players. When costs increase, the players are the firms that are originally in the market. When costs decrease, all the firms, both active and inactive (n) participate in the game.

Assume that the market supports k active firms with c_k. Then the cost increases to $c_h > c_k$, the equilibrium number of firms drops to $h < k$. Now the k firms in the market engage in an exit game where the individual probability of staying in the market is denoted σ_k. The outcome can be any number of firms between zero and k. Whenever the realised number of firms differs from h, the firms re-play the game. Because the equilibrium probability is determined by the indifference condition between entering and not entering, the firms' expected profit is zero. The consumers, however, get a different deal depending on how many firms enter.

Notice that the analysis may look as if the firms were myopic and only considered the next period instead of lifetime profits. But this is not the case, as the probabilities are determined by the zero-profit condition, and the lifetime utilities are also zero.

2.1.1 Exit game

Given the situation described above, with k firms in the market and the periodic cost having increased to $c_h > c_k$, the equilibrium number of firms under this cost level is $h < k$. If there are h firms in the market, each one makes a non-negative profit, whereas with more than h firms, each firms makes negative profit. Consider one of the k firms; if it enters the market it expects profit

$$\sum_{i=0}^{k-1} \binom{k-1}{i} \sigma_k^i \left(1-\sigma_k\right)^{k-1-i} \frac{1}{\left(i+2\right)^2} - c_h = 0 \tag{1}$$

This determines the equilibrium probability of being in the market when there are k players. The exit game is played over and over until there are h firms in the market.

From these probabilities, we construct the transition matrix. State h is an absorbing state, and consequently $p_{hi} = 0$ for $i \neq h$. If the state is $j \neq h$, the transition probability to any other state i is $p_{ji} = \binom{k}{i}\sigma_k^i \left(1-\sigma_k\right)^{k-i}$. We order the states so that the absorbing state occupies the first position in the transition matrix, and the others are in increasing order. The dynamics begin in state k, which is the equilibrium state prior to the cost increase. Denoting the transition matrix by P, the information we are after is given by P^∞ which is explained later on.

2.2 DECREASE IN THE COST

The other exercise consists of figuring out the corresponding matrix when the initial equilibrium is h and the cost decreases to c_k. We conduct an analogous calculation, but in this case there is an entry game among the n potential entrants.

3. ANALYSIS OF ENTRY AND EXIT

In this section, we consider some numerical examples. The idea is to demonstrate how the entry and exit cases become more asymmetric as n grows. We assume that with cost c_k and k active firms, each firm makes exactly zero profit, and likewise with cost c_h and h active firms. The reason is that if in equilibrium the firms make positive profits, one has to take this into account in the calculation of the entry probabilities, and this would complicate the analysis without bringing anything interesting in terms of the results.

Before going to the numerical analysis, we briefly recall the interpretation of P^∞.

To start with, the transition matrix is of the form

$$P = \begin{bmatrix} I & 0 \\ R & Q \end{bmatrix} \tag{2}$$

where I is an identity matrix of dimension equal to the number of absorbing states (here, a single state), R is a rectangular matrix of transition probabilities from non-absorbing states to absorbing states, 0 is a rectangular matrix of zeros representing transition probabilities from absorbing states to non-absorbing states, and Q is a square matrix of transition probabilities from non-absorbing states to themselves. The matrix of interest is

$$P^\infty = \begin{bmatrix} I & 0 \\ NR & 0 \end{bmatrix} \tag{3}$$

where $N = (I - Q)^{-1}$ is known as the fundamental matrix. Its entry (i, j) gives the expected number of periods in non-absorbing state j before absorption when the process starts in state i.[1,2] The purpose is to compare how long it takes to reach absorption with a cost increase to that of cost reduction. Of particular interest is the ratio of time spent in states that are advantageous to the consumer to the states that are disadvantageous.

[1]Note that this is not entry (i, j) of the matrix P^∞ as in N there are only the non-absorbing states.

[2]For more details, see, for example, Charles M. Grinstead (Author), J. Laurie Snell (Author) Introduction to Probability 2nd Edition 2012 American Mathematical Society

3.1 NUMERICAL ANALYSIS OF ENTRY AND EXIT

In this section, we study a setting where the costs first increase such that in the new equilibrium there are fewer firms than in the old equilibrium. Then the costs decrease to the original level. This allows us to compare the adjustment dynamics between the two equilibria. We study two cases which are representative, meaning that whatever equilibrium values for the number of firms are chosen, the results are qualitatively similar. First, we consider the case where the two equilibria consist of five firms and two firms. When there are five firms in equilibrium and costs increase such that in the subsequent equilibrium there are two firms, an exit game emanates. Then the costs decrease to the original level, and an entry game emanates. The cost levels are chosen so that the equilibrium profits are exactly zero. The other case involves equilibria with ten firms and two firms.

When the costs go down, we assume that the number of entrants is either 22 or 42, the point being that there are clearly more firms in the entry game than in the exit game.

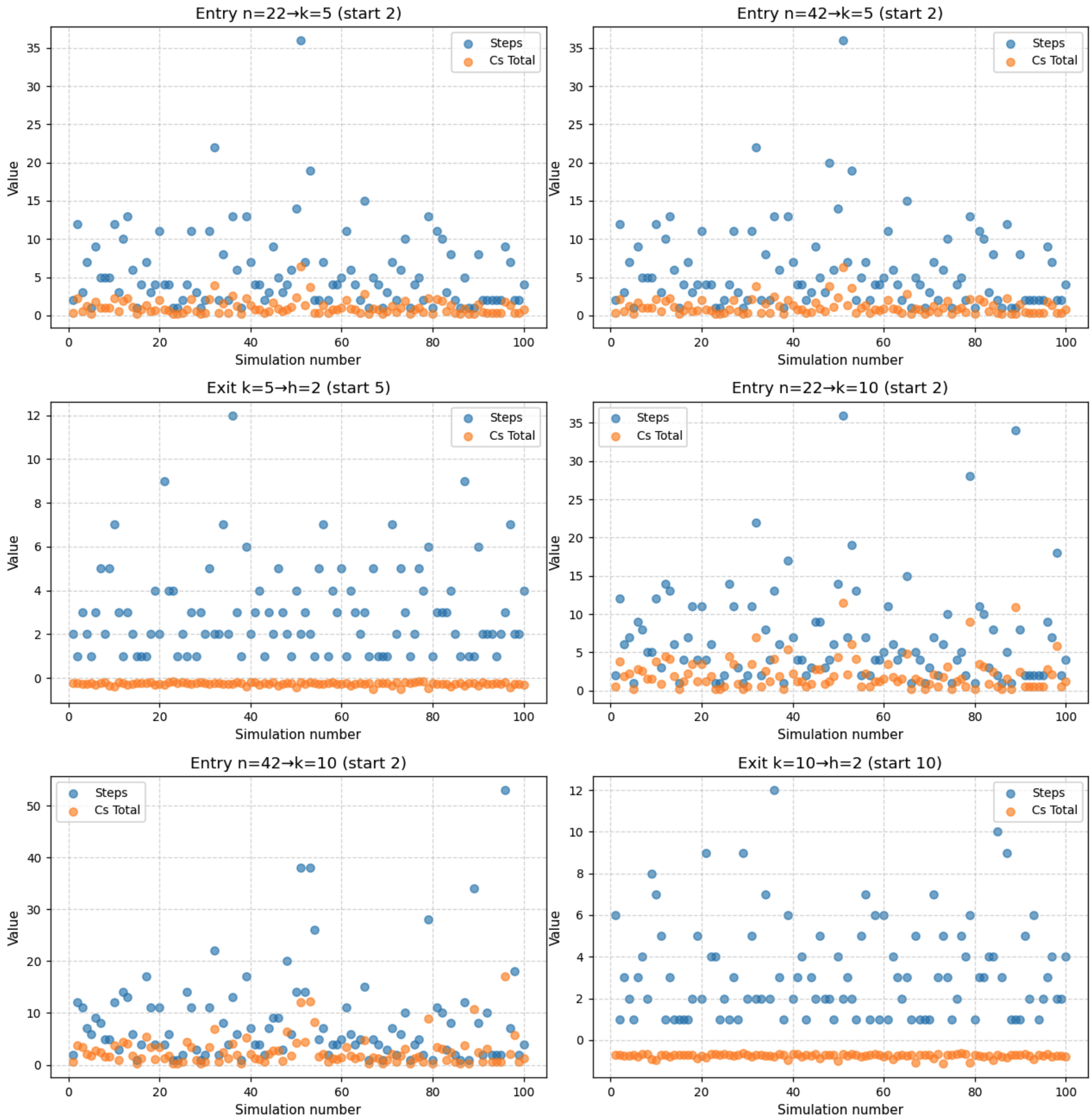

Figure 1: Adjustment outcomes across 100 simulations for each scenario (labels in panel titles). Blue markers ("Steps") show the number of periods required to reach the new equilibrium (absorption). Orange markers ("CS Total") show total consumer surplus accumulated during the adjustment. Entry cases (with many potential participants, $n \gg k$) display longer right tails and greater dispersion in Steps, while exit cases concentrate at lower step counts, indicating faster convergence.

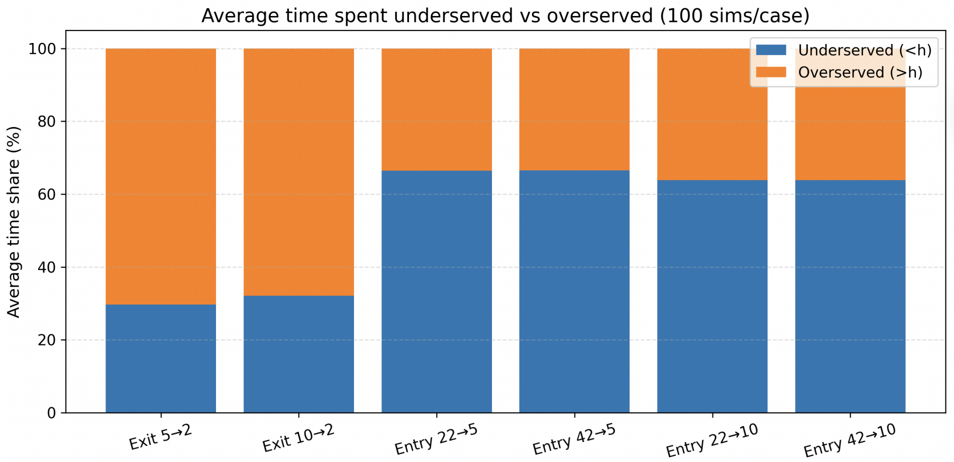

Figure 2: Share of periods spent in underserved ($< h$) and overserved ($> h$) states during adjustment (100 simulations). Entry cases ($2 \to 5,\ 2 \to 10$) spend more time underserved than exit cases ($5 \to 2,\ 10 \to 2$). The asymmetry reflects that, in entry, many more firms participate ($n \gg k$), which disperses outcomes and lengthens adjustment.

3.2 ANALYSIS BETWEEN EQUILIBRIA WITH TWO FIRMS AND FIVE FIRMS

In the tables below, we present some average quantities pertaining to the periods before a new equilibrium is attained after a change in costs. We denote the equilibrium number of firms before the cost change by k and after the change by h, and the number of entrants in an entry game by n.

Table 1: Expected number of periods until equilibrium is attained.

From k=5 to h=2	**From k=2 to h=5, n=22**	**From k=2 to h=5, n=42**
periods	periods	periods
2.97	5.00	5.37

One can immediately see that it takes much longer to attain the equilibrium in the entry games; this is as expected, as we assume that there are many more players, and concurrently many more states, in the entry game. To get a more informative view about what happens, it may be interesting to record how long is spent in states with fewer/more firms than in equilibrium. To scale, we divide the expected number of periods with fewer than the equilibrium number of firms by the total time until equilibrium is reached, and analogously for the case of more than the equilibrium number of firms. We say that the market is underserved/overserved if there are fewer/more firms than the equilibrium.

Table 2: Proportion of time spent in underserved market.

From k=5 to h=2	**From k=2 to h=5, n=22**	**From k=2 to h=5, n=42**
underserved market	underserved market	underserved market
43%	58%	58%

In the exit game, the market spends considerably less time in an underserved market than in the entry game. As the underserved market is undesirable for the consumers, it seems that the entry game is worse from their point of view than the exit game, given that the expectation is the new equilibrium. This may be somewhat misleading, as, with many states, being very close to the equilibrium is almost like being in equilibrium, but this is not taken into account in any way. With this in mind, we next present the expected consumer surplus. The idea is to relate the consumer surplus during the adjustment to the consumer surplus in the new equilibrium. Assuming that consumers' expectation is the level of consumer surplus in the new equilibrium, we interpret lower surplus during adjustment as a negative outcome and higher surplus as a positive outcome for the consumers. In the table below, there is the absolute consumer surplus as well as the proportion of consumer surplus of the equilibrium consumer surplus.

Table 3: Average consumer surplus during adjustment.

From k=5 to h=2	**From k=2 to h=5, n=22**	**From k=2 to h=5, n=42**
average CS	average CS	average CS
0.23	0.31	0.31
105%	89%	89%

The adjustment in the entry games appears less favourable from the consumers' point of view than the adjustment in the exit game.

To get a picture of the economic value created, the next table reports the consumer surplus from which the costs of being in the market are deducted. Our measure is the average consumer surplus with costs subtracted, i.e., the average welfare during the adjustment.

Table 4: Average welfare during adjustment.

From k=5 to h=2	**From k=2 to h=5, n=22**	**From k=2 to h=5, n=42**
average welfare	average welfare	average welfare
-0.08	0.18	0.18

It is seen that in the exit game, negative surplus is created until the equilibrium is reached, whereas in the entry game the surplus remains positive. Moreover, the surplus in the entry game does not change up to the two decimal places when the number of players is varied, even though this number is large relative to that in the exit game. The surplus created in equilibrium, which equals the consumer surplus because firms earn exactly zero profits, is 0.22 with two firms and 0.35 with five firms in equilibrium. The path to the equilibrium is clearly inefficient, and even more so in the exit game.

3.3 ANALYSIS BETWEEN EQUILIBRIA WITH TWO FIRMS AND TEN FIRMS

This section replicates the analysis of the previous one, with the assumption that the original equilibrium features ten firms. Then costs increase such that in equilibrium there are two firms in the market, and again costs decrease such that the ten-firm equilibrium is the new one. The first table records the expected number of periods until the equilibrium is reached.

Table 5: Expected number of periods until equilibrium is attained.

From k=10 to h=2	From k=2 to h=10, n=22	From k=2 to h=10, n=42
periods	periods	periods
3.44	5.95	7.02

Once again, the only thing that can be said is that in entry games, where there are more players than in exit games, it takes longer to reach the equilibrium. The following table records the proportion of underserved periods in the market.

Table 6: Proportion of time spent in underserved market.

From k=10 to h=2	From k=2 to h=10, n=22	From k=2 to h=10, n=42
underserved market	underserved market	underserved market
0.48	0.55	0.54

The consumer surplus is recorded in the next table.

Table 7: Average consumer surplus during adjustment.

From k=10 to h=2	From k=2 to h=10, n=22	From k=2 to h=10, n=42
average CS	average CS	average CS
0.23	0.38	0.38
105%	90%	90%

Once again, relative to the equilibrium, the consumer surplus during the adjustment period is above it in the exit game and below it in the entry game.

The next table shows the economic value created during the adjustment.

Table 8: Average welfare during adjustment.

From k=10 to h=2	From k=2 to h=10, n=22	From k=2 to h=10, n=42
average welfare	average welfare	average welfare
-0.22	0.31	0.31

The surplus created in equilibrium, which equals the consumer surplus because firms earn exactly zero profits, is 0.22 with two firms and 0.41 with ten firms in equilibrium. The path to the equilibrium is clearly inefficient, and even more so in the exit game.

4. DISCUSSION

We treat the entry and exit games in the most symmetric manner to keep the analysis clear and simple. It is not a big complication to consider different ways of modelling if they are regarded as more realistic. First, in both the entry and exit game one could assume that if the outcome in any round is fewer firms than the equilibrium number, then those firms stay in the market, and only the remaining firms participate in the next period. One could also assume that if in some round more firms than the equilibrium number enter the market, then the firms that did not enter do not participate in the game in the succeeding round. For both of these assumptions, one would need to determine different entry probabilities for different outcomes which would change matrices R and Q.

Introducing asymmetric firms, here firms with different costs, would complicate things; as long as the cost differences are moderate in the sense that the equilibrium strategy is mixed, different firms would still have different entry probabilities, and the situation would be much more burdensome from the computational point of view.

Assuming linear demand is, of course, a restriction. One could consider other functional forms, but in order to construct the transition matrix P one should be able to determine explicit numerical values for the entry probabilities.

5. CONCLUSION

We have presented only a few illustrative examples, but (unpublished) numerical analysis indicates that these are representative. A general property of the numerical exercises is that adjustment following a cost increase is faster than adjustment following a cost decrease. This is because in the latter case there is, by assumption, a larger number of firms participating in the entry game than in the exit game.

While firm dynamics is of independent interest, in the end, the bottom line is economic efficiency. With this in mind, we calculate the expected consumer surplus from which the costs of operating in the market are subtracted. This shows that, during the adjustment process, the economic value created can become negative in exit games, indicating that the firms exit the market too slowly.

REFERENCES

Allan Collard-Wexler. Demand fluctuations in the ready-mix concrete industry. *Econometrica*, 81(3):1003–1037, 2013.

Peter A Diamond. A model of price adjustment. *Journal of economic theory*, 3(2): 156–168, 1971.

Ulrich Doraszelski and Mark Satterthwaite. Computable markov-perfect industry dynamics. *The RAND Journal of Economics*, 41(2):215–243, 2010.

Richard Ericson and Ariel Pakes. Markov-perfect industry dynamics: A framework for empirical work. *The Review of economic studies*, 62(1):53–82, 1995.

Drew Fudenberg and Jean Tirole. Preemption and rent equalization in the adoption of new technology. *The Review of Economic Studies*, 52(3):383–401, 1985.

Drew Fudenberg and Jean Tirole. A theory of exit in duopoly. *Econometrica: Journal of the Econometric Society*, pages 943–960, 1986.

Pankaj Ghemawat and Barry Nalebuff. Exit. *The RAND Journal of Economics*, pages 184–194, 1985.

Pauli Murto. Exit in duopoly under uncertainty. *RAND Journal of Economics*, pages 111–127, 2004.

Stephen P Ryan. The costs of environmental regulation in a concentrated industry. *Econometrica*, 80(3):1019–1061, 2012.

Nikolaos Vettas. On entry, exit, and coordination with mixed strategies. *European Economic Review*, 44(8):1557–1576, 2000.