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Socially Responsible Lobbying

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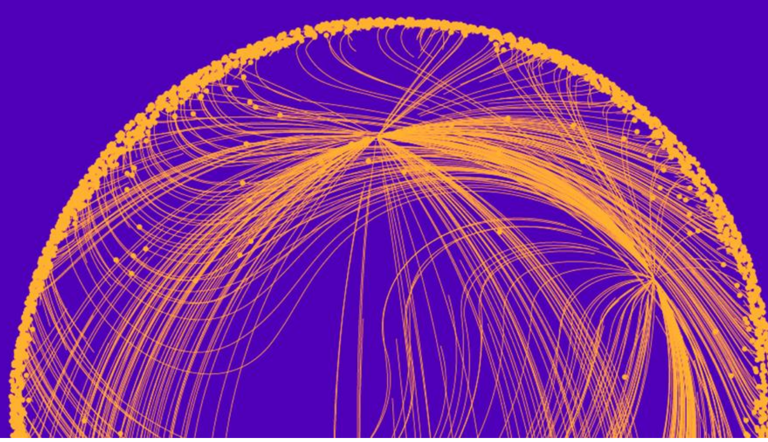

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Socially responsible lobbying*

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Abstract

Lobbyists are nowadays increasingly involved in promoting businesses through societal investment, producing what has been paraphrased as "corporate beauty contests" due to the uncertain winning criteria. To understand what motivates firm participation in these contests and their effects on welfare, our paper analyzes lobbying contests in which firms can boost their competitiveness in a market for political favors through either monetary contributions or societal investments. We establish that the responsiveness of political favors to social lobbying (i) alleviates lobbying competition, (ii) decreases total lobby spending, and (iii) shifts spending from monetary to social lobbying. This is generally welfare improving. Our results thus suggest that the ongoing transition to societal lobbying is a move toward a more efficient lobbying standard.

Keywords: *Lobbying contest, Corporate social responsibility, Beauty contest, Multidimensional expenditure, Differentiation, Endogenous competitiveness, Obfuscation.* **JEL-codes:** D43, D83.

*Section 6 and Appendix A of this manuscript extend our article "Socially responsible procurement", printed as a chapter of Yi Zheng's doctoral dissertation (University of Helsinki, 2020). We thank the excellent anonymous reviewers and the pre-examiners Tuomas Takalo and Mitri Kitti, as well as Panu Poutvaara, Klaus Kultti, and Hannu Vartiainen, and various seminar audiences for helpful comments. S. Hämäläinen acknowledges the financial support of the OP Group research foundation (Grants No. 20180119, and 20170107) and Finnish Cultural Foundations (Grant No. 00190358). Any shortcomings are our own.

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1 Introduction

Lobbyists are nowadays increasingly involved in promoting businesses through social agendas. The winning criteria in these "corporate beauty contests" often remain vague to observers.[1] In particular, in calculating the widely applied KLD ratings to quantify firms' investment in corporate social responsibility (CSR), KLD Research & Analytics Inc. evaluates firms in terms of the following five criteria: (i) environment, (ii) community and society, (iii) employees and supply chain, (iv) customers, and (v) governance and ethics. Because diverse CSR investments are therefore recognized as socially valuable and the costs of specific CSR investments presumably differ across firms, the priorities of companies' social business agendas often vary quite significantly.

For instance, *Coca-Cola* company envisions 'a world without waste' by "collecting and recycling a bottle or can for every one we sell by 2030". In contrast, *Google* has made the promise that it will "decarbonize its electricity supply and operate on 24/7 carbon-free energy, everywhere, by 2030", whilst the global pharmaceutical company *Johnson & Johnson* has set the goal of "sourcing all its electricity from renewables by 2025". *Netflix* and *Spotify* have a more worker-focused social agenda of providing their employees 52 and 24 weeks of paid parental leave, respectively. *Wells Fargo* donates up to 1.5 % of its revenue in charity to "more than 14,500 nonprofits".[2] The same firms are also known for playing a major role in influencing public policy in the making.

> The Coca-Cola Company's public policy agenda is built around our mission to refresh the world and make a difference. In the U.S. and Canada, our policy priorities include environmental sustainability, consumer preference, tax and trade, and workplace and economic inclusion. Our advocacy often involves collaboration and thought leadership in the public and private sectors. When significant to our business interests, we may also advocate through lobbying and coalitions.
>
> Coca-Cola Company's website (2023)

The promotion of a social agenda is thus skillfully intertwined with corporate strategies for advancing business interests. Why is this an attractive public relations strategy for a firm? What are the implications for profits and competition in lobbying? So far, little is known about the effects of integrating social objectives in the preferences of private firms and public agents. Currently evidence only shows that firms that invest more in CSR receive a higher return on their lobbying (Garcia, 2016), insurance against value damage

[1]For a discussion of CSR as a beauty contest, see the handbook chapter in Ihlen et al. (2011).

[2]Numerous similar examples are covered in the article "16 Brands Doing Corporate Social Responsibility Successfully" (`https://digitalmarketinginstitute.com/blog/corporate-16-brands-doing-corporate-social-responsibility-successfully`, accessed Mar 6, 2023)

Liu et al. (2020), and more public tendered contracts (Flammer, 2018).[3] No consensus about the mechanism exists.

The purpose of this article is to take the arguable "beauty contest" nature of CSR seriously and investigate its effects on the competition for policy favors. As the key modeling assumption, our research thereby acknowledges that societal lobbying is likely to confuse the lobbying process relative to the standard monetary lobbying, especially due to the multidimensional nature of societal lobbying vs. the unidimensional character of monetary lobbying.[4] One firm may invest mostly in environmental issues while another focuses on community service and charitable giving.[5] Sometimes a particular decision-maker (DM) assigns a higher weight, e.g., to green production than to business ethics, but typically firms only learn this afterwards.

This entails that firms can successfully employ CSR as a differentiation strategy. To incorporate this natural feature into a basic rent-seeking model, our paper analyzes Tullock contests where firms can boost their competitiveness in a market for political favors through either targeted monetary contributions to a relevant DM (DM-lobbying) or general societal investments in CSR (CSR-lobbying). Both DM-lobbying and CSR-lobbying help a firm to win but CSR renders the contest more random, resulting in a multi-dimensional Tullock success function (CSF) where Tullock noise is endogenous. We explore how the choice between the lobbying modes affects the structure of lobbying spending to influence the outcome of the contest.

Because CSR confuses the competition among firms, we establish that investment in CSR (i) alleviates lobbying competition, (ii) decreases total lobby spending, and (iii) shifts spending from monetary to social lobbying. This is welfare improving insofar as CSR investment is socially more beneficial than contributions to DM. Our results thus suggest that the transition from DM- to CSR-lobbying is a move toward a more efficient lobbying standard.

To present a complete analysis of the strategic effects of CSR in lobbying, we also consider setups where CSR-lobbying is chosen before DM-lobbying and where CSR- and DM-lobbying expenditures are complementary. In these cases, CSR-lobbying does not fully crowd out DM-lobbying but instead reduces it efficiently and reinforces its effects. This may explain why the highest CSR-lobbyists can also be the strongest DM-lobbyists at the same time (e.g., *Coca-Cola*, *Google*, *Johnson & Johnson*, etc.). An extension where contest noise is uncertain and the contest thus akin to a noisy all-pay contest, however,

[3]For a complementary paper investigating competition in responsible procurement, see Hämäläinen and Zheng (2022).

[4]Favotto and Kollman (2021) find that most corporate CSR reports are highly superficial while Chatterji et al. (2009) show that KLD ratings may not optimally capture public information.

[5]As documented by Bertrand et al. (2020), charity is special in being tax-free and easily turned into a form of political influence. We regard it as a borderline case between CSR- and DM-lobbying.

suggests that the highest-spending lobbyists invest more in DM-lobbying and less in CSR-lobbying than their counterparts who spend the least. Being less noisy, DM-lobbying is a more competitive strategy than CSR-lobbying and thereby favored by likely winners.

Our analysis contributes to the literature sparked by Baron (2001), McWilliams and Siegel (2001), and Bagnoli and Watts (2003), who seek to understand CSR investments as a rational firm's strategy, e.g., enhancing a firm's reputation or signaling the quality of its product (Feddersen and Gilligan, 2001; Siegel and Vitaliano, 2007; Bagnoli and Watts, 2017; Bénabou and Tirole, 2010). However, Besley and Ghatak (2007) argue that firms lack a comparative advantage in CSR due to a free-riding problem in private provision.[6] We observe that incentives to supply CSR arise as a side-product of lobbying. Further, our paper shows for lobbying contests that the welfare effects of (privately supplied, publicly demanded) CSR depend on firms' alternative competition strategies: CSR is welfare-improving in lobbying contests as it substitutes for more wasteful lobbying.

A broader lesson derived from our investigation is that we show that companies gain an advantage from CSR-lobbying because it differentiates their political strategies and alleviates lobbying competition.[7] This may help to reconcile the Tullock (2001) paradox, which asks why firms spend so little in lobbying for often substantial rents.[8] Explanations have previously been concentrated on, e.g., asymmetric payoffs (Hillman and Riley, 1989) and war of attrition -type competition (Riley, 1999). When competition is uncertain, we find that by investing in CSR, competing firms not only exaggerate contest noisiness but also introduce a consolation prize for the loser. In the literature, the availability of a consolation prize and noise are known to limit contest spending Barut and Kovenock (1998); Long (2013). The tradeoff between the first prize and the consolation prize that firms need to solve in our model is novel. The question is related to contest design (Kirkegaard, 2012; Siegel, 2014; Haan, 2016). The crux of our paper is that payoffs are determined by the strategies of the *contestants*, not by a designer.[9]

Connecting (i) the competition economics literature on differentiation and obfuscation, and (ii) the public choice literature on political rent-seeking and lobbying contests, our paper also shows, for the first time, to our knowledge, that firms have incentives to obfuscate lobbying processes. CSR is an efficient lobbying strategy in our model because it enables a firm to differentiate from its rival and make the choice among firms more confus-

[6] The recognition of the public good nature of CSR dates back at least to (Olson, 1965; Stigler, 1974).

[7] Bagnoli and Watts (2003) find that CSR is higher under Bertrand rather than Cournot competition. Bagnoli and Watts (2020) and Fernández-Kranz and Santaló (2010) provide evidence that more relaxed competition allows firms to invest more in CSR – funding it via higher prices and green bonds.

[8] Ansolabehere et al. (2003) pose a related question asking why there is so little money in US political lobbying. Borisov et al. (2015); Servaes and Tamayo (2013) estimate that lobbying increases a firm's stock market value, suggesting that political rents are not fully dissipated in the process of lobbying.

[9] Contests with endogenous entry (Gu et al., 2019) or alliances (Konishi and Pan, 2021) also feature endogenous payoffs.

ing to the DM. Differentiation is shown to relax political competition, similarly to what happens in markets (Shaked and Sutton, 1982; Perloff and Salop, 1985). There is also a link to the burgeoning literature on strategic complexity in product markets, e.g., Ellison (2005), Gabaix and Laibson (2006), Carlin (2009), and Gamp and Krähmer (2022). The literature shows that firms have incentives to present their products in ways that impede direct comparisons. This obfuscation raises prices, with generally adverse effects on consumers and market welfare.[10] Our paper shows that the effects of strategic obfuscation could be positive in contests because noise decreases lobbying. The explanation is that competition is non-productive in lobbying, unlike in markets.[11,12]

The paper is organized as follows. The model is set up in Section 2. The analysis of a benchmark lobbying contest is presented in Section 3. This is followed by an analysis of a sequential lobbying contest in Section 4 and a synergistic lobbying contest in Section 5. Section 6 extends the analysis to contests with uncertainty about competition. Section 7 concludes by discussing alternative interpretations of our analysis. Longer proofs are delegated to the Appendix. The analysis of asymmetric contests with competition uncertainty is relegated to the supplementary Online Appendix A.

2 Model

There are two firms $i, j = 1, 2$ competing for a favor, which benefits one firm over the other, hence providing the winner a profit T (e.g., regulation, subsidies, or taxes that favor particular firms, elevated entry barriers, local monopoly position, etc.). The decision to grant the favor is delegated to DM (decision-maker). DM is either (i) sensitive to the social reputation of businesses, justifying its actions to voters and stakeholders, or (ii) works under explicit guidelines that require prioritizing socially responsible firms.[13] This allows firms to influence the contest by investing in CSR (corporate social responsibility).

We first study (in Section 3) a benchmark, where CSR- and DM-lobbying are independent and simultaneous, and later focus (in Sections 4 and 5) on sequential and synergistic contests, where CSR- and DM-lobbying are intertwined.

[10]But see Taylor (2017) and Gamp and Krähmer (2022), where obfuscation furthers screening and improves welfare.

[11]As known since Tullock (1967), the possibility of acquiring a monopoly by influencing public choice not only reduces welfare by alleviating competition ("Harberger triangle") but also generates losses because of wasteful lobbying ("Tullock square").

[12]To our knowledge, our paper is the first to consider firms' incentives to obfuscate lobbying processes. Only a single concurrent paper (Fremeth et al., 2022) touches upon this question, showing that larger industries have noisier political environments.

[13]The EU has published voluntary criteria for green public procurement (on Nov 12, 2019, for public space maintenance; on Oct 2, 2019, for food, catering services, and vending machines; etc.), and the US has several programs that promote environmental-friendly procurement (environmentally preferable purchasing, comprehensive procurement guidelines, green procurement program, etc.).

In the first benchmark model, each firm i simultaneously chooses a budget $e_i = c_i + l_i$ (for expenditure), which specifies its total spending in the contest, e_i, the investment in CSR-lobbying, $c_i \geq 0$, and its expenditure on DM-lobbying $l_i \geq 0$.

Conditional on winning, the payoff of the firm thus equals

$$T - l_i - c_i.$$

The payoff conditional on losing in the lobbying contest is

$$-l_i - c_i.$$

The key distinction between the expenditures l_i (DM-lobbying) vs. c_i (CSR-lobbying) is that, while both boost the likelihood of winning the contest, DM-lobbying as a monetary contribution influences the authority more directly and therefore has a more predictable effect on the winning probability than CSR-lobbying. As discussed previously, public institutions and stakeholders often recognize various forms of CSR, ranging from cleaner production to business ethics and social programs, rendering the comparison of disparate CSR investments quite challenging.

To model this idea in a tractable general framework, we thus assume that the probability of winning a favor w_i (i.e., the contest success function, CSF) has the noisy logit form (see Corchón and Serena (2018) for common CSFs)

$$w_i\left(e_i(c_i, l_i), e_j(c_j, l_j)\right) = \begin{cases} \frac{e_i^r}{e_i^r + e_j^r}, & \text{if } e_i^r + e_j^r \neq 0, \\ \frac{1}{2}, & \text{if } e_i^r + e_j^r = 0, \end{cases} \tag{1}$$

where the noise parameter $r := r(c_i, c_j)$ is determined (symmetrically) by investments $c := (c_i, c_j)$. We suppose that larger investments in CSR will introduce more randomness into CSF, rendering the choice of the winner less dependent on firms' total expenditure $e := (e_i, e_j)$: $\frac{\partial r}{\partial c_i} = \frac{\partial r}{\partial c_j} < 0$.

We presume that $r(c_i, c_j)$ is continuously differentiable and decreasing in (c_i, c_j), its maximum being positive: $r(0, 0) > 0$.

The model gives the Tullock CSF as a special example for $r = 1$. For $r \to 0$ (the least competitive case), the winner is chosen at random: $w_i = 1/2$ for all e_i, e_j. For $r \to \infty$ (the most competitive case), the highest spending always wins: $w_i = 1$ if $e_i > e_j$ – similar to an all-pay auction (APA).

To emphasize the novelties, our employed model assumes two main deviations from the standard Tullock model. First, expenditure is no longer unidimensional, but is the sum of two choice variables: a standard lobbying expenditure component and – as an additional novel component – a CSR investment expenditure. The firms choose both expenditure levels at the same time and the total expenditure determines the probability of winning in the contest success function. Second, the noise, which represents the (lack of) precision with which the DM can evaluate expenditures, is no longer a parameter. Instead it is endogenous and increasing in the sum of CSR investment levels chosen by the firms.

3 Benchmark lobbying contests

3.1 Lobbying without CSR

We start by reviewing the benchmark where CSR-lobbying is not regarded favorably by DM. All expenditure is thus DM-lobbying spending, $e_i = l_i$, which gives the standard in the literature.

Lemma 1 *In a lobbying contest without CSR,*

1. *A symmetric pure equilibrium exists for $r \in [0, 2]$ being unique for $r \in [0, 1)$.*

2. *A symmetric mixed equilibrium exists for $r \in (2, \infty)$ being unique for $r \to \infty$.*

As known, the symmetric equilibrium strategies are easily solved for $r \in [0, 2]$ and in the case of $r \to \infty$ for which firms' expenditures are randomized over the interval $[0, T]$ (APA). Cases where $r \in (2, \infty)$ are seemingly more difficult. However, Ewerhart (2015) shows that firm expenditures are randomized over a countably infinite set on $[0, T]$ with zero payoffs. Moreover, Baye et al. (1994) prove the existence of a symmetric mixed equilibrium with complete rent dissipation, thus characterizing equilibrium payoffs. Our main focus being on payoffs, this suffices for our purposes.

Lemma 2 *In a lobbying contest without CSR,*

1. *Firms' symmetric equilibrium payoffs are $\frac{T(2-r)}{4}$ for $r < 2$.*

2. *Firms' symmetric equilibrium payoffs equal zero for $r \geq 2$.*

We sketch the proof, which is brief, to facilitate the comparison to contests with CSR.

Proof. A firm's best response e_i to e_j is given by the payoff maximization problem $\max_{e_i} Tw_i - e_i$. The first-order condition is

$$T\frac{re_i^{r-1}e_j^r}{(e_i^r + e_j^r)^2} - 1 = 0 \tag{2}$$

and the second-order condition is

$$T\frac{-r(r+1)e_i^{2(r-1)}e_j^r + r(r-1)e_i^{r-2}e_j^{2r}}{(e_i^r + e_j^r)^3} < 0. \tag{3}$$

In a symmetric pure equilibrium, $e_i = e_j =: e$, the first-order condition simplifies into

$$T\frac{r}{4e} - 1 = 0 \Longleftrightarrow e = T\frac{r}{4},$$

showing a firm's payoff to be positive *iff* $r < 2$

$$Tw_i - e_i = T\frac{1}{2} - T\frac{r}{4} > 0 \Longleftrightarrow r < 2,$$

as both firms win with equal probability and both pay their expenditure with probability one. The second-order condition simplifies into

$$T\frac{-r}{4e^2} < 0,$$

which confirms a local maximum at $e_i = e_j$.

If the profit is negative in the tentative pure equilibrium, i.e., $r > 2$, a firm increases its payoff by deviating to zero expenditure, which gives zero profit. However, if the rival has zero expenditure, a firm has a profitable deviation to positive expenditure, and its winning probability jumps. Balancing these joint incentives results in a symmetric mixed equilibrium where the expenditure is randomized in equilibrium.

The limit model $r \to \infty$ (APA) is straightforward to analyze, as the symmetric equilibrium strategy is continuous and has interval support $[0, T]$. Consistent with mixing across $[0, T]$, the payoff of spending zero is 0 as $w_i(0) = 0$ (the rival always wins) and the payoff of spending T is zero as $w_i = 1$ (the firm always wins). For any intermediate expenditure level $e \in (0, T)$, the payoff is the same but $w_i(e) = F(e)$ is given by

$$0 = TF(e) - e \Longleftrightarrow F(e) = \frac{e}{T},$$

i.e., the distribution function of the uniform distribution. For $r \in (2, \infty)$, Ewerhart (2015) shows, using complex analysis, that expenditure is randomized over a countably infinite set on $[0, T]$, with zero as the unique accumulation value. Similar to the more familiar case of $r \to \infty$, the symmetric equilibrium strategy thus also implies zero payoffs to firms when $r \in (2, \infty)$. $\square$

3.2 Lobbying with CSR

When CSR determines the noisiness of the contest, r, the nature of the equilibrium no longer hinges on (i) how competitive the contest originally was without investment in CSR, $r(0,0)$, but instead on (ii) how much a firm may profitably invest in CSR to relax competition, $r(\frac{T}{2},\frac{T}{2})$. As shown below, the value $r(\frac{T}{2},\frac{T}{2})$ gives the highest contest noise that would guarantee both firms a non-negative payoff in a symmetric pure equilibrium – entering as a threshold in Propositions 1 and 2.

A firm i's best response $e_i=(c_i,l_i)$ to firm j's strategy $e_j=(c_j,l_j)$ is given by

$$\max_{c_i,l_i} T\frac{(c_i+l_i)^{r(c_i,c_j)}}{(c_i+l_i)^{r(c_i,c_j)}+(c_j+l_j)^{r(c_i,c_j)}}-c_i-l_i$$

The first-order condition w.r.t. l_i is[14]

$$T\frac{re_i^{r-1}e_j^r}{\left(e_i^r+e_j^r\right)^2}-1=0, \tag{4}$$

and the first-order condition w.r.t. c_i is

$$T\frac{re_i^{r-1}e_j^r}{\left(e_i^r+e_j^r\right)^2}-1+T\frac{e_i^re_j^r}{\left(e_i^r+e_j^r\right)^2}\log\left(\frac{e_i}{e_j}\right)\frac{\partial r}{\partial c_i}=0. \tag{5}$$

The first lines of (4) and (5) equate the effects of increasing the total expenditure on the likelihood of winning the contest and the firm's costs. These effects are equal for both DM-lobbying l_i and CSR-lobbying c_i. The second line of (5) captures the effect of increasing CSR on the probability of winning. The effect is positive if the total expenditure is below the rival's, and negative if total expenditure is above. This novelty effect of CSR, which allows firms to relax their lobbying competition by investing in CSR, will make spending during the contest less lucrative to firms than previously.

To see why, note that, starting from pure DM-lobbying $e=l=T\frac{r(0,0)}{4}$, as in the symmetric equilibrium without CSR, a firm could now lower its expenditure profitably from $e=l$ to $e=l-\epsilon$ by decreasing DM-lobbying l_i by $(1+a)\epsilon$ while increasing CSR-lobbying c_i by $a\epsilon$. As $e=l$ satisfies (2) and (3), the immediate effect on a firm's payoffs of reducing spending by ϵ is clearly negative. However, its total expenditure being thereafter below its rival's, the residual effect of shifting spending by $a\epsilon$ units from DM- to CSR-lobbying is positive. Altogether, we observe that a firm benefits from decreasing l_i and increasing c_i, in suitably designed proportions $(1+a)/a$ while slightly reducing spending e_i.

Extending this reasoning, we can easily see, by comparing equations (4) and (5), that

[14] We suppress the argument (c_i,c_j) of r and replace the summation c_i+l_i by e_i from here on.

the incentive to alter expenditure in this manner remains intact as long as DM-lobbying remains positive in a candidate equilibrium. In particular, because the first terms are identical in (4) and (5) but the second term of (5) is missing from (4), it is impossible to satisfy the conditions of an interior optimum concurrently for both c_i and l_i.[15] In a symmetric pure equilibrium, a firm will thereby only expend on CSR-lobbying – which gives it a higher expected payoff by increasing the noisiness of the contest r.

Specifically, because the second term in (5) will vanish in a symmetric equilibrium, $e_i = e_j = e$, the equilibrium expenditure $e = c$ is given by the first term of (5)

$$T\frac{r(c,c)}{4c} = 1.$$

Thus, solving the equilibrium CSR investment c requires solving a fixed point problem

$$c = T\frac{r(c,c)}{4}$$

Note that $T\frac{r(c,c)}{4} - c$ is continuous and decreasing in c. Therefore, because $Tr(0,0)/4 - 0 > 0$ and $Tr/4 - T/2 < 0$ for any $r < 2$, there exists a unique fixed point $\hat{c} \in [0, T/2]$ satisfying $\hat{c} = T\frac{r(\hat{c},\hat{c})}{4}$ provided $r(T/2, T/2) < 2$.

Because firms' winning probabilities equal and CSR is a sunk cost, a firm's payoff becomes

$$Tw_i - e_i = T\frac{1}{2} - T\frac{r(\hat{c},\hat{c})}{4},$$

which is positive provided $r(\hat{c}, \hat{c}) < 2$. The existence condition of a unique fixed point $r(T/2, T/2) < 2$ will thus agree with that of a symmetric pure equilibrium $r < 2$ and $c < T/2$, necessary for positive firm payoffs.

Proposition 1 (Pure strategy equilibria) *In a lobbying contest with CSR,*

1. *A symmetric pure equilibrium exists for* $r(\frac{T}{2}, \frac{T}{2}) \in [0, 2]$.
2. *Firms' equilibrium expenditure levels are given by the unique solution of the fixed point problem* $e = \hat{c} = \frac{Tr(\hat{c},\hat{c})}{4}$
3. *Firms' equilibrium payoffs equal* $\frac{T(2 - r(\hat{c},\hat{c}))}{4}$.

As we noticed earlier, the nature of equilibrium in lobbying with CSR hinges on how much firms are willing to relax their competition by investing in CSR. The upper bound on a profitable CSR investment in a symmetric equilibrium is given by $T/2$:

If $r(\frac{T}{2}, \frac{T}{2}) > 2$, a firm would need to invest more than $c = T/2$ in CSR to alleviate

[15] The Hessian of second-order effects is developed in the Appendix.

competition to the point where $r < 2$. As a result, a higher firm's payoff is attainable with zero expenditure – resulting by standard logic in a symmetric mixed equilibrium, where expenditure is randomized over $[0, T]$.[16]

As a novelty, with CSR investment affecting the contest outcome, the composition of expenditure, however, depends on its total level e. Namely, we find a firm has an incentive to intensify competition (increase r) when its expenditure is high and alleviate competition (decrease r) when its expenditure is low.[17]

Assuming a level of spending e_i and a distribution F of expenditure e_j, the marginal benefit of transferring expenditure from DM-lobbying to CSR is

$$T \int_0^T \frac{e_i^r e_j^r}{\left(e_i^r + e_j^r\right)^2} \underbrace{\log\left(\frac{e_i}{e_j}\right)}_{><0} \underbrace{\frac{\partial r}{\partial c_i}}_{<0} dF(e_j).$$

The effect is clearly positive for low levels of expenditure ($e_i \in [0, \epsilon)$) and negative for high levels of expenditure ($e_i \in (T - \epsilon, T]$), for "small" $\epsilon > 0$.

Proposition 2 (Mixed strategy equilibria) *In a lobbying contest with CSR,*

1. *A symmetric mixed equilibrium exists for* $r(\frac{T}{2}, \frac{T}{2}) > 2$.
2. *Firms' equilibrium expenditures are randomized over infinite (countable or uncountable) support in* $[0, T]$.[18]
3. *Firms' equilibrium payoffs equal zero.*

Corollary 1 *recognizing CSR investments in lobbying will (i) relax competition, (ii) lower expenditure, and (iii) shift expenditure* e_i *from DM-lobbying* l_i *to CSR-lobbying* c_i.

To sum up, our analysis suggests that the application of CSR in lobbying improves social welfare if either of the following two conditions hold: DM-lobbying spending is socially wasteful or CSR-lobbying is more beneficial than DM-lobbying. The former condition is satisfied, e.g., if the authority values DM-lobbying at αl_i for $\alpha < 1$.[19] The latter condition is fulfilled, e.g., if the total social value of CSR-lobbying is βc_i for $\beta > \alpha$.

Figure 4 depicts the effects of CSR on payoffs for different r for $T = 1$.[20] When the

[16] Extending the analysis of Ewerhart (2015), the support is non-convex in $[0, T]$ for $r(T/2, T/2) > 2$ and, by the standard analysis of APA, the interval $[0, T]$ itself for $r(T/2, T/2) \to \infty$.

[17] Similar to what we describe in the more tractable framework of Section 6.

[18] With cutoffs $\hat{e}_c < \hat{e}_l \in (0, T)$ such that $e_i = c_i$ for $e_i < \hat{e}_c$ and $e_i = l_i$ for $e_i > \hat{e}_l$, and CSR investment increasing in $(\hat{e}_c, \hat{e}_l)$ to support mixing.

[19] Assuming DM-lobbying has no other social value than its valuation to DM.

[20] To make the comparison meaningful, noise is r in the contest exclusive of CSR and $r(c_i, c_j) = r - c_i - c_j$ in the contest inclusive of CSR, where r gives the *basic noise* parameter.

competition among firms is milder ($r(\frac{T}{2}, \frac{T}{2}) < r(0,0) < 2$), CSR-lobbying allows firms to relax contest competition while decreasing their expenditure to increase their profits. When the competition becomes fiercer ($r(\frac{T}{2}, \frac{T}{2}) < 2 < r(0,0)$), CSR enables relaxing the otherwise strong competition to the point that firms derive positive profit.[21]

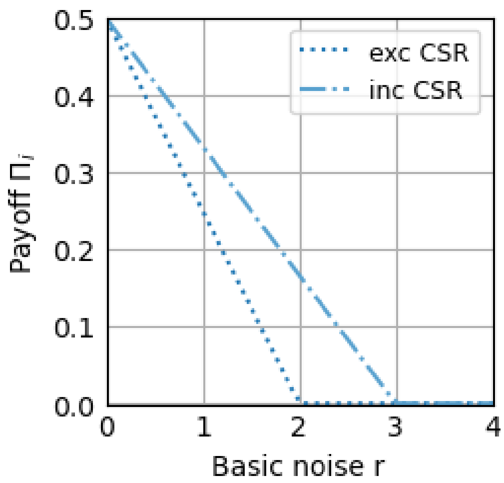


Figure 1: Payoff effect of CSR.

Broadly speaking, our equilibrium analysis demonstrates that endorsing CSR-lobbying in public choice crowds out more wasteful DM-lobbying spending. The mechanism for why this occurs is as follows: DM-lobbying and CSR-lobbying are substitutes in the strategy of the firm, since they both unilaterally increase the probability of winning the contest. However, a higher expenditure in CSR-lobbying relative to DM-lobbying increases the noise in the contest, thereby making the success probability less responsive to expenditures. This results in firms lowering total equilibrium expenditure. Therefore, the firms strategically substitute DM-lobbying expenditures with CSR ones, akin to strategic consumer obfuscation studied by Ellison and Wolitzky (2012), which here leads to lower total lobbying spending in equilibrium. By standard reasoning that lobbying is wasteful, we thus conclude that CSR is welfare-improving.

4 Sequential lobbying contests

Our previous analysis assumes that firms make decisions about their entire lobbying budget at the same time by, e.g., convening to a meeting that allows them to simultaneously set both DM-lobbying expenditure and CSR-lobbying investment. However, if CSR-lobbying involves a fixed cost absent from DM-lobbying, a more reasonable assumption might be to think that CSR-lobbying is fixed at stage 1 and DM-lobbying only later at stage 2. This game can be solved by backward induction. To distinguish it from a

[21] If $r(\frac{T}{2}, \frac{T}{2}) > 2$, CSR has no effect on firms' payoffs, which remain fixed at zero.

simultaneous contest, where equilibrium strategies are referenced by a hat, we denote by tilde the choices in a sequential contest.

Stage 2. In stage 2, symmetric CSR-lobbying investments $\tilde{c}$ are fixed and common knowledge. Our previous analysis then shows that, if a pure equilibrium is sustainable, optimal DM-lobbying is given by

$$\tilde{e} = \tilde{l} + \tilde{c} = \frac{Tr(\tilde{c}, \tilde{c})}{4}$$

and the related profits are

$$\tilde{\pi} = \frac{T(2 - r(\tilde{c}, \tilde{c}))}{4}.$$

Stage 1. This information is incorporated into the expectations of firms in stage 1. The problem of a firm thus becomes that of choosing the optimal $\tilde{c}$ that maximizes the following objective function

$$\max_c \frac{T(2 - r(c))}{4} - c.$$

By solving the first-order and second-order conditions, we thus obtain that optimal CSR-lobbying is characterized by

$$-\frac{\partial r(\tilde{c}, \tilde{c})}{\partial \tilde{c}} = \frac{4}{T}.$$

This contrasts with the previous case where DM-lobbying and CSR-lobbying were determined simultaneously.

The main differences are that now firms invest in both DM- and CSR-lobbying and, since they invest less in CSR-lobbying, they invest more in lobbying in total because lobbying competition remains fiercer. The details of the mechanism are as follows. As the optimal DM-lobbying is the residual $\tilde{l} = \frac{Tr(\tilde{c},\tilde{c})}{4} - \tilde{c}$, we observe that firms invest in DM-lobbying less in industries where they invest more in CSR-lobbying. The optimal CSR-lobbying is also determined differently, changing from being determined by the fixed point condition $\frac{r(\tilde{c},\tilde{c})}{\tilde{c}} = \frac{4}{T}$ (average effect of CSR associated with optimal expenditure) in a simultaneous contest to being characterized by the differential equation $-\frac{\partial r(\tilde{c},\tilde{c})}{\partial \tilde{c}} = \frac{4}{T}$ (marginal effect of CSR associated with optimal expenditure) in a sequential contest.

To proceed, we suppose $-\frac{\partial r(c,c)}{\partial c}$ is decreasing in c. Because the marginal effect of CSR is decreasing, the average effect of CSR exceeds the marginal one, i.e., $-\frac{\partial r(\tilde{c},\tilde{c})}{\partial \tilde{c}} < \frac{r(\tilde{c},\tilde{c})}{\tilde{c}}$. We thus observe that firms choose lower investment in CSR-lobbying if they are able to *commit* to CSR, but not to DM-lobbying, being determined later. Namely, if commitment is impossible, a firm has the previously delineated incentive to decrease total spending while increasing CSR. Thereby, the incentives for CSR-lobbying are weaker in a sequential contest than in a simultaneous contest. Because the contest is hence less noisy, total lobbying expenditure will increase. This will be to the detriment of firm profits. Figure 2a depicts the marginal and average effects of CSR on contest noisiness.

Result 1 *Suppose* $-\frac{\partial r(c,c)}{\partial c}$ *is decreasing. Sequential CSR- and DM-lobbying differs from simultaneous CSR- and DM-lobbying as follows:* $0 < \tilde{c} < \hat{c}$, $\tilde{l} > \hat{l} = 0$, $\tilde{e} > \hat{e} > 0$ *and* $0 < \tilde{\pi} < \hat{\pi}$.

5 Synergistic lobbying contests

If CSR enables a firm to expand its production, the profit from winning the favor, such as preferential tax treatment, can also depend directly on CSR. Investigating the implications of this possibility is straightforward in the previous sequential lobbying framework. We assume that the payoff of winning the favor is given by $T(c) > T$ with $T'(c) > 0$.

The problem of choosing CSR-lobbying can thus be expressed as

$$\max_c \frac{T(c)(2 - r(c))}{4} - c.$$

which gives the following first-order condition

$$-\frac{\partial r(\bar{c}, \bar{c})}{\partial \bar{c}} = \frac{4}{T(\bar{c})} - \frac{2 - r(\bar{c}, \bar{c})}{\bar{c}} T'(\bar{c}) \frac{\bar{c}}{T(\bar{c})} =: \frac{4}{T(\bar{c})} - L(\bar{c}) e_T(\bar{c}). \tag{6}$$

where the equilibrium values are denoted by bar.

Expected profits remain positive presuming $r(\bar{c}, \bar{c}) < 2$. Leveraging on our previous analysis, we thus see that synergies unambiguously increase CSR for decreasing $-\frac{\partial r(c,c)}{\partial c} > 0$. Both a *competition-alleviating* and a *competition-intensifying* effect are present.

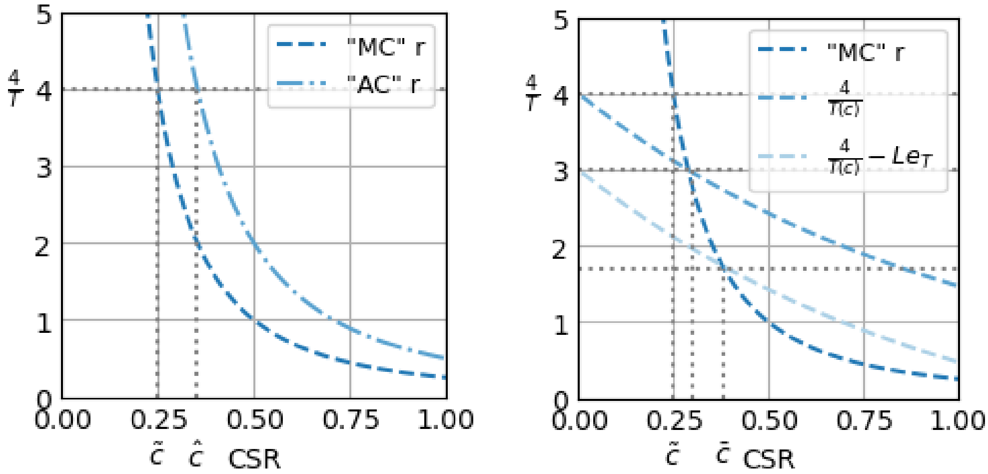
The competition-alleviating effect is incorporated in $\frac{4}{T}(\bar{c}) < \frac{4}{T}$, on the *rhs* of Eq. (6). It demonstrates that firms have augmented incentives to invest in CSR-lobbying, which alleviates competition, because the prize is larger. The working channel is the same as for the previously covered sequential lobbying.

The competition-intensifying effect is represented by $-L(\bar{c})e_T(\bar{c}) < 0$ on the *rhs* of Eq. (6). It captures the incentive to increase CSR because it elevates the prize T. The incentive is amplified by the profit margin, $L(c) := \frac{2-r(c,c)}{c}$, and by the elasticity of prize to investment in CSR, $e_T(c) := T'(c)\frac{c}{T(c)}$; further analysis thus requires additional information on L and e_T. For simplicity we proceed by assuming that $L(c) = 1/e_T(c)$, keeping $L(c)e_T(c)$ constant. Both effects are jointly illustrated by Figure 2b(b).

Interestingly, due to the opposite effects of increasing CSR on competition we here observe that the effect on total lobbying spending is ambiguous. This is shown by $\bar{e} = \bar{c} + \bar{l} = \frac{T(\bar{c})r(\bar{c},\bar{c})}{4}$, where $T(c)$ is increasing and $r(c)$ decreasing. As a result, DM-lobbying may either contract or augment while CSR-lobbying expands.

Result 2 *Presume* $-\frac{\partial r(c,c)}{\partial c}$ *is decreasing and* $-L(c)e_T(c)$ *constant. Synergistic CSR- and DM-lobbying differs from sequential CSR- and DM-lobbying as follows:* $0 < \tilde{c} < \bar{c}$, $\tilde{l} \lesseqgtr \bar{l}$, $\bar{e} \lesseqgtr \tilde{e} > 0$ *and* $0 < \tilde{\pi} < \bar{\pi}$.

We thus see that CSR investment is particularly lucrative to a firm when it plays the dual role of serving as a lobbying strategy and scaling up the stakes in a contest – a highly likely scenario. While total lobbying may well increase, in contrast to previous cases, prize increases boost profit and welfare. This adds to our previous analysis by demonstrating that CSR may be beneficial in lobbying contests not only for its (i) *non-productive* role in reducing rent dissipation but also due to its (ii) *productive* role in amplifying contest value.



(a) Sequential lobbying.

(b) Synergistic lobbying.

Figure 2: The determination of optimal CSR-lobbying ($T = 1$).

6 Uncertainty about competition in contests with CSR

So far we have only studied contests where the noise r generated by CSR-lobbying remains common knowledge. In many real contests, however, this is very unlikely – particularly as rival firms may also have difficulties in evaluating CSR investments. To consider the effects of CSR in a simple framework where uncertainty about the CSF affects spending, we next move to a setting where the firms expect that $r = 0$ with probability δ and $r = \infty$ with probability $1 - \delta$.[22]

As it turns out, this gives rise to a highly tractable setting, generating precise predictions

[22] An interesting variation is developed by Feng (2020), who studies optimal disclosure of Tullock contest r by a better informed authority.

about, e.g., (i) how CSR investments differ across the distribution of expenditure, and (ii) how CSR investments vary between a more and less efficient firm, and thus the welfare properties of contests designed to improve efficiency in regulated markets. As in the previous sections, we continue to suppose that CSR increases the randomness of the contest. Thus, δ is presumed to increase in (c_i, c_j), i.e., $\frac{\partial \delta}{\partial c_i} = \frac{\partial \delta}{\partial c_j} < 0$.

To further simplify the analysis, we also assume that, if neither firm invests in CSR, the winner is chosen via an APA, i.e., $\delta(0,0) := \delta_0 = 0$.[23] However, if either firm invests in CSR, the winner is determined partly at random and partly through an APA. The probability of a draw is thus δ_1 if one firm invest in CSR and δ_2 if both firms invest in CSR, where $\delta_1 < \delta_2$.[24] Random winner selection captures the difficulty of valuing diverse CSR investments.[25]

Table 1 shows δ as a function of CSR investments.

		c_j L	c_j H
c_i	L	δ_0	δ_1
c_i	H	δ_1	δ_2

Table 1: $\delta(c_i, c_j)$

Above, c_i is classified as H ("high") for positive values of CSR $c_i > 0$ and L ("low") for zero investment in CSR $c_i = 0$.

The profit of a firm can thus be written as follows.

$$\Pi_i(e_i, e_j, c_i, c_j) = \begin{cases} T\left(\delta(c_i, c_j)\frac{1}{2} + (1 - \delta(c_i, c_j))\right) - e_i, & \text{if } e_i > e_j, \text{ for } j \neq i \\ T\delta(c_i, c_j)\frac{1}{2} - e_i, & \text{if } e_i < e_j, \text{ for } j \neq i \\ T\frac{1}{2} - e_i, & \text{if } e_i = e_j, \text{ for } j \neq i. \end{cases} \tag{7}$$

In the language of contest theory, the CSF corresponds to a contest with a winning prize of $W_i = T(1 - \delta/2)$ and a consolation prize of $C_i = T\delta/2$ (Barut and Kovenock, 1998). The total expenditure level e_i and the division between, l_i and c_i, are chosen simultaneously by firms.[26]

[23] The assumption is not necessary for the results but shortens payoff formulae.

[24] In equilibrium, a firm only invests either in DM-lobbying or in CSR-lobbying.

[25] This section is likely interesting to market economists as it exemplifies the close linkages between our analysis of endogenous noise in lobbying and price obfuscation in the spirit of Carlin (2009), where obfuscation in financial product markets increases the number of uninformed consumers who choose the firm at random, irrespective of prices. The natural connection between APA lobbying contests and the models of Varian (1980) style price competition is established in Baye et al. (1992) and Baye et al. (1996).

[26] Contest theorists should note that the above CSF is essentially a noisy APA, which is shown in Michelsen (2022) to satisfy the axioms of Skaperdas (1996).

The payoff can also be expressed more compactly as

$$\Pi_i(e_i, e_j, c_i, c_j) = T\left(\delta(c_1, c_2)\frac{1}{2} + (1 - \delta(c_1, c_2))F_j(e_i)\right) - e_i, \tag{8}$$

where F_j denotes the (cumulative) distribution of e_j.[27]

Lemma 3 shows that under competition uncertainty both the size and the type of expenditure are randomized in equilibrium.

Lemma 3 *The equilibrium is in randomized strategies:*

1. *Firms mix their total expenditure strategies e_i over $[0, \bar{e}]$.*
2. *Firms mix their CSR investment strategies c_i over $\{L, H\}$.*

Intuitively, firms have incentives to beat their competition in expenditure because the probability that the contest is competitive is positive, $1 - \delta > 0$. Yet, reducing spending is also attractive as expenditure is a sunk cost. Furthermore, because the possibility of no competition, $\delta \geq 0$, remains, firms have a chance to obtain a positive profit by spending nothing. These opposite expenditure incentives result in randomized expenditure strategies in equilibrium.

Interestingly, this entails that firms also benefit from randomizing their investments in CSR. On the one hand, when firms spend at the upper end of the expenditure distribution, they benefit from stronger competition (decreased δ). Thus, firms optimally cut investment in CSR. On the other hand, when expenditure lies at the lower end of the distribution, relaxing competition benefits firms (increasing δ). This encourages positive investments in CSR.

This shows that uncertainty about competition $\delta_3 > \delta_2 > 0$ alone gives rise to a somewhat similar mixed equilibrium as intensive competition $r(\frac{T}{2}, \frac{T}{2}) > 2$ in our benchmark lobbying contest.

Note that, for $\delta_0 = 0$[28], if no CSR investments are made, the contest reduces to an APA, where at least one firm makes zero expected profit. This is because it competes away its profits in the contest. However, if either firm invests in CSR, the choice of the winner is partly random because comparing firms becomes more difficult. This entails that a firm can expect a positive profit even when it spends almost nothing, i.e., for $e_i = \epsilon \to 0$.[29] Thus, at least one of the firms has an incentive to alleviate competition by investing in CSR with a positive probability.[30]

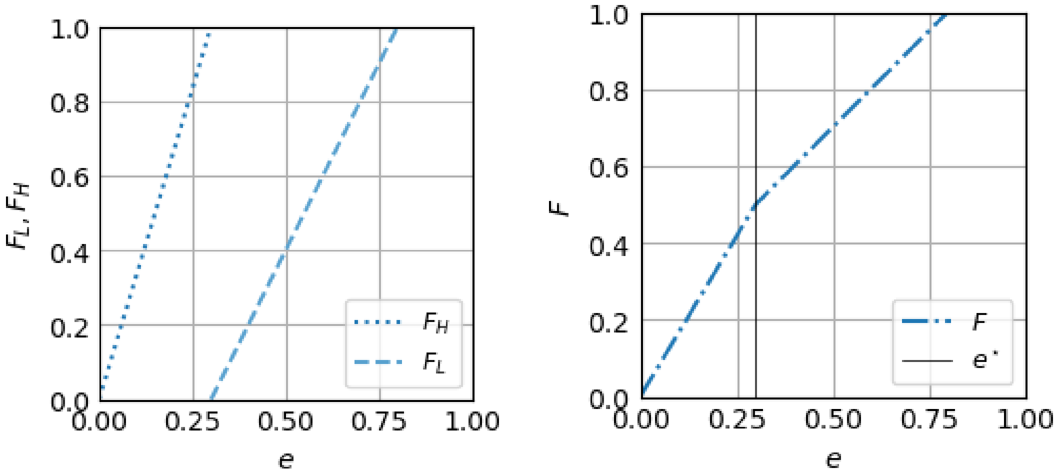
[27]Payoffs (7) and (8) are equivalent providing $e_i = e_j$ occurs with probability zero.
[28]This convenience assumption is not needed for Lemma 4 nor our economic results.
[29]We could think that a firm simply vividly advertises its adherence to law.
[30]Investing $(c_i, c_j) = (H, H)$ with probability one is an equilibrium iff $\delta_1 = \delta_2$.

It might be more natural to assume that at least some positive CSR spending $\underline{c} > 0$ is required to qualify as a contribution to the lobbying contest. Then firms need to tradeoff the benefit of supplying this CSR-lobbying level and the cost of supplying zero DM-lobbying level. If $\underline{c}$ remains low enough, the lowest-spending firms offer this level of CSR-lobbying, $e = c = \underline{c}$, and the highest-spending ones provide a higher DM-lobbying level, $e = l > \underline{c}$. Otherwise, all firms apply only DM-lobbying, ranging from zero to $\bar{e}$.

As previously discussed, the total expenditure level e_i and the division between, l_i and c_i, are chosen simultaneously by firms. However, because equilibrium strategies are randomized, it is often convenient to study conditionals, i.e., equilibrium expenditure $e_i|c_i$ for a given level of CSR investment (e.g., Figure 3(a)) and equilibrium investment $c_i|e_i$ in CSR conditional on expenditure (e.g., Figure 3(b)) – without assuming that one decision precedes the other. We find that high (low) total lobbying expenditure is associated with low (high) CSR investment in equilibrium.

Lemma 4 *There is a level $e^\star$ such that in equilibrium a firm chooses $c_i = L$ for $e_i > e^\star$ and $c_i = H$ for $e_i < e^\star$. The conditionals $F_L := F(e|c = L)$ and $F_H := F(e|c = H)$ are supported on $S_H = [\underline{e}_H, e^\star] = [0, e^\star]$ and $S_L = [e^\star, \bar{e}_L]$.*

The expenditure distributions (23) and (24) in the Appendix are illustrated in Figure (3)



(a) Conditionals F_H and F_L. (b) Unconditional F and $e^\star$.

Figure 3: Expenditure distribution ($T = 1, \delta_1 = 0.2, \delta_2 = 0.4$).

Proposition 3 *If $T_1 = T_2$, there exists a unique symmetric mixed equilibrium where firms' payoffs are $\frac{T}{2}\left[\delta_1 + (\delta_2 - \delta_1)\frac{\delta_1}{\delta_2}\right]$.*

Firms' mixed strategies in c_i are given by (20) and e_i by (23) for c_L and by (24) for c_H.

The logic behind the payoff $\frac{T}{2}\left[\delta_1 + (\delta_2 - \delta_1)\frac{\delta_1}{\delta_2}\right]$ is that a CSR-centered strategy allows a firm to win the contest with negligible expenditure. CSR increases the likelihood of a non-competitive contest outcome. When CSR investment is recognized, the contest is non-competitive with probability δ_1, when the other firm does not invest in CSR (probability δ_1/δ_2), and with probability δ_2, when the other firm is also investing in CSR (probability $(\delta_2 - \delta_1)/\delta_2$).

Note that $\left(\delta_1 + (\delta_2 - \delta_1)\frac{\delta_1}{\delta_2}\right)\frac{1}{2} = \delta_1\left(2 - \frac{\delta_1}{\delta_2}\right)\frac{1}{2}$, which shows that the probability of no competition is a downward opening parabola with roots at $\delta_1 = 0$ and $\delta_1 = 2\delta_2$. For a given level of δ_2, the payoff of a firm is thus maximized at $\delta_1 = \delta_2$, the upper bound of $\delta_1 < \delta_2$. Likewise, the payoff of a firm is higher if δ_2 is larger for a given level of $\delta_1 < \delta_2$.[31] The economic intuition for the result is that firms benefit from a reduced likelihood of competition.

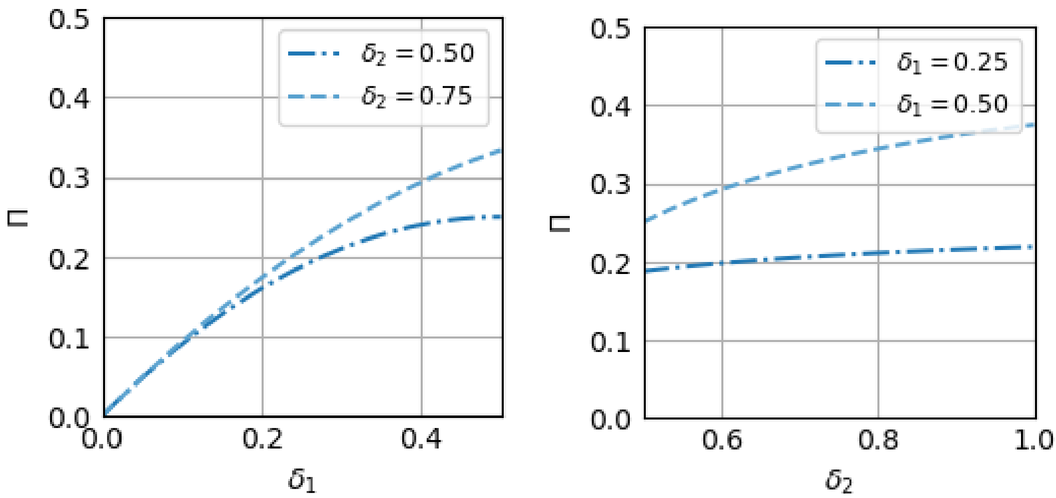


Figure 4: Payoff effects of δ's ($T = 1$).

To summarize, our findings show that

1. Uncertainty about competition results in a mixed equilibrium, where expenditure is randomized over an interval.

2. Firms deploy investments in CSR strategically to alleviate competition when their other contributions are low.

7 Concluding discussion

In this paper, we study how the possibility of participating in lobbying contests with contributions that differ in their observability and comparability affects welfare. The

[31]Payoffs are maximized at $(\delta_1, \delta_2) = (1, 1)$, which gives $r = 0$ (random contest), and minimized at $(\delta_1, \delta_2) = (0, 0)$, giving $r = \infty$ (APA).

leading examples outlined in the paper involve DM-lobbying, targeting a decision-maker more or less directly, and CSR-lobbying, which serves a more general advertising function.

As our main literature contribution, we demonstrate that harnessing CSR for lobbying purposes alleviates competition among firms by transferring the nexus of lobby competition from a well-specified monetary dimension to a more obscure qualitative dimension. Because firms are better off when they no longer need to compete their profits away in lobbying, our paper helps to reconcile why companies may invest significant amounts in CSR "corporate beauty contests".

Our research can corroborate the firmly-held notion that the heart-warming CSR-lobbying is indeed the *socially more responsible lobbying mode* relative to the rent-seeking DM-lobbying. CSR-lobbying is shown to limit wasteful spending.[32] This channel through which CSR improves social welfare is novel, and vitally complements more typical explanations for why CSR investments are beneficial to society, such as positive externalities and the public good nature of CSR.

We close by discussing some alternative interpretations of our model and certain non-obvious literature precursors to the ideas elaborated more carefully in this work and in our other paper Hämäläinen and Zheng (2022).

7.1 Optimal contest design

Contest noise depends on firm strategies here. Understanding this, a designer may obviously want to design a different contest that better advances its goals. In Letina et al. (2023) the designer maximizes the sum of expenditures minus the prize. It is then shown that the optimal contest for two firms has intermediate noise level. This suggests for our framework that if the equilibrium noise is too large, DM is better off acknowledging CSR only up to a given (optimal) point.

7.2 Transparency and obfuscation in lobbying

Our analysis could be exploited to shed light on the desirability of transparency in lobbying. Lobbying can take many forms, some being more transparent.[33] Monetary contributions to political campaigns are perhaps the easiest for their targets to compare.[34]

[32]Firms could invest their lobbying money either internally or externally, while the DM might just enjoy more consumption (e.g., a free lunch with the lobbyist) but not necessarily in its preferred way (e.g., the lunch venue/company/timing).

[33]Many public acts have been pursued to make lobbying spending more transparent. The EU transparency register for lobbyists is just one such example.

[34]At the same time, these might not be the easiest for outsiders to compare. Increasing private transparency can decrease public transparency.

On the other hand, a firm may also lobby, e.g., by providing the authority valuable but potentially biased information or advertising the features of its product.

To map the application to our model, we can thus study a case where a firm's total lobbying spending e_i is divided between more transparent lobbying l_i and less transparent lobbying c_i. Building on our previous analysis, we conclude that (i) rival firms have incentives to obfuscate their lobbying, (ii) transparency is associated with higher lobbying spending, and (iii) more efficient firms are more transparent in equilibrium.

This can be easily understood by noting that lobbying is just a form of selling, not to consumers but to politicians or regulators. Similar to consumer markets, lobbyists thus have incentives to economize on the costs of competition by obfuscating their lobbying strategies. As a consequence, in so far as lobbying is wasteful, opacity can be welfare improving – serving the purpose of limiting rent-seeking or reducing bribe payments.[35] Interestingly, statistics show that lobbying expenditure multiplied in 1998–2021, from \$1.45 to \$3.73 billion in the US, at the same time as technological advances increased transparency.[36] This begs for careful judgment from transparency regulators.

7.3 Procurement with price obfuscation

Previous literature documents that obfuscation in consumer markets is prevalent, ranging from the use of complex price frames to multiproduct price proliferation (Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013; Petrikaitė, 2018; Hämäläinen, 2018, 2022). Similar practices are also used in procurement.

For example, a multiproduct firm renting industrial equipment may win a procurement with a menu of rental prices and an estimate of the total payment based on, say, X hours on a small excavator and Y hours on a large excavator – a common market practice. However, the hours can be purposely distorted to the cheaper machine, which might turn out to be, e.g., "unavailable", less convenient, or less efficient to use. Thus, the public may, in reality, need to rent the cheaper digger for $X - Z$ hours and the more expensive one for $Y + Z$ hours, thus resulting in a much higher bill.

To apply our model to procurement obfuscation, we can consider a framework, such as Carlin (2009), where firms choose a price $p = -l$ and a (free-of-cost) frame c_L, c_H. Frame c_L is simple (e.g., the final price) while c_H is complex (e.g., a menu of fees). Carlin (2009) finds that firms benefit from choosing a simple frame for relatively low prices but a complex frame for higher prices. The additional results we derive demonstrate that incentives to obfuscate hinge on the possible complementarity between pricing and

[35] Apart from alleviating competition, the shadiness of lobbying could also serve the purpose of hiding influence chains and allowing secret dealings, which is generally not desirable.

[36] See, e.g., `opensecrets.org` and`statista.com` (accessed Sep 7, 2022).

framing. The Appendix shows that efficient firms obfuscate less.

7.4 Procurement with quality competition

Our model could also be interpreted as a procurement auction, in which the expenditure assumes the *score* form $e_i = q_i - p'_i$, where $q_i = c_i$ is the firm's quality investment and $p'_i = -l_i$ is its price mark-up proposal. Presuming that qualities are more difficult to compare than prices, the application immediately maps to our model (for a more usual treatment of score auctions, see Che (1993) for winner-pay quality bids and Kovenock and Lu (2020) for all-pay quality bids).[37]

Our earlier findings suggest that (i) firms have incentives to compete on quality rather than on prices to alleviate competition, (ii) higher quality is generally associated with higher prices, and (iii) more efficient firms are more likely to compete on prices. A tax-sensitive authority may thus instead want to employ a scoring rule that places a higher weight on price.

The closest counterpart to this idea in the related literature is perhaps Burguet and Che (2004). They observe that the optimal scoring rule in corrupt contests de-emphasizes quality relative to price. Incidentally, corruption in their model acts as a confusing ingredient, as CSR here, although corruption is generally harmful while CSR is regarded as positive for society.

No data are associated with this article.

Author contribution clarification. S. Hämäläinen: Idea, analysis, writing, and figures in Sections 1-7. S. Hämäläinen and S. Y. Zheng jointly: proofs of Lemma 3 and Proposition 5, analysis in Appendix A. S. Y. Zheng: proof of Lemma 4, figures in Appendix A.

Appendix

Proof for Proposition 1 (missing details)

We derive the Hessian

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$

[37]The immediate connection between lobbying and advertising in procurement is discussed more in Burguet and Sákovics (2022).

First, we differentiate (4) w.r.t. l_i

$$a = T\frac{r(r-1)e_i^{r-2}e_j^r}{\left(e_i^r+e_j^r\right)^2} - 2T\frac{r^2e_i^{2(r-1)}e_j^r}{\left(e_i^r+e_j^r\right)^3} \tag{9}$$

Then, we differentiate (5) w.r.t. l_i

$$\begin{aligned}
b =& T\frac{r(r-1)e_i^{r-2}e_j^r}{\left(e_i^r+e_j^r\right)^2} - 2T\frac{r^2e_i^{2(r-1)}e_j^r}{\left(e_i^r+e_j^r\right)^3}+\\
&\left(T\frac{re_i^{r-1}e_j^r}{\left(e_i^r+e_j^r\right)^2} - 2T\frac{re_i^{2r-1}e_j^r}{\left(e_i^r+e_j^r\right)^2}\right)\log\left(\frac{e_i}{e_j}\right)\frac{\partial r}{\partial c_i}+\\
&T\frac{e_i^re_j^r}{\left(e_i^r+e_j^r\right)^2}\frac{e_j}{e_i}\frac{\partial r}{\partial c_i}
\end{aligned} \tag{10}$$

Finally, we differentiate (5) w.r.t. c_i

$$\begin{aligned}
c =& T\frac{r(r-1)e_i^{r-2}e_j^r}{\left(e_i^r+e_j^r\right)^2} - 2T\frac{r^2e_i^{2(r-1)}e_j^r}{\left(e_i^r+e_j^r\right)^3}+\\
&T\left(\frac{re_i^{r-1}e_j^r}{\left(e_i^r+e_j^r\right)^2} - 2\frac{re_i^{2r-1}e_j^r}{\left(e_i^r+e_j^r\right)^2}\right)\log\left(\frac{e_i}{e_j}\right)\frac{\partial r}{\partial c_i}+\\
&T\frac{e_i^re_j^r}{\left(e_i^r+e_j^r\right)^2}\frac{e_j}{e_i}\frac{\partial r}{\partial c_i}+T\frac{e_i^re_j^r}{\left(e_i^r+e_j^r\right)^2}\log\left(\frac{e_i}{e_j}\right)\frac{\partial^2 r}{\partial c_i^2}+\\
&T\left[1+rlog(e_i)+rlog(e_j)-2r\frac{e_i^rlog(e_i)+e_j^rlog(e_j)}{e_i^r+e_j^r}\right]\frac{e_i^{r-1}e_j^r}{\left(e_i^r+e_j^r\right)^2}\frac{\partial r}{\partial c_i}+\\
&T\left[log(e_i)+log(e_j)-2\frac{e_i^rlog(e_i)+e_j^rlog(e_j)}{e_i^r+e_j^r}\right]\frac{e_i^re_j^r}{\left(e_i^r+e_j^r\right)^2}\log\left(\frac{e_i}{e_j}\right)\frac{\partial r}{\partial c_i}+\\
&T\frac{e_i^re_j^r}{\left(e_i^r+e_j^r\right)^2}\log\left(\frac{e_i}{e_j}\right)\frac{\partial^2 r}{\partial c_i^2}
\end{aligned} \tag{11}$$

In a symmetric equilibrium the elements will collapse into

$$\begin{aligned}
a =& T\frac{-r}{4e^2} < 0,\\
b =& T\frac{-r}{4e^2} + T\frac{1}{4}\frac{\partial r}{\partial c},\\
c =& T\frac{-r}{4e^2} + T\frac{1}{4e}\frac{\partial r}{\partial c},
\end{aligned}$$

which gives

$$
\begin{aligned}
(ac-b^2)/T^2 &= \frac{-r}{4e^2}\left(\frac{-r}{4e^2}+\frac{1}{4e}\frac{\partial r}{\partial c}\right)-\left(\frac{-r}{4e^2}+\frac{1}{4}\frac{\partial r}{\partial c}\right)\left(\frac{-r}{4e^2}+\frac{1}{4}\frac{\partial r}{\partial c}\right)\\
&= \frac{r^2}{4^2e^4}+\frac{-r}{4^2e^3}\frac{\partial r}{\partial c}-\frac{r^2}{4^2e^4}+2\frac{r}{4^2e^2}\frac{\partial r}{\partial c}-\frac{1}{4^2}\left(\frac{\partial r}{\partial c}\right)^2\\
&= \frac{r}{4^2e^2}\frac{\partial r}{\partial c}-\frac{1}{4^2}\left(\frac{\partial r}{\partial c}\right)^2<0.
\end{aligned}
$$

Negative principal minors show that we have a saddle point in any symmetric (candidate) equilibrium for interior values of l and c. Thus, a maximum is located at the lowermost boundaries for at least one class of expenditure: $e=l\geq c=0$ or $e=c\geq l=0$ ($l>T$ and $c>T$ are obviously suboptimal). We sketch a proof in the main text for why a profitable deviation from a tentative equilibrium with positive lobbying always exists. Comparing the payoffs in the candidate equilibria, $\frac{T(2-r(0,0))}{4}$ in the former case and $\frac{T(2-r(\hat{c},\hat{c}))}{4}$ for $\hat{c}=\frac{Tr(\hat{c},\hat{c})}{4}>0$ in the latter, we can also see that (only) investing in CSR dominates (only) investing in lobbying. □

Proof for Proposition 2 (missing details)

We can approach the problem by initially considering finite strategy spaces $(c_i,l_i)\in\left\{0,\frac{1}{k}T,...,\frac{k-1}{k}T,T\right\}^2$ for k, a large positive integer. In this finite symmetric payoff game, the existence of a symmetric mixed equilibrium is guaranteed by Dasgupta and Maskin (1986).

A firm's payoff, obtained from using strategy $(c_i,l_i)\in\left\{0,\frac{1}{k}T,...,\frac{k-1}{k}T,T\right\}^2$, can be written as follows

$$
\Pi(c_i,l_i)=\sum_{m=0}^{k}\sum_{n=0}^{k}p_c\left(\frac{k-m}{k}T\right)p_l\left(\frac{k-n}{k}T\right)w_i(c_i,l_i;\frac{k-m}{k}T,\frac{k-n}{k}T)T-c_i-l_i,
$$

where p_c and p_l denote the firms' (symmetric) probability distributions of c_i and l_i, respectively.

We embark to derive boundaries for equilibrium payoffs Π. In a symmetric mixed-strategy equilibrium, $p_c\left(c_i\right)p_l\left(l_i\right)>0$ implies

$$
\sum_{m=0}^{k}\sum_{n=0}^{k}p_c\left(\frac{k-m}{k}T\right)p_l\left(\frac{k-n}{k}T\right)w_i\left(c_i,l_i;\frac{k-m}{k}T,\frac{k-n}{k}T\right)=\frac{c_i+l_i+\Pi}{T},
$$

which gives the complementary-slackness condition

$$p_c\left(c_i\right) p_l\left(l_i\right)\left[\frac{c_i+l_i+\Pi}{T}-\sum_{m=0}^{k} \sum_{n=0}^{k} p_c\left(\frac{k-m}{k} T\right) p_l\left(\frac{k-n}{k} T\right) w_i\left(c_i, l_i ; \frac{k-m}{k} T, \frac{k-n}{k} T\right)\right]=0.$$

Part I. If we assume that $p_c(0)p_l(0) > 0$, we can immediately show that

$$\sum_{m=0}^{k} \sum_{n=0}^{k} p_c\left(\frac{k-m}{k} T\right) p_l\left(\frac{k-n}{k} T\right) w_i\left(0,0 ; \frac{k-m}{k} T, \frac{k-n}{k} T\right)= p_c(0) p_l(0) \frac{1}{2}=\frac{\Pi}{T}, \tag{12}$$

because $w_i(0,0;\frac{k-m}{k}T,\frac{k-n}{k}T) = 1/2$ for $m = n = k$ but $w_i(0,0;\frac{k-m}{k}T,\frac{k-n}{k}T) = 0$ for $m < k$ or $n < k$, for any $r \in (0,\infty)$. From the complementry-slackness condition, we also obtain that for $c_i = 0$ and $l_i = \frac{T}{k}$

$$\sum_{m=0}^{k} \sum_{n=0}^{k} p_c\left(\frac{k-m}{k} T\right) p_l\left(\frac{k-n}{k} T\right) w_i\left(0, \frac{1}{k} T ; \frac{k-m}{k} T, \frac{k-n}{k} T\right)= p_c(0) p_l(0)+\alpha \leq \frac{\Pi}{T}+\frac{1}{k}, \tag{13}$$

where $0 \le \alpha \le \frac{1}{2}$. Joining Eqs. (12) and (13) results in

$$\alpha \leq \frac{1}{k}+\frac{\Pi}{T}-p_c(0) p_l(0)=\frac{1}{k}+\frac{\Pi}{T}-2 \frac{\Pi}{T}=\frac{1}{k}-\frac{\Pi}{T}. \tag{14}$$

Satisfying the condition implies $\Pi \le T/k \to 0$ as $k \to \infty$.

Part II. If $p_c(0)p_l(0) = 0$, we focus instead on the highest values m' for which $p_c\left(\frac{k-m'}{k}T\right) p_l\left(\frac{k-n'}{k}T\right) > 0$. By complementary-slackness conditions, we can now show that

$$\Pi(\frac{k-m'}{k} T, \frac{k-n'}{k} T)=$$

$$\sum_{m=0}^{m'} \sum_{n=0}^{n'} p_c\left(\frac{k-m}{k} T\right) p_l\left(\frac{k-n}{k} T\right) \frac{\left(\frac{2k-n'-m'}{k} T\right)^{r\left(\frac{k-m'}{k} T, \frac{k-m}{k} T\right)}}{\left(\frac{2k-n'-m'}{k} T\right)^{r\left(\frac{k-m'}{k} T, \frac{k-m}{k} T\right)}+\left(\frac{2k-n-m}{k} T\right)^{r\left(\frac{k-m'}{k} T, \frac{k-m}{k} T\right)}}= \tag{15}$$

$$\frac{\Pi+\frac{k-m'}{k} T+\frac{k-n'}{k} T}{T}, \tag{16}$$

and

$$\Pi(\frac{k-m'}{k} T, \frac{k-n'-1}{k} T)=$$

$$\sum_{m=0}^{m'} \sum_{n=0}^{n'} p_c\left(\frac{k-m}{k} T\right) p_l\left(\frac{k-n}{k} T\right) \frac{\left(\frac{2k-n'+1-m'}{k} T\right)^{r\left(\frac{k-m'}{k} T, \frac{k-m}{k} T\right)}}{\left(\frac{2k-n'+1-m'}{k} T\right)^{r\left(\frac{k-m'}{k} T, \frac{k-m}{k} T\right)}+\left(\frac{2k-n-m}{k} T\right)^{r\left(\frac{k-m'}{k} T, \frac{k-m}{k} T\right)}} \leq \tag{17}$$

$$\frac{\Pi+\frac{k-m'}{k} T+\frac{k-n'+1}{k} T}{T}=\frac{\Pi+\frac{k-m'}{k} T+\frac{T}{k}+\frac{k-n'}{k} T}{T}. \tag{18}$$

In slightly simpler notation, (15)=(16) reads as

$$p_e(\underline{e})\frac{1}{2} + \underbrace{\cdots + p_e(T)\frac{\underline{e}^{r(\cdot)}}{\underline{e}^{r(\cdot)} + T^{r(\cdot)}}}_{=:\beta} = \frac{\Pi + \underline{e}}{T},$$

while (17)≤(18) gives us the following inequality

$$p_e(\underline{e})\frac{(\underline{e}+\frac{T}{k})^{r_0}}{(\underline{e}+\frac{T}{k})^{r_0}+\underline{e}^{r_0}} + \underbrace{\cdots + p_e(T)\frac{(\underline{e}+\frac{T}{k})^{r(\cdot)}}{(\underline{e}+\frac{T}{k})^{r(\cdot)}+T^{r(\cdot)}}}_{=:\gamma} \leq \frac{\Pi + \underline{e} + \frac{T}{k}}{T},$$

where $r_0 := r(\underline{e}+\frac{T}{k}, \underline{e}) > 2$ for large enough values of k as long as $\underline{e} < \frac{T}{2}$.

Solving $p_e(\underline{e})$ from the former equation and inserting $p_e(\underline{e})$ to the latter inequality gives

$$\gamma - 2\beta\frac{(\underline{e}+\frac{T}{k})^{r_0}}{(\underline{e}+\frac{T}{k})^{r_0}+\underline{e}^{r_0}} \leq \frac{\Pi + \underline{e} + \frac{T}{k}}{T} - 2\frac{\Pi+\underline{e}}{T}\frac{(\underline{e}+\frac{T}{k})^{r_0}}{(\underline{e}+\frac{T}{k})^{r_0}+\underline{e}^{r_0}} \tag{19}$$

The rhs of (19) is negative if

$$\begin{aligned}
\Pi + \underline{e} + \frac{T}{k} &< 2(\Pi+\underline{e})\frac{(\underline{e}+\frac{T}{k})^{r_0}}{(\underline{e}+\frac{T}{k})^{r_0}+\underline{e}^{r_0}} \\
\left(\left(\underline{e}+\frac{T}{k}\right)^{r_0} + \underline{e}^{r_0}\right)\left(\Pi+\underline{e}+\frac{T}{k}\right) &< 2(\Pi+\underline{e})\left(\underline{e}+\frac{T}{k}\right)^{r_0} \\
\underline{e}^{r_0}\left(\Pi+\underline{e}+\frac{T}{k}\right) &< \left(\Pi+\underline{e}-\frac{T}{k}\right)\left(\underline{e}+\frac{T}{k}\right)^{r_0}
\end{aligned}$$

The rhs is of this condition is increasing in Π. Thus, if the condition holds for $\Pi = \frac{T}{k}$, then it continues to hold for all $\Pi \geq \frac{T}{k}$. Assuming $\Pi = \frac{T}{k}$ results in the following inequality, which is easily proved to be satisfied for all $\frac{\underline{e}}{\frac{T}{k}} > 0$ for any $r_0 > 2$.

$$\left(\frac{\frac{\underline{e}}{\frac{T}{k}}}{1+\frac{\underline{e}}{\frac{T}{k}}}\right)^{r_0} < \frac{\frac{\underline{e}}{\frac{T}{k}}}{2+\frac{\underline{e}}{\frac{T}{k}}}$$

The lhs of (19) is non-negative if

$$\gamma - 2\beta\frac{(\underline{e}+\frac{T}{k})^{r_0}}{(\underline{e}+\frac{T}{k})^{r_0}+\underline{e}^{r_0}} \geq 0,$$

which holds if the term-wise conditions are satisfied for $x = \frac{T}{k}, \ldots, T$

$$\underbrace{\frac{(\underline{e}+\frac{T}{k})^{r_2}}{(\underline{e}+\frac{T}{k})^{r_2}+(\underline{e}+x)^{r_2}}}_{\leq 1/2} \geq 2 \underbrace{\frac{\underline{e}^{r_1}}{\underline{e}^{r_1}+(\underline{e}+x)^{r_1}}}_{\leq 1/2} \underbrace{\frac{(\underline{e}+\frac{T}{k})^{r_0}}{(\underline{e}+\frac{T}{k})^{r_0}+\underline{e}^{r_0}}}_{\geq 1/2}$$

where $r_2 \leq r_1 \leq r_0$. Because $x^r/(x^r+y^r)$ is increasing in r for $x > y$ and decreasing in r for $x < y$, and $\frac{(\underline{e}+\frac{T}{k})^{r_0}}{(\underline{e}+\frac{T}{k})^{r_0}+\underline{e}^{r_0}} \to 1/2$ for any r_0 as $k \to \infty$, it suffices to show that

$$\frac{(\underline{e}+y)^r}{(\underline{e}+y)^r+(\underline{e}+x)^r} > 2\frac{\underline{e}^r}{\underline{e}^r+(\underline{e}+x)^r}\frac{(\underline{e}+y)^r}{(\underline{e}+y)^r+\underline{e}^r}$$

for $0 < y \leq x$. Rearranging the inequality provides the following condition

$$\underline{e}^{2r} - \underline{e}^r(\underline{e}+y)^r - \underline{e}^r(\underline{e}+x)^r + (\underline{e}+y)^r(\underline{e}+x)^r > 0.$$

Because $-\underline{e}^r + (\underline{e}+y)^r > 0$, the condition is satisfied for all $x \geq y > 0$, provided it is satisfied for $x = y > 0$. Inserting x into y results in the square of a binomial

$$\underline{e}^{2r} - 2\underline{e}^r(\underline{e}+y)^r + (\underline{e}+y)^{2r} = (\underline{e}^r - (\underline{e}+y)^r)^2 > 0.$$

Altogether, we thus find that the requirement for (19) in Part II is $\Pi \leq T/k \to 0$ as $k \to \infty$, which is equal to the requirement for (14) in Part I.

Part III. It remains to show that $\underline{e} < T/2$. In a symmetric equilibrim, the profit of a firm that chooses $\underline{e} \geq T/2$ has the upper bound

$$p_e(\underline{e})\frac{T}{2} + (1-p_e(\underline{e}))T\alpha - \frac{T}{2} < 0$$

where $\alpha < 1/2$. Because the profit is negative, a firm can increase its profit by choosing $e = 0$, thus representing a profitable deviation.

Demonstrating the existence of an equilibrium in a continuous state space is now a simple limit taking exercise. An equilibrium can be shown to exist by checking the conditions presented in Theorem 6 of Dasgupta and Maskin (1986). □

Proof of Lemma 3

Part 1. The proof of the lemma follows the usual logic in standard all-pay auctions. Consider a candidate equilibrium with pure expenditure strategies (e_i, e_j) and equilibrium investments (c_i, c_j). First, if $e_i < e_j$, there is a profitable deviation for firm j to a lower expenditure level $e' \in (e_i, e_j)$, keeping δ constant, which reduces its costs without affecting its winning probability. Second, if $e_i = e_j$, there is a profitable deviation for firm i to a marginally higher expenditure level $e' > e_j$, increasing its probability of winning clearly, from 1/2 to $\delta(c_i, c_j)\frac{1}{2} + 1 - \delta(c_i, c_j)$, with only a slight positive effect on its cost.

Part 2. The proof is by contradiction. To begin, suppose that firms apply pure strategies (c_i, c_j), allowing to express the profit of a firm thus by

$$\Pi_i = T\left[\delta(c_i, c_j)\frac{1}{2} + (1 - \delta(c_i, c_j))F_j(e_i)\right] - e_i.$$

For completeness, we first delineate some basic properties of a firm's expenditure distribution. Denote the support of F_i by $S_i = cl\,\{e|f_i(e) > 0\}$.[38]

1. The upper and lower bounds of the supports must overlap. Namely, if $\sup S_j < \sup S_i$, the higher spender i can profitably lower its expenditure level from $e = \sup S_i$ to $e' = \sup S_j$ without affecting its winning probability. Further, if $\inf S_j < \inf S_i$, at least one of the firms profits from lowering expenditure: If there is $e \in (\inf S_j, \sup S_i)$ in S_j, then firm i benefits from lowering expenditure from e to $\inf S_j$ (w_i is unaffected). If there is no $e \in (\inf S_j, \sup S_i)$ in S_j, then firm j benefits from lowering expenditure from $\inf S_j$ to e (w_j is unaffected).

2. The supports must be dense and convex. Otherwise, the rival can decrease its spending from a point above an isolated point without affecting its winning probability. This together implies interval supports: $S_i = S_j = [\underline{e}, \overline{e}]$.

3. The supports cannot have mass points $F(e) - \lim_{x \to e-} F(x) = a(e) > 0$. If there were a mass point $e \in S_j$, firm i could increase its winning probability by $a(e) > 0$ by slightly increasing its expenditure from $e_i = e - \epsilon a(e)$ to $e_i = e + \epsilon a(e)$.

4. The lower bound of the support must equal zero. Having $\underline{e} > 0$ implies a firm can decrease its expenditure without a reduction in its winning probability. For example, a deviation from $e_i = \underline{e}$ to $e_i = \underline{e} - \epsilon$ raises the profit by ϵ.

We move to consider deviations in CSR.

5. Suppose firms apply strategy $(c_i, c_j) = (L, L)$ or $(c_i, c_j) = (L, H)$ with equilibrium expenditure $e_i = \epsilon \in S_i(c_i, c_j)$. Now, the profit of firm i for $(c_i, e_i) = (L, \epsilon)$ equals $T\left[\delta_n\frac{1}{2} + (1 - \delta_n)F_j(e_i)\right] - e_i \to T\delta_n\frac{1}{2}$ for $\epsilon \to 0$. Yet, the profit of firm i for $(c_i, e_i) = (H, \epsilon)$ equals $T\left[\delta_{n+1}\frac{1}{2} + (1 - \delta_{n+1})F_j(e_i)\right] - e_i \to T\delta_{n+1}\frac{1}{2}$ for $\epsilon \to 0$. Because $\delta_{n+1} > \delta_n$ for $n = 0, 1$, the deviations are profitable.

6. Suppose firms apply strategy $(c_i, c_j) = (H, H)$ with equilibrium expenditure $e_i =$

[38] Note that presuming that firms apply pure CSR strategies, we need not distinguish between the expected distribution, F_i, and the marginals, F_i, L and F_i, H – but see the proof of Lemma 4.

$\overline{e}(H,H) - \epsilon \in S_i(H,H)$. Consider a deviation from $c_i = H$ to $c_i = L$. The former choice yields the profit $T\left[\delta_3 \frac{1}{2} + (1-\delta_3)F_j(e_i|c_j = H)\right] - e_i$ whereas the latter would give the firm $T\left[\delta_2 \frac{1}{2} + (1-\delta_2)F_j(e_i|c_j = H)\right] - e_i$. Because $\delta_3 > \delta_2$ and $F_j(\overline{e}(H,H) - \epsilon|c_j = H) \to 1$ as $\epsilon \to 0$, the latter profit is clearly larger. $\square$

Proof of Lemma 4

We denote the support associated with c_L and $e_i > e^\star$ at the equilibrium as $S_L = [\underline{e}_L, \overline{e}_L]$ and the support associated with c_H and $e_i < e^\star$ as $S_H = [\underline{e}_H, \overline{e}_H]$. We first show that

1. $\min\{\underline{e}_L, \underline{e}_H\} = 0$

   Suppose not. $\min\{\underline{e}_L, \underline{e}_H\} = \underline{e} > 0$ such that $F_L(\underline{e}) = 0$ and $F_H(\underline{e}) = 0$. Thus, there would be a profitable deviation $e' < \underline{e}$ s.t. $F(e') = 0$ and $\Pi(\underline{e}) < \Pi(e') < 0$ for $e' > 0$ and $\Pi(\underline{e}) < \Pi(e') = 0$ for $e' = 0$.

2. $S_L \cap S_H \neq \emptyset$

   (i) suppose $\underline{e}_L > \overline{e}_H > 0$, we get $F_H(e_L) = 1, \forall e_L \in S_L$. By choosing $c = L$ and $e_L = \underline{e}_L$, firm 1 gets the profit $\Pi(L, \underline{e}_L) = (1-\lambda)T(1-\delta_1/2) - \underline{e}_L$. The profit is positive only if firm 2 chooses the investment H. Firm 1 can do better by deviating to $e' = \underline{e}_L - \epsilon > \overline{e}_H$ where $\epsilon > 0$.

   (ii) suppose $\underline{e}_H > \overline{e}_L > 0$, we get $F_H(e_L) = 0, \forall e_L \in S_L$. By choosing $c = L$ and $e_L = \underline{e}_L$, firm 1 gets the profit $\Pi(L, \underline{e}_L) = (1-\lambda)T\delta_1/2 - \underline{e}_L$. Firm can do better by choosing H instead and gets $\Pi(H, \underline{e}_L) = \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e}_L > \Pi(L, \underline{e}_L)$.

3. $S_L \cap S_H = \{e\}$ for some e

   Suppose $S_L \cap S_H = [\underline{e}, \overline{e}]$, then for any $e \in [\underline{e}, \overline{e}]$, the indifference condition $\Pi(L, e) = \Pi(H, e)$ should hold.

$$\begin{aligned} &\lambda[TF_L(e)] + (1-\lambda)[T\delta_1/2 + T(1-\delta_1)F_H(e)] - e \\ =\ &\lambda[T\delta_1/2 + T(1-\delta_1)F_L(e)] + (1-\lambda)[T\delta_2/2 + T(1-\delta_2)F_H(e)] - e \\ &\Rightarrow F_L(e)\{\lambda\delta_1\} + C = F_H(e)\{(1-\delta_2)(\delta_1 - \delta_2)\} \end{aligned}$$

   Since $F_L(e)$ and $F_H(e)$ increase in e, LHS is increasing in e while RHS is decreasing. Contradiction.

4. $S_H = [\underline{e}_H, e^\star]$ and $S_L = [e^\star, \overline{e}_L]$

   Suppose not. $S_L = [\underline{e}_L, e^\star]$ and $S_H = [e^\star, \overline{e}_H]$. Clearly $\underline{e}_L$ must be zero. Firm's profit by choosing low investment L and low expenditure $\underline{e}_L$ is $\Pi(L, \underline{e}_L) = (1-\lambda)T(1-\delta_1/2) - \underline{e}_L$ by $F_L(\underline{e}_L) = F_H(\underline{e}_L) = 0$. It only gets positive profit if the other firm chooses high investment H. Now consider a deviation to $(H, \underline{e}_L)$. $\Pi(H, \underline{e}_L) = \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e}_L > \Pi(L, \underline{e}_L)$. It is a profitable deviation. Contradiction. $\square$

Proof of Proposition 3

We proceed to solve the mixed equilibrium. Denote the probability of choosing low CSR by λ. Because a firm earns equal profit at $e^\star$ whether it chooses high or low CSR, λ can be solved from a firm's indifference condition $\Pi(L, e^\star) = \Pi(H, e^\star)$. As $F_L(e^\star) = 0$ and $F_H(e^\star) = 1$, we obtain

$$\begin{aligned} &\lambda[TF_L(e^\star)] + (1-\lambda)[T\delta_1/2 + T(1-\delta_1)F_H(e^\star)] - e^\star \\ =\ &\lambda[T\delta_1/2 + T(1-\delta_1)F_L(e^\star)] + (1-\lambda)[T\delta_2/2 + T(1-\delta_2)F_H(e^\star)] - e^\star \\ &\Rightarrow \lambda = (\delta_2 - \delta_1)/\delta_2 \text{ and } 1-\lambda = \delta_1/\delta_2. \end{aligned} \tag{20}$$

A firm's payoff can now be derived by inserting λ into $\Pi(H, \underline{e}_H)$

$$\begin{aligned} \Pi(H, \underline{e}_H) = \Pi(H, 0) &= \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e}_H \\ &= \frac{T}{2}\left[\delta_1 + (\delta_2 - \delta_1)\frac{\delta_1}{\delta_2}\right] > 0, \end{aligned}$$

which immediately shows that the payoff becomes higher with CSR.

The upper bound of S_H $e^\star$ can be obtained from $\Pi(H, e^\star) = \Pi(H, 0)$

$$\begin{aligned} \lambda T\delta_1/2 + (1-\lambda)T(1-\delta_2/2) - e^\star &= \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e}_H \\ \Rightarrow e^\star = T(1-\lambda)(1-\delta_2) &= T\delta_1/\delta_2(1-\delta_2). \end{aligned} \tag{21}$$

The upper bound of S_L $\overline{e}_L$ can be calculated from $\Pi(L, \overline{e}_L) = \Pi(H, 0)$.

$$\begin{aligned} \lambda T + (1-\lambda)T(1-\delta_1/2) - \overline{e}_L = \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e}_H \\ \Rightarrow \overline{e}_L = T(1-\delta_1). \end{aligned} \tag{22}$$

This allows deriving the expenditure distribution associated with high CSR $F_H(e)$ by applying $\Pi(H, e^\star) = \Pi(H, \underline{e}_H)$ and $F_L(e^\star) = 0$.

$$\begin{aligned} &\lambda T\delta_1/2 + (1-\lambda)\{T\delta_2/2 + T(1-\delta_2)F_H(e)\} - e \\ &\qquad = \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e}_H \\ &\Rightarrow F_H(e) = \frac{e}{(1-\lambda)T(1-\delta_2)} = \frac{\delta_2}{\delta_1(1-\delta_2)}\frac{e}{T} \end{aligned} \tag{23}$$

The expenditure distribution associated with low CSR $F_L(e)$ can similarly be obtained from $\Pi(L, e^\star) = \Pi(H, e^\star) = \Pi(H, \underline{e}_H)$ and $F_H(e^\star) = 1$.

$$\begin{aligned} \lambda[TF_L(e)] + (1-\lambda)[T\delta_1/2 + T(1-\delta_1)] - e& \\ = \lambda T\delta_1/2 + (1-\lambda)T\delta_2/2 - \underline{e}_H& \\ \Rightarrow F_L(e) = \frac{T\delta_1(1-1/\delta_2)+e}{\lambda T} = \frac{\delta_1(\delta_2-1)}{\delta_2-\delta_1} + \frac{\delta_2 e}{(\delta_2-\delta_1)T}.& \quad \square \end{aligned} \tag{24}$$

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