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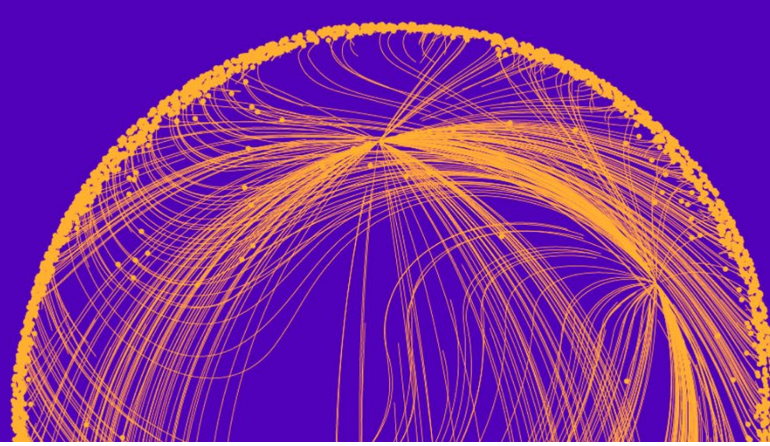
Information requirement for efficient decentralized screening

Saara Hämäläinen



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Information requirement for efficient decentralized screening

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Abstract

We establish new efficiency results for decentralized markets with quality uncertainty. Buyers encounter a succession of passing trade opportunities and related asset information, allowing them to screen the quality of assets by conditioning pricing on informative signals. We link key equilibrium properties with the intensity of screening. This innovative approach delivers conditions under which efficient equilibria exist, characterizes efficient and inefficient equilibria in terms of asset screening and trade dynamics, and presents a new measure for the information required for efficient trade and asset screening. Trade dynamics may manifest as either *standard* or *reversed*.

Keywords: *Decentralized lemons market; Screening; Buyer signals; Trade dynamics; Efficient equilibrium; Equilibrium existence.* **JEL-codes:** D82, D83.

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1 Introduction

Asset markets are large and global. Trades are regularly executed *over-the-counter* in multiple *decentralized* exchanges. Some assets are clearly "lemons" as defined by Akerlof (1970), e.g., a firm might have issues with information security or customer management, just waiting to surface. However, even these assets often generate positive value for their owner, new trading opportunities arrive constantly, and buyers can inspect assets before trading. Indeed, the law requires *due diligence* in acquisitions and *caveat emptor* applies. How do such decentralized markets with informative signals fare? Will the market remain inefficient, as in Moreno and Wooders (2010), without signals? Or will the lemons problem resolve on its own with time and the market settle into an efficient equilibrium? Which dynamic trade patterns, as characterized by Kaya and Kim (2018), are sustained in the long run? How do frictions and signals contribute to market performance?

In this article, we address these questions by investigating the effect of information on a decentralized lemons market, where i. traders are small, numerous, and anonymous, ii. frictions of trade are negligible, and iii. average asset quality has settled into its steady-state level. [1] The setup adheres loosely to the seminal model of dynamic trade by Moreno and Wooders (2010): asset sellers enter the market with different asset qualities, meet a sequence of random buyers, and exit the market upon trading. To incorporate asset information into this model, we introduce the assumption that a buyer can obtain a signal of a seller's asset quality before making the seller a price offer. This provides an extended version of canonical models for decentralized trade[2] where traders face not only a constant flow of trade opportunities, as in the previous literature, but also an incessant flow of asset information. The setup emulates information-rich financial markets.

We establish new efficiency results for this formerly neglected class of markets that has recently garnered great interest from financial economists.[3] In particular, we find that all key properties of an equilibrium – existence, efficiency, and dynamics – derive from the *screening intensities of different asset qualities*, representing the *difficulty of obtaining a high price for the asset*. In the model, signal distributions differ between assets, with lower signals suggesting lower quality. As a result, it is possible for buyers to screen the quality of assets by offering high prices only for high enough signals, above a chosen cutoff. Furthermore, assuming that signals are sufficiently informative in comparison to trading frictions, the screening intensity of low-quality assets can be freely adjusted relative to that of high. To equate the costs of waiting with the benefits of increased quality assurance, a buyer could thus make obtaining a high price offer either equally hard for all assets, infinitely harder for low quality, or anything in between. This insight

[1] This case is particularly interesting as a decentralized counterpart of the static market à la Akerlof (1970).

[2] See Wolinsky (1990); Serrano and Yosha (1993, 1996); Blouin and Serrano (2001); Blouin (2003)

[3] For examples of high impact work, see Rostek and Yoon (2021) and Azevedo and Gottlieb (2017).

permits us to characterize equilibria by focusing on screening.

Our main result is that a limit-efficient steady-state equilibrium, where payoffs approach the first-best as trade frictions disappear, exists in the market for an extensive range of parameter values. The range is partly characterized by *the severity of the lemons problem* and partly by *the relative trade surpluses* among different asset qualities, which is novel. Specifically, we find that there are (essentially) two trade patterns that sustain a limit-efficient equilibrium: i. standard dynamics (low quality trades faster), which mandate that the trade surplus of low quality is larger, and ii. reversed dynamics (high quality trades faster), which require that the static lemons problem is not severe. These new limit-efficiency results contrast with the persistence of trading problems described in the literature (Blouin and Serrano, 2001; Camargo and Lester, 2014; Guerrieri and Shimer, 2014; Moreno and Wooders, 2010).[4]

Efficiency hinges on adjusting screening to market conditions: In the limit-efficient equilibrium i., where trade dynamics are standard, the screening of low quality is strong enough to make the seller accept a low price, rather than waiting for high signals. In the limit-efficient equilibrium ii., where the dynamics are reversed, the screening of both qualities is relaxed, encouraging a low-quality seller to wait for a high price if the signal is low. Interestingly, there is also a limit-inefficient equilibrium with standard dynamics. It arises because exacerbating screening of *low-quality assets* increases buyer rents from trading at low prices. This results in further escalation in the screening of *high-quality assets* as buyers require equal rents with them. Excessive screening eventually stifles trading. We find that a common marker that distinguishes efficient trade patterns is that screening of high-quality assets is lenient.

Our second major result is that the conditions for the existence of a limit-efficient equilibrium are necessary in the sense that, if there exists no *efficient* equilibrium, there exists *no* equilibrium in the market. This occurs when trading high quality is both more difficult (i.e., the lemons problem is severe) and more valuable (i.e., the trade surplus is larger). Non-existence derives from a discrepancy between the required trade dynamics and the relative trade surpluses. We can show that, if the lemons problem is severe, only standard dynamics prevail.[5] Because low quality thus trades faster, the quality of unsold assets increases, while vanishing frictions entail a higher opportunity cost of trade. Thereby, we find that buyers only offer high prices when they are almost certain about high asset quality, which gives them the high trade surplus.[6] However, this implies that

[4]For positive efficiency results in decentralized markets, see Golosov et al. (2014) for divisible assets and aggregate uncertainty and Asriyan et al. (2017) for correlated values and information spillovers. Camargo et al. (2020) find that non-steady-state equilibria with aggregate uncertainty become efficient as frictions vanish.

[5]Otherwise, buyers should only offer high prices and only trade for high signals but, then, average asset quality decreases so much that buyers only offer low prices – a contradiction.

[6]Here, buyers obtain positive rents under high and possibly also low signals, unlike in Moreno and Wooders (2010), where buyers mix between high and low prices and receive no payoffs.

buyers and low-quality sellers cannot agree on a low price under low expectations, since the low trade surplus is smaller – thus contradicting standard dynamics. The existence and efficiency of an equilibrium thus depend not only on the *severity of the lemons problem*, as known since Akerlof (1970), but also on the *relative trade surpluses* across traded assets.

Our focus on screening allows us to quantify the information requirements of limit-efficient trade, by inverting the functions that map screening with information requirements for given trade frictions. As our third key contribution, we can thus demonstrate that almost efficient trading can be approached for boundedly informative signals and positive frictions as long as *signals are sufficiently informative relative to frictions*. This resolution of a lemons problem relies on both i. the existence of frictions and ii. the presence of signals, confirming the hypothesis of Moreno and Wooders (2010) that "decentralized trade mitigates the lemons problem".

In general, we find that the information requirement for a high offer elevates as frictions decrease because costs of waiting vanish and the quality of future traded assets initially surpasses the quality for which a high price offer is made. High prices thus require exceedingly informative signals, $y \to 1$, as frictions disappear, $\delta \to 1$. While this limit is invariant across equilibria, our results show that it can approached via alternative paths, $(y^i, \delta^i)_i \to (1, 1)$, entailing different screening of low- and high-quality assets. Because payoffs depend on screening, welfare and dynamics are not defined by limit information but by asset screening over the equilibrium path $(y^i, \delta^i)_i$.

Our research contributes to the growing literature that studies adverse selection in decentralized market environments with random sequential search. There is also a large body of literature about dynamic trading with incomplete information in directed search markets, e.g., Inderst and Müller (2002); Inderst (2005); Guerrieri et al. (2010); Camargo and Lester (2014), and in competitive lemons markets, e.g., Janssen and Roy (2002, 2004); Daley and Green (2012); Fuchs and Skrzypacz (2019).

A voluminous body of literature studies whether decentralized trade results in equal payoffs to its centralized counterpart if trade frictions are small. Gale (1986a,b, 1987) and Binmore and Herrero (1988) investigate this question under complete information, finding efficient payoffs. Moreno and Wooders (2010) extend the analysis to markets with a lemons problem where no efficient one-price equilibrium may exist. They find that payoffs approach the highest payoffs in a static market, which makes them inefficient *iff* the lemons problem is severe. Unlike our current case, buyers can only separate sellers by randomizing between different prices, which leaves the surplus to low-quality sellers and screens all assets with the same intensity – fostering limit-inefficient outcomes.

Our work contributes to this literature by showing that efficient decentralized screening can outperform inefficient centralized trade.[7] Previously, efficient trade mechanisms in a lemons market have been related to sorting. In Hendel et al. (2005), observed asset vintages allow the establishment of approximately efficient rental markets for all assets. In Inderst and Müller (2002), different assets are traded in separate markets with distinct prices and liquidity conditions. Interestingly, in Inderst and Müller (2002), the expected quality in markets adjusts to support the Riley separating equilibrium outcome, whereas here, only the cutoff for high prices adjusts to support efficient trade while prices remain semi-pooling as in Moreno and Wooders (2010) or Cho and Matsui (2018).[8]

Another impressive body of literature considers dynamic trading with adverse selection. Due to the different time-preferences of high- and low-quality sellers, standard dynamics are derived in almost all articles in the literature. The few exceptions that feature reversed dynamics (Taylor, 1999; Zhu, 2012; Kaya and Kim, 2018; Palazzo, 2017; Hwang, 2018; Martel et al., 2022) are characterized by a non-steady-state setup and observable time-on-market.

For example, Kaya and Kim (2018) explore a dynamic model where an asset seller meets a sequence of buyers who offer prices after observing the marketing time and a private quality signal of the asset. They find that trade dynamics depend on exogenous prior beliefs. If the prior is low, dynamics are standard. However, reversed dynamics prevail when buyers have inflated prior beliefs, which alleviate screening to the point that no seller accepts low prices. Our approach is different as it focuses on a steady-state setup where the average asset quality is endogenous and constant.[9,10] Because assets exit the market upon trading, reversed dynamics mean that low quality remains in the market longer, decreasing the average market quality and buyers' quality expectations. As a consequence, because buyer beliefs at the cutoff are bounded above by entry quality, we show that reversed dynamics cannot prevail if the lemons problem is severe and the average entry quality low. Our work also connects trade dynamics to asset screening and delivers a new measure of the information required in efficient trading. Previous work remains mute about the relationship between trade dynamics and efficiency.

The article is organized as follows. The model is outlined in Section 2 and its basic features in Section 3. Section 4 describes limit equilibria first with unbounded information and later with bounded information. Section 5 concludes by discussing extensions and alternative model assumptions. Most proofs are relegated to the Appendix.

[7]When frictions remain positive, Moreno and Wooders (2010) also demonstrate that the surplus created by trade can be higher in the decentralized equilibrium than in the centralized equilibrium. However, the described payoffs remain inefficient in the limit. Moreover, as noted by Kim (2017), the result does not survive extension to continuous time trading.

[8]Contrary to our balanced market approach, Inderst and Müller (2002) assume that buyers outnumber sellers, which gives buyers zero payoffs, fostering an efficient outcome.

[9]In the sequential adverse selection experiment of Araujo et al. (2021), the majority of players applied stationary responses in contrast to optimal time varying ones.

[10]We also dispense with the assumption in Kaya and Kim (2018) that time-on-market is observable, our focus being on over-the-counter markets where assets sell quietly.

2 Model

The model closely follows that of Moreno and Wooders (2010) except for the added buyer signals. The general setup emulates modern information-rich over-the-counter markets, where buyers face a steady flow of new trade opportunities and asset information.

Time is discrete and horizon infinite. A unit mass of buyers and a unit mass of sellers enter the market in each period. Thereafter, all buyers and sellers in the market are randomly matched in pairs in order to trade. A buyer and a seller who trade exit the market. If there is no trade, the match dissolves. The buyer and the seller will thus return to the market where they will be matched with someone else in the next period.

Buyers and sellers discount future payoffs by the common discount factor $\delta < 1$. This discount factor captures trade frictions by showing how much payoffs are reduced if opportunities for trading are delayed. We will focus on the limit $\delta \to 1$ where frictions of trade disappear.

Every seller holds an indivisible asset whose quality, $\theta = h, l$, can be high or low. Quality is private information to the seller. The payoff from an asset of quality θ to the seller is denoted as C_θ and the payoff to the buyer as U_θ, evaluated at the period of trading. The buyer's payoff exceeds the seller's payoff, and gains from trade therefore arise: $U_\theta > C_\theta$.

We assume that one-half of the entering sellers have a high-quality asset ($\theta = h$) and the rest a low-quality asset ($\theta = l$). The assumption is innocuous. It delivers a tractable parametrization that will help to highlight the drivers of our results. We relax it later without substantial changes.

We assign the following magnitudes to the payoffs, which allow for the presence of a lemons problem.

$$U_h > C_h > U_l > C_l$$

Properties of equilibria will depend on the relative trade surpluses of assets and the "gap", defined as follows

$$\begin{aligned}
\Delta_h &:= U_h - C_h, \\
\Delta_l &:= U_l - C_l, \\
\Delta_g &:= C_h - U_l.
\end{aligned}$$

Note that, although high-quality assets are always more valuable to both buyers and sellers, the low trade surplus Δ_l can still exceed the high Δ_h if the spread between a buyer's and a seller's payoff is higher.

The gap $\Delta_g = C_h - U_l$ represents the temptation for low-quality sellers to trade for a high price, $p \geq C_h$, instead of a low price, $p \leq U_l$. The minimum price a high-quality seller accepts is C_h; the maximum price that a buyer pays for a low-quality asset is U_l.

In a static centralized market, a lemons problem always arises if a buyer's payoff for buying a random asset, $\frac{U_h+U_l}{2}$, remains below a high-quality seller's payoff for holding his asset, C_h, which gives

$$\begin{aligned}
\overline{U} := \frac{U_h + U_l}{2} &< C_h, \\
U_h + U_l &< 2C_h, \\
U_h - C_h &< C_h - U_l, \\
\Delta_h &< \Delta_g.
\end{aligned}$$

We can thus see that only low-quality assets can be traded in a static centralized market if the gap exceeds the high trade surplus. However, if the gap remains smaller, $\Delta_h \geq \Delta_g$, a lemons problem may not arise. In that case, the static market has both an efficient equilibrium where trade occurs at $p \geq C_h$ (both qualities are traded) and an inefficient equilibrium where trade occurs at $p \leq U_l$ (only low quality is traded).

Our dynamic decentralized model extends the static centralized model in that trade takes place in private meetings between a buyer and a seller. Thus, i. there could be trade at different prices in different meetings with different signals, and ii. trade could be postponed if the current terms of trade are not sufficiently attractive.

Furthermore, unlike many papers which presume that the static lemons condition holds, our paper studies markets with both $\Delta_h < \Delta_g$ ("severe lemons problem") and $\Delta_h \geq \Delta_g$ ("non-severe lemons problem"). We are also agnostic about the ranking of trade surpluses, $\Delta_h < \Delta_l$ or $\Delta_h \geq \Delta_l$. These cases lead to different equilibria.

Over-the-counter trades between a buyer and a seller proceed as follows. After a buyer and a seller are randomly matched, the buyer obtains a signal s of the seller's asset quality and makes the seller a take-it-or-leave-it-offer p about the price. If the seller accepts the price p, the asset is traded to the buyer, and both traders exit the market. Otherwise, the buyer and seller separate and wait until the next trade opportunity arises in the following period with someone else. The market is so large that the same buyer and seller are almost never matched again.

To investigate how much information is required for efficient decentralized trading, we allow the informativeness of signals to span all values from uninformative to revealing. Signals s are distributed according to distribution functions $F_\theta : [0,1] \to [0,1]$, which are continuous and supported on the unit interval $[0,1] = \text{cl}\{s|f_\theta(s) > 0\}$, where f_θ denotes the density function related to F_θ.[11] For simplicity, we assume that higher signals indicate higher quality. Extreme signals at the limits of $[0,1]$ approach being perfectly revealing. [12] Assumption 1 captures these ideas.

[11] The set closure cl$A$ is the smallest closed set which contains the original set A.

[12] As F_θ are continuous, the likelihood of observing a revealing signal is almost nil.

Assumption 1

$$\frac{f_h(s)}{f_l(s)} \in (0,\infty)\,, \ \textit{for all } s \in (0,1)\,,$$
$$\frac{\partial}{\partial s}\frac{f_h(s)}{f_l(s)} \in (0,\infty)\,, \ \textit{for all } s \in (0,1)\,,$$
$$\lim_{s\to 0}\frac{f_h(s)}{f_l(s)} = 0,$$
$$\lim_{s\to 1}\frac{f_h(s)}{f_l(s)} = \infty.$$

The first two lines just state that signals $s \in (0,1)$ satisfy the standard monotone likelihood ratio property (MLRP). The two latter lines entail more specifically that any likelihood ratio $\frac{f_h(s)}{f_l(s)} \in (0,\infty)$ is attainable for an appropriate signal $s \in (0,1)$.

To focus on decentralized environments and simple trading strategies, we further assume that i. the signals and actions in a pairwise meeting are not observable by outsiders, and ii. strategies do not condition on the signals observed in earlier meetings.

We study simple steady-state equilibria in behavioral strategies $\boldsymbol{\sigma} = (p, a_h, a_l)$. The strategy of a buyer is a function $p : [0,1] \to \Delta\,[0,\infty)$ mapping a signal s to the probability distribution $G(s)$ of offers $p(s)$. The strategy of a seller is a function $a_\theta : \mathbb{R} \to [0,1]$ that maps a price p to the probability of acceptance $a_\theta(p)$.

The solution concept employed is a perfect Bayesian equilibrium (PBE). A PBE is a pair $(\boldsymbol{\sigma}, \boldsymbol{\pi})$ consisting of a strategy profile $\boldsymbol{\sigma}$ and a belief system $\boldsymbol{\pi}$ such that i. the strategy profile $\boldsymbol{\sigma}$ is optimal for the beliefs $\boldsymbol{\pi}$, and ii. the belief system $\boldsymbol{\pi}$ is derived from the strategy $\boldsymbol{\sigma}$ with Bayes' rule whenever possible.

Our focus on a steady-state market, maintaining constant proportions of high- and low-quality assets, enables us to endogenize buyers' expectations of traded assets. Subsequently, we find that our steady-state setup imposes significant restrictions on market quality and future payoffs, mitigating a buyer's tendency for excessive asset screening for vanishing trade frictions. This finding is pivotal for supporting limit equilibria.

The existence of an equilibrium is not immediately evident. In general, low frictions render buyers selective, to the point where they might only offer low prices accepted by low-quality sellers. In contrast, the implied surge in asset quality suggests they should only offer high prices – indicating the possibility of a contradiction. Our analysis demonstrates how this contradiction can be avoided by adjusting asset screening.

The key intuitions for the analysis are summarized as follows. As we demonstrate later, in a steady-state setup, the average quality of future traded assets surpasses the lowest cutoff quality level for which a buyer makes a high price offer. Therefore, as the costs of waiting vanish, buyers become more discerning, requiring exceeding information and assurance of quality to make a high price offer (Lemma 6).

On the other hand, in a steady state, non-traded assets amass in the market, making their later trade more probable. Since high- and low-quality assets enter the market in equal proportions, buyers expect to trade them with equal probabilities in future matches. The benefit of waiting being bounded (Eqs. (6) and (7)), a buyer becomes more willing to offer a high price, paving the way for the discovery of an equilibrium.

In the following sections, we link the properties of equilibria with screening, that is, the time cost of obtaining a high price for the asset. The details of the analysis depend on whether $\Delta_h > \Delta_g$ and $\Delta_h < \Delta_l$.

3 Preliminaries

Any strategies (p, a_h, a_l) define continuation values, V_b for a buyer and V_h and V_l for the sellers of each quality type. Sequential rationality requires that the equilibrium strategies employed by a buyer and a seller in a meeting are optimal given (V_b, V_h, V_l).[13]

After observing a buyer's price offer, p, a seller chooses whether to accept it. The optimal choice satisfies the Bellman equation:

$$V_\theta(p) = \max_{a_\theta} \quad a_\theta(p - C_\theta) + (1 - a_\theta)\delta V_\theta. \tag{1}$$

By accepting the offer, the seller obtains the price p but loses the value of holding the asset C_θ. Instead, by rejecting the price, the seller keeps the asset and retains the value of selling it later, δV_θ. The problem of the seller does not depend on whether the seller can observe the signal.

We can see immediately that the optimal strategy of a seller is a cutoff strategy: a seller accepts any price above a cutoff but rejects lower offers. The cutoff equals the sum of a seller's reservation value and continuation value $C_\theta + \delta V_\theta$, denoting the opportunity cost of accepting the price.

Lemma 1 (Seller's cutoffs) *For any V_θ, the optimal strategy of a seller of quality $\theta = h, l$ is a cutoff strategy, defined as follows*

$$a_\theta(p) = \begin{cases} 1, & \text{if } p \geq C_\theta + \delta V_\theta, \\ 0, & \text{if } p < C_\theta + \delta V_\theta. \end{cases}$$

Conditional on observing a signal, s, a buyer offers the seller a price. The optimal

[13] These continuation values are derived in the Appendix.

price offer satisfies the Bellman equation:

$$\begin{aligned} V_b(s) = \max_p \quad & q(s)a_h(p)(U_h - p) + (1 - q(s))a_l(p)(U_l - p) + \\ & (q(s)(1 - a_h(p)) + (1 - q(s))(1 - a_h(p)))\delta V_b, \end{aligned} \tag{2}$$

where $q(s)$ denotes the probability, conditional on signal s, that the asset has high quality. If the asset quality is high and the price is accepted, the buyer receives a payoff of $U_h - p$. Conversely, if the price is accepted by a low-quality seller, the buyer secures a payoff of $U_l - p$. Otherwise, if neither scenario occurs, the buyer returns to the market, obtaining the valuation of δV_b.

Knowing that a seller of quality θ accepts any price above a cutoff, a buyer who targets this seller never offers more than $C_\theta + \delta V_\theta$. In general, a buyer either offers i. a high price p_h that targets a high-quality seller, ii. a low price p_l that targets a low-quality seller, or iii. an even lower price p_0 that neither seller accepts. A buyer may offer either two or three prices in equilibrium.

Lemma 2 (Buyer's cutoffs) *For any (V_b, V_l), there is a cutoff signal $y \in [0, 1]$ that allows expression of the optimal strategy of a buyer as follows*

$$p(s) = \begin{cases} p_h, & \text{if } s \geq y, \\ p_l, & \text{if } s < y \text{ and } \Delta_l \geq \delta(V_l + V_b), \\ p_0, & \text{if } s < y \text{ and } \Delta_l \leq \delta(V_l + V_b). \end{cases}$$

where $p_0 < p_l = C_l + \delta V_l \leq U_l < p_h = C_h + \delta V_h = C_h$.

The optimal price strategy of a buyer is subtle as it depends on the endogenous valuations V_b and V_l. Without showing the existence of an equilibrium and without yet knowing the exact values of V_b and V_l, we can prove that a buyer will offer p_h for signals that exceed a cutoff y, when the buyer is sufficiently certain of high quality. Instead, for signals below the cutoff y, a buyer offers lower prices p_l or p_0.

Which of these offers is made for low signals depends on the joint continuation values of a buyer and a low-quality seller, $\delta(V_l + V_b)$. In a static centralized market, a buyer can offer a low price p_l and trade low quality if the expected asset quality is low. However, in a dynamic setting, buyers and low-quality sellers can also wait for higher signals, which suggest higher quality and allow for trade at a high price p_h. This prospect can increase the continuation values $\delta(V_l + V_b)$ to the point where they exceed the gains from trade Δ_l. When this is so, it is impossible for a buyer and a low-quality seller to agree on a low price $p \leq U_l$ that would cover δV_b to the buyer and $C_l + \delta V_l$ to the seller.

It can be shown that the offer to a low-quality seller, p_l, lies below the offer to the high-quality seller, p_h, because the seller's reservation value is lower, $C_l < C_h$. Additionally, we observe that the offer that targets the high-quality seller, p_h, cannot exceed C_h. This is because a holdup problem arises in a meeting, and it allows the buyer to reduce the offer from p_h to $(1-\delta)C_h + \delta p_h$ unless p_h already equals the seller reservation value C_h. This entails that the continuation value of a high-quality seller V_h must be zero. The gains from trade thus accrue only to buyers and low-quality sellers. The non-accepted price offer p_0 is indeterminate, but it has to lie below the low price offer p_l.

3.1 Expected quality

Because gains from trade are positive with both qualities, buyers are willing to pay higher prices for higher quality, but are reluctant to do so if the expected quality remains low. Buyers' optimal price strategies hence depend on their beliefs. Specifically, a buyer will offer a high price $p_h = C_h$ which each seller will accept if and only if the probability $q(s)$ that the seller has a high-quality asset reaches a cutoff $q(y)$, i.e., $q(s) \geq q(y)$. The cutoff $q(y)$ solves the following equality, requiring that a buyer is indifferent between offering a high price, p_h, and either p_l or p_0 – whichever provides a higher buyer payoff:

$$q(y)\underbrace{(U_h - C_h)}_{>0} + (1-q(y))\underbrace{(U_l - C_h)}_{<0} = \max\left\{(1-q(y))\,(U_l - p_l) + q(y)\delta V_b, \delta V_b\right\}. \quad (3)$$

If a buyer offers a high price, $p_h = C_h$, the buyer's payoff is positive, $U_h - C_h = \Delta_h$, if the seller has a high-quality asset but, if the seller has a low-quality asset, the buyer's payoff is negative, $U_l - C_h = -\Delta_g$. Instead, the payoff for offering a low price p_l is $(1-q(y))\,(U_l - p_l) + q(y)\delta V_b$ (only low-quality sellers accept the offer) and the payoff for offering a low price p_0 is δV_b (neither of the sellers accepts this offer). If the seller does not accept a price, the buyer's continuation value is δV_b.

Buyer beliefs about quality $q(s)$ are shaped by both market composition and signal information. First, buyers take into account the endogenous market composition, that is, how many assets of each quality circulate in the market. These buyers' prior beliefs, which reflect the equilibrium trade probabilities of assets, are called *unconditional* beliefs q_u. Second, buyers consider information conveyed by the signal they obtain in the meeting. These *conditional* (posterior) beliefs are denoted by $q_c(s)$.

Because sellers enter the market in equal proportions and exit the market upon trading, the market composition is determined solely by the sellers' relative trading probabilities. In a steady state, the mass of assets of quality θ in the market remains constant, denoted as M_θ, and the inflow of each quality to the market has to equal outflow:

$$1/2 = M_\theta(1 - G_\theta(p_\theta -)).$$

On the left-hand side (lhs), 1/2 denotes the entry of assets of each quality in the market. In each time period, a unit mass of assets enters, half of each quality. The right-hand side (rhs) represents asset exits, with M_θ assets of each quality in the market. Each asset trades with a probability of $1 - G_\theta(p_\theta-)$, of a buyer offering at least p_θ.[14]

Solving for the measures M_θ and using Bayes' rule, q_u and $q_c(s)$ can be derived as follows

$$q_u = \frac{M_h}{M_h + M_l} = \frac{1}{1 + \frac{1-G_h(p_h-)}{1-G_l(p_l-)}}, \tag{4}$$

$$q_c(s) = \frac{M_h f_h(s)}{M_h f_h(s) + M_l f_l(s)} = \frac{1}{1 + \frac{1-G_h(p_h-)}{1-G_l(p_l-)}\frac{f_l(s)}{f_h(s)}}, \tag{5}$$

where $q_c(s)$ is derived from q_u by incorporating the information about the likelihood ratio $\frac{f_l(s)}{f_h(s)}$ of receiving the observed signal s from a low-quality asset *versus* high.

We observe that beliefs about asset quality increase under three conditions: when low-quality assets trade more quickly, when high-quality assets trade more slowly, or when the observed signal increases. Namely, if one asset quality is traded more slowly than the other asset quality, it amasses in the market in relative terms, increasing a buyer's expectation of meeting a seller with this quality. Further, because the likelihood ratio $f_h(s)/f_l(s)$ is by Assumption 1 increasing in s, buyers' conditional beliefs $q_c(s)$ are clearly increasing in the observed signal. Consequently, a buyer will offer a high price if and only if the signal is above the cutoff, i.e., $q(s) \geq q(y)$ *iff* $s \geq y$.

Our framework deviates from earlier approaches in that buyers observe continuous signals of variable information content. Price strategies hence differ from those in Moreno and Wooders (2010), without signals, and Kaya and Kim (2018), with binary signals, where buyers mix between low and high prices. Mixed strategies allow for the adjustment of screening, similar to signals, albeit less finely – resulting in inefficient limit equilibria in Moreno and Wooders (2010). Here, signals allow the purification of buyer strategies (Harsanyi, 1973), and buyers set low prices for $s < y$ and offer a high price for $s \geq y$;[15] we find that y can adjust to sustain limit-efficient trade.

3.2 Trading dynamics

Whether trade dynamics are standard, reversed, or what we call "knife-edge" depends on the endogenous valuations V_b and V_l.

Lemma 3 *Feasible equilibrium dynamics can be classified into three main patterns:*

[14] Technically, $G_\theta(p_\theta-) = \lim_{p \to p_\theta} G_\theta(p)$ denotes the left derivative of a buyer's unconditional (marginal) offer distribution G_θ to sellers of quality θ at p_θ.

[15] "Knife-edge" dynamics defined below allow for both pure and mixed price strategies as buyers can either randomize between p_0 and p_l for $s < y$ or offer p_0 for $s \in [0, z)$ or p_l for $s \in [z, y)$. Any equilibrium payoffs can be sustained by pure strategies.

1. *If $\Delta_l > \delta(V_l + V_b)$, trade dynamics are standard and low-quality assets trade faster:* $p(s) = C_h$ *for* $s \geq y$, $p(s) = p_l$ *for* $s < y$, *and* $q_u = \frac{1}{1+(1-F_h(y))} > 1/2$.

2. *If $\Delta_l < \delta(V_l + V_b)$, trade dynamics are reversed and high-quality assets trade faster:* $p(s) = C_h$ *for* $s \geq y$, $p(s) = p_0$ *for* $s < y$, *and* $q_u = \frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}} < 1/2$.

3. *If $\Delta_l = \delta(V_l + V_b)$, "knife-edge" trade dynamics arise:* $p(s) = C_h$ *for* $s \geq y$, $p(s) = p_l$ *for* $s \in [z, y)$ *and* $p(s) = p_0$ *for* $s \in [0, z)$, *and* $q_u = \frac{1}{1+\frac{1-F_h(y)}{1-F_l(z)}} \lesseqgtr 1/2$.

Under standard and reversed dynamics, two prices are offered by buyers, whereas under knife-edge dynamics, buyers offer three prices in equilibrium. We focus on standard and reversed trade dynamics in the main text; the analysis of knife-edge dynamics is delegated to the Appendix. We can show immediately that reversed dynamics cannot arise under a severe lemons problem due to the deteriorating market quality.

Lemma 4 *A necessary condition for reversed dynamics is $\Delta_h \geq \Delta_g$.*

Proof. The proof is by contradiction. Consider the beliefs of a buyer who has observed the cutoff signal y. Assuming reversed dynamics, this buyer must be indifferent between offering p_h and p_0. As assets are traded only for high signals, Lemma 3 shows that the conditional beliefs of a buyer are given by

$$q_c(y) = \frac{1}{1 + \frac{1-F_h(y)}{1-F_l(y)} \frac{f_l(y)}{f_h(y)}}.$$

But now, our assumption of MLRP implies a monotone hazard rate, $\frac{1-F_h(y)}{1-F_l(y)} \frac{f_l(y)}{f_h(y)} > 1$, indicating that the asset is more likely of low quality, $q_c(y) < 1/2$. Thus, a buyer is not willing to make a high price offer p_h after observing the cutoff signal y because the payoff for doing so is negative if $\Delta_h < \Delta_g$, as shown by

$$E(U|y) - C_h = q_c(y)\Delta_h - (1 - q_c(y))\Delta_g < \frac{1}{2}\Delta_h - \frac{1}{2}\Delta_g < 0.$$

This contradicts the assumption that a buyer is willing to offer p_h at y, demonstrating that reversed dynamics cannot arise under $\Delta_h < \Delta_g$. □

Lemma 4 shows that our steady-state setup places new restrictions on equilibrium dynamics, absent from setups such as Kaya and Kim (2018), where reversed dynamics arise when buyer beliefs start from above the long run level of beliefs. Our model assumptions do not grant such flexibility. Indeed, the analysis in the following Section 4 shows that the steady-state market composition not only places limitations on the quality at the cutoff $q_c(y)$, as shown in the proof of Lemma 4, but also notably restricts buyer continuation value V_b, with significant effects on search incentives.

4 Equilibrium

4.1 Positive frictions

To evaluate market welfare in a steady-state equilibrium, we use the measure applied by Moreno and Wooders (2010),

$$W = V_b + \frac{1}{2}V_h + \frac{1}{2}V_l = V_b + \frac{1}{2}V_l,$$

denoting the expected present discounted value of the trade surplus accruing to a single entry-cohort of buyers and sellers. The maximum trade surplus is given by the first-best complete information benchmark, $\frac{\Delta_h+\Delta_l}{2}$, which is reached if all assets are traded in the period they enter the market. Lemma 5 shows that the maximum is generally unattainable due to positive asset screening ($y > 0$) and positive trade frictions ($\delta < 1$).

Lemma 5 *$y > 0$ for $\delta < 1$.*

According to Lemma 5, the cutoff y is positive in dynamic markets with signals. This indicates that, although the surplus of trading is positive with both qualities, some meetings are not conductive to trade as would be efficient. By Lemma 2, high-quality sellers only trade for high prices $p_h = C_h$, which a buyer offers to them with probability $1 - F_h(y)$. Because the cutoff y is positive, this probability is less than one.

The result is notable in showing that screening reduces efficiency even when the lemons problem is not severe. Namely, in the absence of signals and a severe lemons problem, all sellers could trade in their first meeting for C_h. As no unsold assets would remain in the market, average quality in the market would stay high, as required for immediate trade. However, Lemma 5 shows that signals destroy this efficient pooling equilibrium, as in Hirshleifer (1971). By Assumption 1, there is a positive probability $F_h(s(\epsilon)) > 0$ of observing such a low signal $s < s(\epsilon)$ that a buyer's beliefs in (5) collapse to $q_c(s) < \epsilon$ for any $M_h, M_l, \epsilon > 0$. Almost certain about low asset quality, a buyer thus makes a low price offer, which a high-quality seller rejects. An endogenous lemons problem therefore arises.

Previously, Daley and Green (2012) observe in a model with news that trade could be delayed without a severe lemons problem because traders wait for news to accumulate in order to trade. The reason for the trade delay is much like here, that information renders buyer beliefs noisy. This will make it harder for a buyer and a seller to agree on a price as the noise can take a buyer's belief about an asset far from its seller's belief.

4.2 Vanishing frictions

We move to investigate markets where trade frictions are negligible. Decreasing frictions increase the information requirements of buyers. Buyers thus require exceeding quality confirmation before a high offer is made.

Lemma 6 *$y \to 1$ as $\delta \to 1$.*

Lemma 6 shows that the cutoff approaches its upper bound as trading frictions decrease. This is because the buyer continuation value of waiting for higher signals, V_b, increases relative to the benefit of trading at a high price given the current signal, $E(U|s) - p_h$. A buyer thus needs to be more strongly convinced about high asset quality to terminate search by offering p_h.

The mechanism is mediated by the MLRP. Specifically, because faster trading assets accumulate in a steady state, a monotone hazard rate implies that the quality that the buyer expects to trade at the cutoff (proportional to $\Delta_h - (1-F_h)\frac{f_l}{f_h}\Delta_g$ or $\Delta_h - \frac{1-F_h}{1-F_l}\frac{f_l}{f_h}\Delta_g$) is initially surpassed by the quality that the buyer expects to trade in later meetings (proportional to $\Delta_h - (1 - F_l)\Delta_g$ or $\Delta_h - \Delta_g$). As frictions diminish, an incentive to postpone trade thus arises, driving up the cutoff.

While buyers become more selective as frictions disappear, the efficiency properties of limit equilibria are uncertain. On the one hand, buyers obtain cheaper information when frictions decrease since it costs less to wait for highly informative signals. On the other hand, buyers also become more selective, possibly forgoing valuable trades. Interestingly, we find that characteristics of equilibria are not driven by either of these tendencies alone but by the *proportions* of $\delta \to 1$ and $y \to 1$.

In particular, we find that there are different paths satisfying $(y, \delta) \to (1, 1)$ that correspond with three potential limit equilibria.

1. In the first tentative equilibrium, the odds ratio of high asset quality $\frac{f_h(y)}{f_l(y)}$ remains low with respect to discounting δ. The general ease of trading at high prices thus entails that dynamics are reversed and efficient pooling prevails.

2. In the second equilibrium candidate, $\frac{f_h(y)}{f_l(y)}$ increases relative to discounting δ. The time cost of waiting for a high price offer p_h becomes high for low-quality assets but remains low for high-quality assets. Dynamics are standard and screening efficient.

3. In the third possible equilibrium, $\frac{f_h(y)}{f_l(y)}$ is even higher with respect to discounting δ. All sellers thus face extremely high costs of waiting for a high price offer. This excessive screening is inefficient. Trade dynamics remain standard.

Equilibrium existence hinges on the severity of the lemons problem (whether $\Delta_h > \Delta_g$) and the relative gains from trade (whether $\Delta_h < \Delta_l$).

4.3 Screening with unbounded signal information

4.3.1 Screening

We proceed to describe conditions when each of the described equilibrium candidates represents a steady-state limit-equilibrium. This is done by partitioning the signal space by screening, i.e., the time cost of trading different assets at a high price. Lemma 7 formalizes the notion of screening.

Lemma 7 *For any $M > 1$ and $\delta \geq 1 - \frac{1}{M}$, there exist signals $s_0 < s_l < s_h < 1$ and functions $\nu_h(y,\delta) < \nu_l(y,\delta) < \infty$ such that*

$$\nu_h(y,\delta) := \frac{1-\delta F_h(y)}{1-F_h(y)} - 1 = (1-\delta)\frac{F_h(y)}{1-F_h(y)},$$
$$\nu_l(y,\delta) := \frac{1-\delta F_l(y)}{1-F_l(y)} - 1 = (1-\delta)\frac{F_l(y)}{1-F_l(y)},$$

$$\nu_l(s_0,\delta) = \nu_h(s_l,\delta) = \frac{1}{M} < M \leq \nu_l(s_l,\delta) = \nu_h(s_h,\delta),$$

and $s_0 \to 1$ as $M \to \infty$.

Lemma 7 introduces screening functions ν_θ, which quantify the difficulty associated with selling an asset of quality θ for a high price. The inverse of $\nu_\theta + 1$ denotes the probability of receiving a high price offer for the asset in either this period or any future one.

$$\left(\frac{1-\delta F_\theta(y)}{1-F_\theta(y)}\right)^{-1} = (1-F_\theta(y))(1+\delta F_\theta(y)+\delta^2 F_\theta(y)^2+\ldots)$$

Both ν_h and ν_l are increasing in y and decreasing in δ because waiting for a high price has a higher cost if either the cutoff signal y (representing screening) is higher or the discount factor δ (representing frictions) is lower. In general, the screening function of high quality ν_h always stays below that of low quality ν_l because higher signals $s \geq y$ are observed more frequently with high-quality assets. In addition, screening functions are continuous in both arguments (y,δ), approximating the constant zero function as $\delta \to 1$, for fixed $y \in (0,1)$, and infinity as $y \to 1$, for fixed $\delta \in (0,1)$.

Lemma 7 shows that screening partitions the signal space in four regions: First, if the cutoff y belongs to $\mathrm{I}_0 = [0, s_0)$, it is very easy for all assets to trade for p_h. Second, if $y \in \mathrm{I}_\mathrm{l} = [s_0, s_l)$, obtaining a high price for low quality becomes hard (i.e., as hard as we want) whereas receiving a high price for high quality remains easy (i.e., as easy as we want). Third, presuming the cutoff reaches higher levels, $y \in \mathrm{I}_\mathrm{h} = [s_l, s_h)$, screening also intensifies for high quality. Finally, if $y \in \mathrm{I}_1 = [s_h, 1]$, both qualities are extremely hard to sell at a high price and high quality barely ever trades.

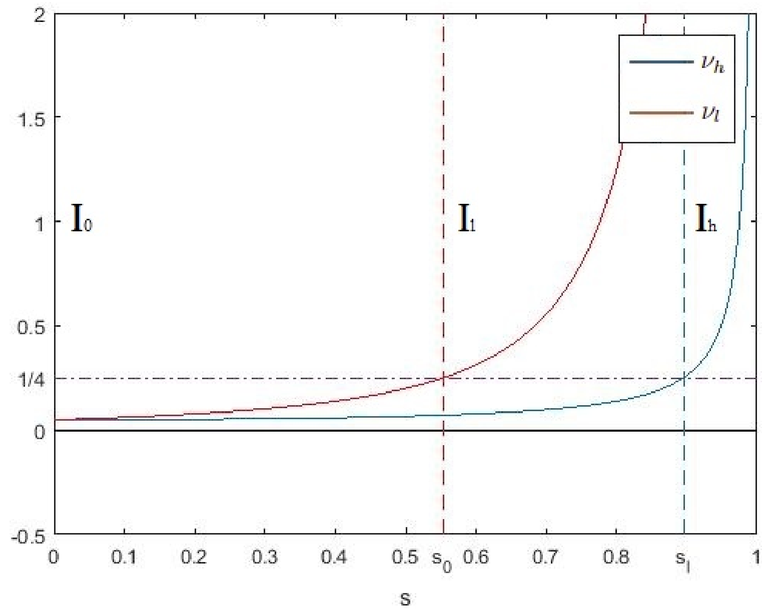


Figure 1: Illustration of screening functions ν_l and ν_h.

Figure 1 illustrates this partitioning by mapping ν_h and ν_l as functions of y, showing the cutoffs s_0 and s_l corresponding to $M = 4$; s_h is so close to unity that it is indiscernible. As do s_0 and s_h, s_l increases if $\delta \geq 1 - 1/M$ increases. To keep the relative screening of low quality above a certain level, $\frac{\nu_l(y,\delta)}{\nu_h(y,\delta)} \geq M^2$, the cutoff signal must be raised if frictions of trade are decreased.

4.3.2 Valuations

Leveraging these basic properties, we can demonstrate the existence of equilibria and characterize them by focusing on the intensity of screening. This involves rewriting payoffs in terms of ν_h and ν_l.

A powerful steady-state property that we discover is that screening cannot increase the likelihood of trading one quality over the other in future matches, irrespective of which dynamics of trade prevails.

For example, if high quality is screened more strongly, its concentration in the market elevates. This accumulation of high-quality assets entails that a buyer will trade them equally often as before, despite stronger screening. As a result, because assets are assumed to enter the market in equal proportions, a buyer will expect to trade both assets with the same probability. This allows a simple expression of continuation values.

Lemma 3 shows that, under standard dynamics, the probability of trading high quality is $q_u(1 - F_h) = \frac{1-F_h}{1+(1-F_h)}$ and the probability of trading low quality is $(1 - q_u) = \frac{1-F_h}{1+(1-F_h)}$, which are equal. We can thus show that the buyer continuation value is

$$V_b(y,\delta) = \frac{\Delta_h - (1-F_l(y))\Delta_g + F_l(y)(\Delta_l - \delta V_l)}{2+\nu_h(y,\delta)}, \tag{6}$$

which is obtained by dividing buyer payoffs (2) by the common trade probability of assets and reorganizing terms. The payoff from trading at p_h is $\Delta_h - (1-F_l(y))\Delta_g$ and that from trading at p_l is $F_l(y)(\Delta_l - \delta V_l)$, where buyer rents $\Delta_l - \delta V_l$ depend on low-quality seller rents V_l. Screening of high-quality assets ν_h reduces buyer payoffs since a buyer is forgoing valuable trade opportunities of high-quality assets for low signals.

For reversed dynamics, Lemma 3 shows that a buyer expects to trade high quality with probability $q_u(1-F_h) = \frac{(1-F_l)(1-F_h)}{(1-F_l)+(1-F_h)}$ and expects to trade low quality with the same probability $(1-q_u)(1-F_l) = \frac{(1-F_l)(1-F_h)}{(1-F_l)+(1-F_h)}$. Thus, the buyer continuation value is

$$V_b(y,\delta) = \frac{\Delta_h - \Delta_g}{2+\nu_h(y,\delta)+\nu_l(y,\delta)}, \tag{7}$$

which can be derived as before by dividing buyer payoffs (2) by the trade probability and reorganizing terms. The payoff from trading at p_h is $\Delta_h - \Delta_g$; all trade takes place at this high price, and there is no trade at the low price, p_0, which is offered for low signals. The payoffs of a buyer are reduced by both ν_h and ν_l because assets only trade for high signals. A buyer hence forgoes trades with both assets for low signals.

In the same vein, we can show that screening also reduces the valuation of a low-quality seller (1) whose rents are $C_h - C_l = \Delta_g + \Delta_l$ from trading at high prices for high signals, which gives

$$V_l(y,\delta) = \frac{\Delta_g + \Delta_l}{1+\nu_l(y,\delta)}. \tag{8}$$

4.3.3 Properties of equilibrium correspondences

Generally, an equilibrium with standard trade dynamics is given by y and (V_b, V_l) satisfying (6), (8), and the following system[16]

$$IC := \Delta_l - \delta\left(V_l + V_b\right) \geq 0, \tag{IC-s}$$

$$FP := q_c(y)\Delta_h + (1-q_c(y))\left(-\Delta_g\right) - q_c(y)\delta V_b - (1-q_c(y))\left(\Delta_l - \delta V_l\right) = 0, \tag{FP-s}$$

$$q_c(s) = \frac{1}{1+\frac{1-F_h(y)}{1}\frac{f_l(s)}{f_h(s)}}, \text{ for } s \in [0,1]. \tag{q-s}$$

Similarly, an equilibrium with reversed trade dynamics is given by y and (V_b, V_l) sat-

[16]See the Appendix for the details and additional commentary.

isfying (7), (8), and the system of conditions

$$IC := \Delta_l - \delta\left(V_l + V_b\right) \leq 0, \tag{IC-r}$$

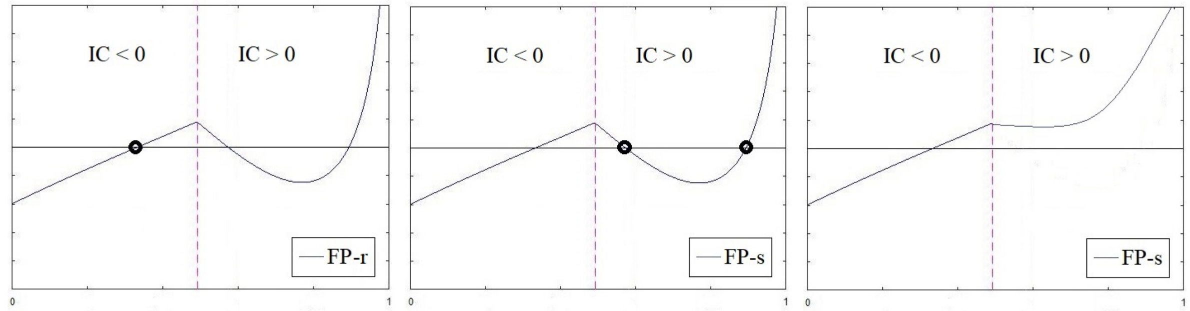
$$FP := q_c(y)\Delta_h + (1 - q_c(y))\left(-\Delta_g\right) - \delta V_b = 0, \tag{FP-r}$$

$$q_c(s) = \frac{1}{1 + \frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(s)}{f_h(s)}}, \text{ for } s \in [0,1]\,. \tag{q-r}$$

The first line in both systems denotes the *incentive condition* IC, which ascertains that dynamics are as presumed. The next line just restates the *fixed point condition* FP (3) that defines the cutoff under these dynamics. The last line defines corresponding beliefs q_c. The rhs of IC is positive if screening is stringent enough to make a low quality seller accept a low price offer. The rhs of FP is positive if buyer beliefs about quality are high enough to encourage a buyer to offer a high price.

Because ICs and the related FPs[17] are continuous in y, we can demonstrate the existence of equilibria and characterize them by locating for the roots of these correspondences for fixed (low) trade frictions. The roots of each such FP correspond to equilibria if the respective IC is satisfied. Equilibria are shown as the black circles in Figure 2.

In each case, FP[17] is negative at the lowest cutoffs for which the expected asset quality is low. A buyer will hence rather return to the market than trade low-quality assets for a high price. Buyer beliefs about traded assets improve as the cutoff is raised. More stringent screening also erodes the buyer benefit from waiting. FP thus turns positive for high enough cutoffs, corresponding with slightly positive screening. IC only becomes positive for higher cutoffs as screening intensifies.



(a) $\Delta_h > \Delta_g$ (b) $\Delta_l > \Delta_h$ (c) $\Delta_g \geq \Delta_h \geq \Delta_l$

Figure 2: Roots of FP-s and FP-r (solid lines), and IC (dashed lines).

Indeed, irrespective of whether beliefs q_c correspond with reversed dynamics (q-r) or standard dynamics (q-s), we find that the original definition of a cutoff (3) always allows for a fixed point that satisfies

$$q_c(y)\Delta_h + (1 - q_c(y))(-\Delta_g) = \delta\frac{\Delta_h - \Delta_g}{2 + \nu_h(y,\delta) + \nu_l(y,\delta)}, \tag{FP-s^0}$$

[17]Obtained by inserting (8) and either (q-s) and (6) or (q-r) and (7) into (3).

corresponding with (FP-r), (IC-r), and (7), for reversed dynamics. This fixed point defines the first roots of FP-s and FP-r in Figure 2. Further, we show that the definition (3) may allow for a higher cutoff, satisfying

$$\begin{aligned} q_c(y)\Delta_h + (1 - q_c(y))(-\Delta_g) = \\ q_c(y)\delta\frac{\Delta_h - (1 - F_l(y))\Delta_g + F_l(y)(\Delta_l - \delta\frac{\Delta_g+\Delta_l}{1+\nu_l(y,\delta)})}{2 + \nu_h(y,\delta)} + \\ (1-q_c(y))\left(\Delta_l - \delta\frac{\Delta_g + \Delta_l}{1 + \nu_l(y,\delta)}\right), \end{aligned} \tag{FP-s^l}$$

corresponding with (FP-s), (IC-s), and (6), for standard dynamics. These fixed points define the second and the third roots of FP-s and FP-r in Figure 2. We describe conditions of equilibrium existence in the following Propositions 1-3.

4.3.4 Reversed dynamics

The first equilibrium, illustrated as the black circle in Figure 2a, has the most relaxed screening and thus reversed dynamics.

Proposition 1 (Reversed dynamics) *If $\Delta_h > \Delta_g$, there exists a limit-efficient equilibrium where $\nu_h \le \nu_l \to 0$,*

$$V_l \to \Delta_l + \Delta_g, V_b \to \frac{\Delta_h - \Delta_g}{2},$$
$$W = V_b + \frac{1}{2}V_l \to \frac{\Delta_h + \Delta_l}{2},$$

as $\delta \to 1$. The equilibrium features reversed dynamics and low average market quality with $q_u = 0$ and $q_c(y) = 1/2$.

We know from Lemma 4 that reversed dynamics cannot arise under a severe lemons problem. A necessary condition for equilibrium existence is thus $\Delta_h \ge \Delta_g$. The existence of the fixed point in (FP-s^0) for a positive buyer valuation (7) demonstrates that this condition is sufficient as well.

The intuition is rather simple. For low cutoffs, the buyer is almost certain of low asset quality. At the same time, the buyer continuation value remains positive. A buyer would thus prefer to return to the market rather than risk receiving a negative payoff by trading low-quality assets for a high price.

Buyer beliefs about the quality of traded assets improve as the cutoff is raised. However, this improvement is subdued as the average asset quality deteriorates at the same time because low-quality assets leave the market more slowly under reversed dynamics. Therefore, a buyer only expects to trade assets with approximately the same probability even for the highest cutoffs.

On the other hand, since there is no severe lemons problem, this gives the buyer a positive payoff. Moreover, as assets undergo more rigorous screening, the buyer's continuation value diminishes. This decline continues until screening reaches a level where the buyer is better off making a high price offer for assets of average quality rather than returning to the market.

We turn to characterize the equilibrium in the limit as trading frictions vanish, $\delta \to 1$. Lemma 6 shows that the cutoff for vanishing frictions approaches its limit $y \to 1$. The fixed-point condition (FP-r) for the cutoff therefore approaches

$$\frac{1}{2}\Delta_h + \frac{1}{2}(-\Delta_g) = \frac{\Delta_h - \Delta_g}{2 + \nu_h + \nu_l} = V_b.$$

This demonstrates that the limit equilibrium under reversed dynamics is approached over a path which keeps the screening mild for both assets: $(\nu_h, \nu_h) \to (0, 0)$ as $(y, \delta) \to (1, 1)$. For positive frictions of trade, screening is positive but low.

Lenient screening is important for reversed dynamics where both qualities only trade for high prices as it encourages low-quality sellers to wait for high price signals. Lemma 7 is a crucial part of equilibrium derivation in showing that it is possible to approach the limit $(y, \delta) \to (1, 1)$ over a path which keeps both $\nu_h(y, \delta)$ and $\nu_l(y, \delta)$ as low as desired by keeping the cutoff below the value s_l. A limit equilibrium with the above properties thus exists. Because a buyer's payoff is $\frac{\Delta_h - \Delta_g}{2}$ and a low-quality seller's payoff is $\Delta_l + \Delta_g$, this equilibrium is also efficient; half the sellers hold a low-quality asset.

These are novel findings, extending reversed dynamics in markets with signals to a limit-efficient steady-state environment.[18] Kaya and Kim (2018) describe reversed dynamics in a non-stationary environment of unknown efficiency properties. A significant caveat to practitioners arising from our research is that, although Kaya and Kim (2018) show that reversed dynamics arise under flexible conditions assuming the prior is above the steady-state beliefs, we observe instead that reversed dynamics cannot be sustained in the long run in steady-state markets under a severe lemons problem.

The restriction may seem unfortunate. There is ample evidence of reversed dynamics in various setups (Hendel et al., 2009; Lei, 2011; Tucker et al., 2013; Albertazzi et al., 2015; Jolivet et al., 2016; Aydin et al., 2019) whereas standard dynamics seem rare (Ghose, 2009). An explanation is suggested by Lemma 5, which shows that an endogenous lemons problem arises with informative signals irrespective of whether the lemons problem is *severe*, i.e., $\Delta_g > \Delta_h$. While the literature has concentrated on severe lemons problems, real-world applications might well be dominated by non-severe ones. The severity of the

[18] Without signals, the standard dynamics in lemons markets derive straight from the *skimming property* (Fudenberg and Tirole, 1991), which states that all prices that are accepted by high-quality sellers are also accepted by low-quality sellers. If the same prices are offered to all sellers, this means that low quality is traded faster. By Lemmata 1 and 2, the skimming property holds also in this article. However, because signals enable buyers to target high prices to high-quality sellers, as accurately as desirable, the property does not suffice to characterize trade dynamics with signals.

problem – the size of the "gap" Δ_g – is hardly known in practice.

4.3.5 Standard dynamics

The remaining equilibria pictured as the black circles in Figure 2b have more intensive screening and thus standard dynamics.

Proposition 2 (Standard dynamics) *If $\Delta_h < \Delta_l$, there exist both a limit-efficient equilibrium where $\nu_h \to 0 < \nu_l \to \frac{\Delta_g+\Delta_h}{\Delta_l-\Delta_h}$*

$$V_l \to \Delta_l - \Delta_h, V_b \to \Delta_h,$$
$$W = V_b + \frac{1}{2}V_l \to \frac{\Delta_h + \Delta_l}{2},$$

as $\delta \to 1$, and a limit-inefficient equilibrium where $\nu_h \to \frac{\Delta_l-\Delta_h}{\Delta_h} < \nu_l \to \infty$

$$V_l \to 0, V_b \to \Delta_h,$$
$$W = V_b + \frac{1}{2}V_l \to \Delta_h.$$

as $\delta \to 1$. These equilibria feature standard dynamics and high average quality with $q_u = q_c(y) \to 1$.

Standard dynamics require at least moderate screening in order to allow low-quality sellers to accept low prices for low signals. However, without further screening of low-quality assets, buyer rents from low-quality trades remain low. On the other hand, the average quality of traded assets increases under standard dynamics more quickly than under reversed dynamics because low-quality assets trade for all signals and high-quality assets only for high signals. This higher quality of traded assets increases the buyer benefit of making a high price offer, while low rents from low quality trades keep the benefit of waiting for future trades subdued. Therefore, as the cutoff reaches a level for which low quality trades for low prices, a buyer is initially better off making a high price offer than a low one. The level of screening is therefore not sufficient for an equilibrium. Next, we discuss how an equilibrium can be attained for higher values of screening.

Payoff from offering p_h. As screening becomes more rigorous, the average asset quality increases. The buyer payoff for making a high price offer will therefore keep increasing with more stringent screening. Ultimately, as screening exacerbates with disappearing frictions, a buyer becomes almost certain of high asset quality. Therefore, the benefit to buyer of offering p_h approaches Δ_h in all limit equilibria where $(y, \delta) \to (1, 1)$.

Payoff from offering p_l. Initially, screening only intensifies for low-quality assets while the screening on high-quality assets remains low. More stringent screening decreases low quality seller payoffs, which leaves higher rents to buyers from trades at a low price. These rents boost the buyer payoff from making a low price offer p_l by augmenting both i. the surplus share, $\Delta_l - \delta V_l$, which a buyer obtains if p_l is accepted by a low quality seller, and ii. the buyer valuation, V_b, which a buyer receives if p_l is rejected by a high quality seller. In the limit, as the market becomes almost exclusively populated by high quality sellers, the benefit of making a low price offer p_l approaches the buyer continuation value V_b.

Buyer continuation value V_b. As discussed in deriving buyer payoffs (6), the effects of higher asset screening ($y \to 1$) and elevating asset quality ($q_u \to 1$) cancel, implying that a buyer expects to trade assets with equal probabilities, $(1 - F_h(y))q_c(y) = 1 - q_c(y)$. As the screening of low-quality assets intensifies and the rents from low trades thus accrue to buyers only, the buyer value will approach the maximum trade surplus of $\frac{\Delta_h+\Delta_l}{2}$.

If $\Delta_l > \Delta_h$, this buyer value of making a low price offer, $\frac{\Delta_h+\Delta_l}{2}$, will exceed the buyer value of making a high price offer, Δ_h. That will allow for an equilibrium where the screening of low-quality assets is stringent but the screening of high-quality assets is lenient. As frictions disappear and screening increases, the fixed-point condition (FP-s), defining the screening of different qualities, approaches

$$\Delta_h = \frac{\Delta_h + \left(\Delta_l - \frac{\Delta_g+\Delta_l}{1+\nu_l}\right)}{2 + \nu_h} = V_b. \tag{9}$$

Closer examination demonstrates that (9) has not one but two solutions that satisfy (FP-s) and (IC-s), an efficient one and an inefficient one. First, there is a solution where $\nu_h \to 0 < \nu_l \to \frac{\Delta_g+\Delta_h}{\Delta_l-\Delta_h}$ as $(y, \delta) \to (1, 1)$. Second, there is another solution with $\nu_h \to \frac{\Delta_l-\Delta_h}{\Delta_h} < \nu_l \to \infty$ as $(y, \delta) \to (1, 1)$. Lemma 7 shows that, presuming $\Delta_l > \Delta_h$, there are paths $(y, \delta) \to (1, 1)$ corresponding with such ν_h and ν_l.

The multiplicity of equilibria originates from the strategic complementarity between the screening of low and high-quality assets

$$\frac{\partial V_b}{\partial \nu_l} > 0, \frac{\partial V_b}{\partial \nu_h} < 0,$$

and the partitioning in Lemma 7 of the signal space into regions where ν_l and ν_h are low, ν_l is high but ν_h is low, and ν_l and ν_h are high.

The effect of higher ν_l on V_b is positive because stronger screening reduces low-quality sellers' payoff, permitting a buyer to capture a larger share of trade surplus Δ_l from trades with low quality sellers conducted at price p_l. This contrasts with the negative effect of ν_h on V_b. A higher ν_h implies both a higher average quality in the market and an increased buyer threshold for offering C_h. Consequently, more meetings involve high-quality assets but more of these meetings end without trade. No trade outcomes erode buyer payoffs.

The inefficient equilibrium has more stringent screening than the efficient one. Intuitively, when the screening of low-quality assets increases, buyer rents obtained from low-price trades elevate. Eventually, these rents from low-price trades, Δ_l, exceed those from high-price trades, Δ_h, making buyers more selective with proposing high prices. At the same time, stronger screening decreases the buyer benefit of offering p_l, V_b, allowing to equate the buyer payoffs from offering p_l and p_h, as required to sustain an equilibrium.

4.3.6 Non-existence

Figure 2c points out that the non-existence of an equilibrium is also a possibility. This is already suggested by our previous analysis, which shows that the maximum of total market surplus and buyer continuation value is

$$V_b = \frac{\Delta_h + \Delta_l}{2}. \tag{10}$$

If high quality is more valuable, $\Delta_h > \Delta_l$, no equilibrium with standard dynamics, where $V_h = \Delta_h$, thereby exists. Adding to this, if the lemons problem is severe, $\Delta_g > \Delta_h$, no equilibrium with reversed dynamics exists either.

Proposition 3 (Non-existence of equilibrium) *If $\Delta_g \geq \Delta_h \geq \Delta_l$, there exists no steady-state limit equilibrium as $\delta \to 1$.*

The intuition for the non-existence of an equilibrium is given by a fundamental discrepancy between i. standard dynamics required to overcome a severe lemons problem, and ii. the higher payoff of trading high quality than of trading low quality.[19] Under standard dynamics, buyers obtain the full surplus of high quality trade Δ_h as information costs vanish and market quality increases. As a result, since the benefit of waiting, Δ_h, exceeds the low trade surplus, Δ_l, buyers are no longer interested in trading low quality for low signals. This contradicts the assumption of standard dynamics.

4.3.7 Discussion

Equilibrium summary. Figure 1 summarizes the existence conditions of equilibria. There are multiple equilibria with different dynamics and efficiency properties when low quality has a higher trade surplus, whereas either a unique equilibrium or no equilibrium exists if there are higher gains from trade with low-quality assets. The relatively neglected trade value of "lemons" thus determines the trade possibilities. A major role is also given to the "gap" between the values of assets that are traded in the market. No severe lemons problem arises if the highest seller value and the lowest buyer value are close.

Refinement of equilibria. The equilibrium set can be refined by focusing on, e.g., i. *undefeated equilibria* with maximal payoffs (primarily) to buyers and (secondarily) to sellers (Mailath et al., 1993) or ii. "simple" and "robust" equilibria. Pareto-ranking clearly advocates limit-efficient equilibria. There exist two such equilibria only for $\Delta_l >$

[19] Golosov et al. (2014) analyze a related model, without proving existence.

parameters	$\Delta_l > \Delta_h$	$\Delta_h \geq \Delta_l$
$\Delta_h > \Delta_g$	reversed & efficient standard & efficient standard & inefficient	reversed & efficient
$\Delta_g \geq \Delta_h$	standard & efficient standard & inefficient	

Table 1: Existence of equilibria for $\delta \to 1$.

$\Delta_h > \Delta_g$. Among those, the equilibrium with reversed dynamics gives $V_b = \frac{\Delta_h + \Delta_l}{2}$ while the equilibrium with standard dynamics yields $V_b = \Delta_h$, which is lower for $\Delta_l > \Delta_h$. Focusing on the optimal equilibrium selection of buyers, who make the first move in each match, therefore advocates reversed dynamics. Low information needs also speak for reversed dynamics.

Comparative statics. Regarding comparative statics, we further observe that buyers' payoffs are increasing in Δ_h (and decreasing in Δ_g) while low-quality sellers' payoffs are increasing in Δ_l (increasing in Δ_g, and decreasing in Δ_h). This arises because of two forces. The first force is that efficiency considerations combined with flexible screening possibilities allow buyers and sellers to enjoy the entire trade surplus $\frac{\Delta_l + \Delta_h}{2}$. The second novel force arises as screening must keep a buyer indifferent between offering p_h and p_l or p_0. This defines buyer payoffs as $V_b = \Delta_h$ under standard dynamics ($q_u = 1$) and $V_b = \frac{\Delta_h - \Delta_g}{2}$ under reversed dynamics ($q_u = 1/2$), while leaving low quality sellers with rents $\Delta_l - \Delta_h$ and $\Delta_l + \Delta_g$, respectively. We are not aware of any counterpart to this result in the literature.

Transition to equilibrium. We find that, under the conditions where a steady-state equilibrium exists, there always exists an *efficient* steady-state equilibrium. This convergence to a limit-efficient equilibrium requires sufficiently rich information (e.g., in Moreno and Wooders (2010) no quality information is observed and in Kaya and Kim (2018) the observed information is coarse). For all that, a remaining problem that we have is that efficient trading only arises in a steady-state equilibrium. Consequently, unless the market has already reached an efficient steady-state equilibrium, the properties of the transition path will be crucial for overall efficiency. Non-steady-state dynamics may also play a key role when no steady-state equilibrium exists for $\Delta_g > \Delta_h > \Delta_l$. These pending issues are left for future studies.

4.4 Screening with bounded signal information

To demonstrate the usefulness of considering rich information structures, we next show how our analysis with unboundedly informative signals informs analyses with bounded signal information. To this goal, we suppose that there exists an upper bound $B < \infty$ on the informativeness of quality signals s, i.e., $\frac{1}{B} \leq \frac{f_h}{f_h}(s) \leq B$.

To transport the idea to our framework, we assume additionally that all more informative signals $\frac{f_h}{f_h}(s) < \frac{1}{B}$ and $\frac{f_h}{f_h}(s) > B$ are replaced by, respectively, the (lowest) signal $\underline{s}$ which gives $\frac{f_h}{f_h}(\underline{s}) = \frac{1}{B}$ and the (highest) signal $\overline{s}$ which gives $\frac{f_h}{f_h}(\overline{s}) = B$.[20] To retain the feature that high signals indicate high quality, we assume that $E(U|\overline{s}) > E(U) = \overline{U}$.

Our previous analysis permits us to derive limits on the information content of signals that suffices to sustain almost efficient trade with positive trade frictions. In other words, we obtain a new measure for bounded signal information $B < \infty$ needed for "constrained efficient screening" of assets with positive frictions $\delta < 1$.

Corollary 1 (of Propositions 1 and 2) *For any (high) $\delta < 1$ there exists $B < \infty$ such that a steady-state equilibrium generates higher welfare than the static market if $\Delta_g \geq \Delta_h$ and $\Delta_l > \Delta_h$ and almost equal payoff if $\Delta_h > \Delta_g$.*

In the limit $\delta \to 1$, equilibrium analysis can be conducted similarly as in the previous section. The upper bound on signal informativeness implies that the payoffs of offering p_h cannot exceed

$$E(U|\overline{s}) - C_h,$$

which gives Δ_h only when market quality is very high $q_u = 1$. Another novelty is that in the limit, screening becomes ineffective with bounded signals, i.e., $\nu_\theta(\overline{s}, \delta) = \frac{1-\delta F_\theta(\overline{s})}{1-F_\theta(\overline{s})} \to 0$ as $\delta \to 1$ for any $\overline{s}$.

Still, if the lemons problem is not severe, $\Delta_h > \Delta_g$, the fixed point condition remains,

$$\frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\Delta_h + \left(1 - \frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\right)(-\Delta_g) = \tag{11}$$
$$\frac{1}{2+\nu_h(y,\delta)+\nu_l(y,\delta)}\Delta_h + \frac{1}{2+\nu_h(y,\delta)+\nu_l(y,\delta)}(-\Delta_g),$$

as with unboundedly informative signals for an equilibrium with reversed dynamics. Because information requirement for this equilibrium is negligible, the equilibrium can be sustained for boundedly informative signals. Specifically, the utility of offering p_h on the lhs of (11) is $U_l - C_h$ at $y = 0$ and $E(u|\overline{s}) - C_h$ at $y = \overline{s}$ whereas the utility from p_0 on the rhs of (11) is $\overline{U} - C_h$ at $y = 0$ through $y = \overline{s}$. Given our assumptions that $\Delta_h > \Delta_g$ and $E(U|\overline{s}) > \overline{U}$, an equilibrium satisfying (11) thus exists for high values of δ. Equilibrium payoffs remain as in Proposition 1 in the limit $\delta \to 1$.

[20]Because only the upper bound is binding, the lower information bound is redundant.

Remark 1 *If $\Delta_h > \Delta_g$, an efficient equilibrium with bounded signals exists for $\delta \to 1$.*

This contrasts with cases where the lemons problem is severe. The ineffectiveness of screening low-quality assets then implies that no limit-efficient equilibrium can be sustained without mixing.

Remark 2 *If $\Delta_g \geq \Delta_h$, no pure equilibrium with bounded information exits for $\delta \to 1$.*

To make it unattractive for low-quality sellers to wait for high prices, a buyer needs to randomize between offering p_l and p_h at $s = \overline{s}$, e.g., in proportions $r_l > 0$ and $r_h > 0$, with $r_l = 1 - r_h$. This mixing is optimal for a buyer at the highest signal $\overline{s}$, which a buyer observes with probability $1 - F(\overline{s})$, if

$$\frac{1}{1 + r_h(1 - F_h(\overline{s}))\frac{f_l(\overline{s})}{f_h(\overline{s})}}\Delta_h + \left(1 - \frac{1}{1 + r_h(1 - F_h(\overline{s}))\frac{f_l(\overline{s})}{f_h(\overline{s})}}\right)(-\Delta_g) =$$
$$\frac{1}{1 + r_h(1 - F_h(\overline{s}))\frac{f_l(\overline{s})}{f_h(\overline{s})}}V_b + \left(1 - \frac{1}{1 + r_h(1 - F_h(\overline{s}))\frac{f_l(\overline{s})}{f_h(\overline{s})}}\right)(\Delta_l - \delta V_l),$$

Note that we can partly mimic either of the equilibria (y, δ) that arise with unboundely informative signals by choosing r_h for $\delta < 1$ and $\overline{s} < y$ such that

$$r_h(1 - F_h(\overline{s})) = (1 - F_h(y)) \text{ or } r_h(1 - F_l(\overline{s})) = (1 - F_l(y))$$

where $r_h \to 0$ as $\delta \to 1$. The former definition of r_h yields the same ν_h (a lower ν_l) while the latter definition delivers the same ν_l (a higher ν_h) as in the original equilibrium.

For vanishing frictions, $\delta \to 1$, the average asset quality exacerbates, $q_u = q_c(\overline{s}) \to 1$, as high prices are offered so rarely, $r_h \to 0$. The limit fixed point condition is thus the same as with unboundedly informative signals, (9).

Now, we can define the values of r_h in a way that allows to satisfy (9) and yields almost efficient payoffs for vanishing frictions. We note first that, at the lower end, if we set r_h at the highest level for which $\nu_l \geq \frac{\Delta_g}{\Delta_l}$ satisfies (IC-s) we obtain too low a ν_h to satisfy (FP-s). On the other hand, if we set r_h to replicate the ν_l in the limit-efficient equilibrium, (IC-s) is satisfied but ν_h is too high to satisfy (FP-s). As the underlying correspondencies are continuous in r_h, a fixed point r_h satisfying both (FP-s) and (IC-s) and yielding high payoffs thus exists.

The above analysis shows that, if the lemons problem is severe and signals boundedly informative, we need mixing both in high price offers and in low price offers, much like in Moreno and Wooders (2010) and Kaya and Kim (2018).

5 Conclusion

The main lessons from our analysis for practical market design are the following.

1. Large enough trade surpluses $\Delta_h > \Delta_g$, for high quality, or $\Delta_l > \Delta_h$, for low quality, are sufficient to guarantee limit-efficient trade in markets with signals.

2. Information requirements supporting limit-efficient trade are negligible for vanishing frictions if $\Delta_h > \Delta_g$ but increase in proportion to $(1-\delta)\frac{\Delta_g+\Delta_h}{\Delta_l-\Delta_h}$ if $\Delta_l > \Delta_h$.

3. With sufficient information and negligible frictions, no limit-efficient equilibrium exists in markets infested, at the same time, by i. assets with high value differences (high Δ_g) and ii. assets with low gains from trade (low Δ_h and Δ_l), for $\Delta_g \geq \Delta_h \geq \Delta_l$. As a consequence, market intervention is necessary to restore efficiency. One possibility involves introducing a *pre-selection mechanism* to exclude from markets the low-quality assets with negative contribution to market performance. This asset sorting could be implemented, for example, by imposing an entry cost to drive away low-quality assets as in Heinsalu (2020) or by providing a liquid market for trading low-quality assets as in Inderst and Müller (2002).

4. When the gains from trade are highest for low quality, $\Delta_l > \Delta_h$, the problem of multiple Pareto-ranked equilibria arises. The possibility of a coordination failure thus exists. Our pivotal finding is that efficient equilibria exhibit lenient screening of high-quality assets while inefficient equilibria feature extreme screening of low-quality assets. This suggests that screening mechanisms should not be evaluated based on whether they *prevent* the trade of low-quality assets at high prices but on whether they *enable* high-quality assets to trade without friction. When this *criterion for efficient trade* is not met, redesign of the mechanism to ensure that high quality can easily pass any market test is required.

We close by discussing some extensions and alternative modeling frameworks.

Coasian payoffs

Because uninformed buyers are given full bargaining power over informed asset sellers, it is also interesting to study whether payoffs become *Coasian* as frictions disappear, i.e., whether buyers lose all commitment power to low prices and there will be efficient trade in the limit. Fuchs and Skrzypacz (2022) argue that a form of the Coase conjecture often survives even if trading is delayed. Our finding of limit-efficient equilibria also testifies to this happening.

As for the payoffs, the Coase conjecture translated to our case could mean, e.g., that buyers trade at prices equal to the highest seller valuation, C_h, , or at prices that equate to the expected buyer utility, $E(U|s) - C_h$, with the buyer continuation value, V_b.

Indeed, when dynamics are reversed, we find that trade only occurs for high prices C_h which both high and low-quality sellers can accept. However, when dynamics are standard, a buyer will price to equate marginal utility for $s = y$ only when indifferent between offering p_l and p_h. Otherwise, buyer rents exceed V_b. In other words, in our model buyers are not always i. pricing at the highest seller valuation C_h nor ii. obtaining only their continuation value V_b. Thus, payoffs are not *Coasian* even when they are efficient.

Payoffs are non-*Coasian* under standard dynamics, in short, because signals grant the buyer an additional degree of commitment power, which is absent from models where no information is available to a buyer. Buyers know that, by waiting for a high signal, they can trade high quality with high certainty whereas, if they prefer not to wait, they also have a chance to buy low quality for low prices. As a result, buyers obtain at least $\Delta_h > 0$ when they trade for p_h and receive $\Delta_l - (\Delta_l - \Delta_h) > 0$ when they trade for p_l.

Sellers offer prices

The signaling version of our model is studied more closely in Hämäläinen (2015). Focusing on seller-optimal equilibria, this article observes that, if $\Delta_l = \lambda$ is relatively high compared to $\Delta_h = 1 - \lambda$, a steady-state equilibrium with standard dynamics exists for $\lambda \geq \underline{\lambda}$ but, if $\Delta_h = 1 - \lambda$ is instead high relative to $\Delta_l = \lambda$, a steady-state equilibrium with reversed dynamics exists for $\lambda \leq \overline{\lambda}$. Between, for $\lambda \in (\underline{\lambda}, \overline{\lambda})$ both kinds of dynamics can be supported in a steady-state equilibrium.

Standard dynamics arise in an equilibrium where sellers are pooling for high signals and separating for low signals. Reversed dynamics arise in an equilibrium where sellers pool for high signals but return to the market for low signals.[21] Seller-optimal prices leave no surplus to buyers, i.e., $V_b = 0$: Pooling prices thus equal $p(s) = E[U|s]$ whereas separating prices are $p_h = U_h$ for high quality and $p_l = U_l$ for low quality. In the seller-optimal case, $p(s)$ and p_l are accepted by buyers with probability one but, to prevent low quality from mimicking high, p_h can only be accepted with probability $\frac{p_l - V_l}{p_h - V_l} < 1$.

Efficiency properties of equilibria are not analyzed for vanishing trade frictions in Hämäläinen (2015). One possible approach would be to employ similar cutoffs as in this article, given by Lemma 7, which allow adjusting asset screening to the severity of the lemons problem. An open question is whether this would allow for limit-efficient trade, where $V_l \to \Delta_l$ and $V_h \to \Delta_h$ as $\delta \to 1$.

[21] In a so called semi-pooling equilibrium, bridging the pooling and separating cases, low-quality sellers mix between offering p_l and p_0 for $s < y$.

Different entry rates

Different entry rates, e_h for high quality, and $e_l = 1 - e_h$ for low quality, alter the market composition through the following steady-state condition

$$e_\theta = M_\theta(1 - G_-(p_\theta)).$$

Because buyers' expectations q_u and $q_c(s)$ of sellers' assets thus change, the fixed point condition under standard dynamics can be transformed into

$$\Delta_h - \frac{1-F_h(y)}{1}\frac{f_l(s)}{f_h(s)}\frac{e_l}{e_h}\Delta_g = \delta V_b' + \frac{1-F_h(y)}{1}\frac{f_l(s)}{f_h(s)}\frac{e_l}{e_h}\left(\Delta_l - \delta V_l'\right),$$

where

$$V_b' = \frac{\Delta_h - (1-F_l)\frac{e_l}{e_h}\Delta_g + F_l\frac{e_l}{e_h}(\Delta_l - \delta V_l')}{1 + \frac{e_l}{e_h} + \nu_h},$$
$$V_l' = \frac{1}{1+\nu_l}\left(\Delta_g + \Delta_l\right).$$

The fixed point condition hence turns into

$$\Delta_h = \frac{\Delta_h + \frac{e_l}{e_h}\Delta_l - \frac{e_l}{e_h}\frac{1}{1+\nu_l}\left(\Delta_g + \Delta_l\right)}{1 + \frac{e_l}{e_h}}.$$

for $\delta \to 1, y \to 1$ and $\nu_h \to 0$ and

$$\Delta_h = \frac{\Delta_h + \frac{e_l}{e_h}\Delta_l}{1 + \frac{e_l}{e_h} + \nu_h}.$$

for $\delta \to 1, y \to 1$ and $\nu_l \to \infty$.

We see that the existence condition, $\Delta_l > \Delta_h$, and the properties of equilibria with standard dynamics are unchanged. Total trade surplus is in the efficient equilibrium

$$V_b + e_l V_l = \Delta_h + e_l(\Delta_l - \Delta_h) = e_h\Delta_h + e_l\Delta_l,$$

and in the inefficient equilibrium $V_b + e_l V_l = \Delta_h + e_l 0 = \Delta_h$.

An equilibrium with reversed dynamics continues to exist if there is no severe lemons problem. Due to unequal entry rates, this existence condition however changes into

$$\Delta_h > \frac{e_l}{e_h}\Delta_g.$$

Ergo, our assumption that different asset qualities enter the market at equal rates is innocuous.

References

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *Quarterly Journal of Economics 84*(3), 488–500.

Albertazzi, U., G. Eramo, L. Gambacorta, and C. Salleo (2015). Asymmetric information in securitization: An empirical assessment. *Journal of Monetary Economics 71*, 33–49.

Araujo, F. A., S. W. Wang, and A. J. Wilson (2021). The times they are a-changing: experimenting with dynamic adverse selection. *American Economic Journal: Microeconomics 13*(4), 1–22.

Asriyan, V., W. Fuchs, and B. Green (2017). Information spillovers in asset markets with correlated values. *American Economic Review 107*(7), 2007–40.

Aydin, E., S. B. Correa, and D. Brounen (2019). Energy performance certification and time on the market. *Journal of Environmental Economics and Management 98*, 102270.

Azevedo, E. M. and D. Gottlieb (2017). Perfect competition in markets with adverse selection. *Econometrica 85*(1), 67–105.

Binmore, K. G. and M. J. Herrero (1988). Matching and bargaining in dynamic markets. *Review of Economic Studies 55*(1), 17–31.

Blouin, M. R. (2003). Equilibrium in a decentralized market with adverse selection. *Economic Theory 22*(2), 245–262.

Blouin, M. R. and R. Serrano (2001). A decentralized market with common values uncertainty: Non-steady states. *Review of Economic Studies 68*(2), 323–346.

Camargo, B., D. Gerardi, and L. Maestri (2020). Efficiency in decentralised markets with aggregate uncertainty. *Economic Journal 130*(626), 446–461.

Camargo, B. and B. Lester (2014). Trading dynamics in decentralized markets with adverse selection. *Journal of Economic Theory 153*, 534–568.

Cho, I.-K. and A. Matsui (2018). Search, adverse selection, and market clearing. *International Economic Review 59*(3), 1437–1467.

Daley, B. and B. Green (2012). Waiting for news in the market for lemons. *Econometrica 80*(4), 1433–1504.

Fuchs, W. and A. Skrzypacz (2019). Costs and benefits of dynamic trading in a lemons market. *Review of Economic Dynamics 33*, 105–127.

Fuchs, W. and A. Skrzypacz (2022). Dynamic bargaining with private information. In *In Bargaining: Current Research and Future Directions (Eds. Karagözoğlu, E. and Hyndman, K.)*, pp. 61–86. Springer.

Fudenberg, D. and J. Tirole (1991). *Game theory.* MIT press.

Gale, D. (1986a). Bargaining and competition part i: characterization. *Econometrica 54*(4), 785–806.

Gale, D. (1986b). Bargaining and competition part ii: existence. *Econometrica 54*(4), 807–818.

Gale, D. (1987). Limit theorems for markets with sequential bargaining. *Journal of Economic Theory 43*(1), 20–54.

Ghose, A. (2009). Internet exchanges for used goods: An empirical analysis of trade patterns and adverse selection. *MIS Quarterly 33*(2), 263–291.

Golosov, M., G. Lorenzoni, and A. Tsyvinski (2014). Decentralized trading with private information. *Econometrica 82*(3), 1055–1091.

Guerrieri, V. and R. Shimer (2014). Dynamic adverse selection: A theory of illiquidity, fire sales, and flight to quality. *American Economic Review 104*(7), 1875–1908.

Guerrieri, V., R. Shimer, and R. Wright (2010). Adverse selection in competitive search equilibrium. *Econometrica 78*(6), 1823–1862.

Hämäläinen, S. (2015). *Essays on Market Dynamics and Frictions. Publications of the Helsinki Center of Economic Research No 2015:5.* Ph. D. thesis, University of Helsinki.

Harsanyi, J. C. (1973). Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. *International Journal of Game Theory 2*(1), 1–23.

Heinsalu, S. (2020). Investing to access an adverse selection market. *International Journal of Industrial Organization 72*, 102660.

Hendel, I., A. Lizzeri, and M. Siniscalchi (2005). Efficient sorting in a dynamic adverse-selection model. *Review of Economic Studies 72*(2), 467–497.

Hendel, I., A. Nevo, and F. Ortalo-Magné (2009). The relative performance of real estate marketing platforms: Mls versus fsbomadison. com. *American Economic Review 99*(5), 1878–98.

Hirshleifer, J. (1971). The private and social value of information and the reward to inventive activity. *American Economic Review 61*(4), 561–574.

Hwang, I. (2018). Dynamic trading with developing adverse selection. *Journal of Economic Theory 176*, 761–802.

Inderst, R. (2005). Matching markets with adverse selection. *Journal of Economic Theory 121*(2), 145–166.

Inderst, R. and H. M. Müller (2002). Competitive search markets for durable goods. *Economic Theory 19*(3), 599–622.

Janssen, M. and S. Roy (2004). On durable goods markets with entry and adverse selection. *Canadian Journal of Economics 37*(3), 552–589.

Janssen, M. C. and S. Roy (2002). Dynamic trading in a durable good market with asymmetric information. *International Economic Review 43*(1), 257–282.

Jolivet, G., B. Jullien, and F. Postel-Vinay (2016). Reputation and prices on the e-market: Evidence from a major french platform. *International Journal of Industrial Organization 45*, 59–75.

Kaya, A. and K. Kim (2018). Trading dynamics with private buyer signals in the market for lemons. *Review of Economic Studies 85*(4), 2318–2352.

Kim, K. (2017). Information about sellers' past behavior in the market for lemons. *Journal of Economic Theory 169*, 365–399.

Lei, Q. (2011). Financial value of reputation: Evidence from the ebay auctions of gmail invitations. *Journal of Industrial Economics 59*(3), 422–456.

Mailath, G. J., M. Okuno-Fujiwara, and A. Postlewaite (1993). Belief-based refinements in signalling games. *Journal of Economic Theory 60*(2), 241–276.

Martel, J., K. S. Mirkin, and B. Waters (2022). Learning by owning in a lemons market. *Journal of Finance 77*(3), 1737–1785.

Moreno, D. and J. Wooders (2010). Decentralized trade mitigates the lemons problem. *International Economic Review 51*(2), 383–399.

Palazzo, F. (2017). Search costs and the severity of adverse selection. *Research in Economics 71*(1), 171–197.

Rostek, M. and J. H. Yoon (2021). Exchange design and efficiency. *Econometrica 89*(6), 2887–2928.

Serrano, R. and O. Yosha (1993). Information revelation in a market with pairwise meetings: the one sided information case. *Economic theory 3*(3), 481–499.

Serrano, R. and O. Yosha (1996). Welfare analysis of a market with pairwise meetings and asymmetric information. *Economic Theory 8*(1), 167–175.

Taylor, C. (1999). Time-on-the-market as a sign of quality. *Review of Economic Studies 66*(3), 555–578.

Tucker, C., J. Zhang, and T. Zhu (2013). Days on market and home sales. *RAND Journal of Economics 44*(2), 337–360.

Wolinsky, A. (1990). Information revelation in a market with pairwise meetings. *Econometrica 58*(1), 1–23.

Zhu, H. (2012). Finding a good price in opaque over-the-counter markets. *Review of Financial Studies 25*(4), 1255–1285.

Appendix

Roadmap of results

1. Lemma 1: Sellers accept prices above a cutoff, given by their opportunity costs of trade.

2. Lemma 2: Buyers offer a high price above a cutoff signal, when the expected asset quality is high, and offer a low price below a cutoff signal, when the expected asset quality is low. All sellers accept a high price. High-quality sellers reject a low price, while low-quality sellers accept a low price if the future expected payoffs of buyers and low-quality sellers remain below their mutual gains from trade; otherwise, they also reject a low price.

3. Lemma 3: Trade dynamics are either standard or reversed, or knife-edge, depending on whether the future expected payoffs of buyers and low-quality sellers are below, above, or equal their mutual gains from trade.

4. Lemma 4: reversed dynamics cannot arise under a severe lemons problem because reversed dynamics reduces market quality, implying that the average asset quality at the cutoff signal remains below (low) entry quality.

5. Lemma 5: the cutoff signal remains positive for positive frictions because buyers are not willing to make high offers for the lowest signals.

6. Lemma 6: the cutoff signal approaches unity as frictions disappear. For standard dynamics, the intuition is that the screening of low quality must remain high enough

as frictions become smaller to discourage low-quality sellers from waiting for high signals in order to trade. For reversed dynamics, the intuition is that the expected quality traded in this period at the cutoff signal is lower than the expected quality traded in the next period. As waiting costs vanish, buyers thus prefer to wait, increasing the cutoff signal.

7. Lemma 7: We can adjust the cutoff continuously to the reducing trade frictions such that either the screening of both assets is low, the screening of low-quality assets is high but the screening of high-quality assets is low, or the screening of both assets is high, or anything between these three boundary options.

   Payoffs depend on screening. Because market quality adjusts to screening, a buyer expects to trade both assets with equal probability in the future. Screening of low-quality assets increases buyer rents from trades at low prices. Asset screening reduces buyer payoffs if the asset is not traded at low prices.

8. Proposition 1: If there is no severe lemons problem, a limit-equilibrium with reversed dynamics arises. While buyers only offer high prices for the highest signals, screening of both assets remains low relative to frictions, encouraging low-quality sellers to wait for high prices as required for reversed dynamics. The expected quality traded at the cutoff signal approaches the average quality of assets traded in future meetings. There is low positive screening of low-quality assets for low positive frictions. As screening decreases the benefits to buyers from waiting, buyers become willing to offer high prices for high signals. In the limit, no screening of either asset quality is required.

9. Proposition 2 If the low trade surplus is larger, a limit-equilibrium with standard dynamics arises. Again buyers only offer high prices for the highest signals, but now the screening of low-quality assets must be high enough, to make the sellers accept low prices for low signals as required for standard dynamics. The expected quality traded at the cutoff signal exceeds the average quality of assets traded in future meetings. This encourages buyers to reduce their cutoff. Buyers only become willing to offer low prices below the cutoff as the benefits of waiting increase with stronger screening of low-quality assets, which entails higher rents to buyers from these low-quality trades.

   There exists both a limit-efficient and a limit-inefficient equilibrum. The efficient equilibrium features low screening of high-quality assets and positive screening of low-quality assets, which increases buyer benefits of waiting until it reaches the benefit of trading high quality at high prices. The inefficient equilibrium features positive screening of high-quality assets and high screening of low-quality assets, which reduces buyer benefits of waiting until it again crosses the benefit of trading high quality at high prices. Buyer uses a more stringent screening cutoff in the inefficient equilibrium. This is because the rents to buyers from trade at low prices increase as the screening of low-quality assets becomes stronger. This implies that buyers require higher rents from trading at high prices, intensifying the screening of high-quality assets.

10. Proposition 3. If there is a severe lemons problem and the high trade surplus is larger, no limit-equilibrium arises. This is because severe lemons problem requires standard dynamics. Standard dynamics increase the average asset quality in the market. As buyer payoffs hence approach the high trade surplus, they are not willing to trade low quality at low prices, as required for standard trade dynamics.

Proof of Lemma 1: Seller cutoff strategies

The problem of a seller is given by (1). By accepting a price p, seller obtains $p - C_\theta$ and, by rejecting a price, the seller obtains δV_θ. As a result, it is optimal for the seller to accept the price if

$$p - C_\theta \geq \delta V_\theta.$$

Otherwise, it is optimal for the seller to reject the price. □

Proof of Lemma 2: Buyer cutoff strategies

The problem of a buyer is given by (2). A buyer knows that the seller strategy is a cutoff strategy, given by Lemma 2. Thus, a seller of quality θ accepts any price equal or above a cutoff $p_\theta = C_\theta + \delta V_\theta$.

Step 1. Ranking of cutoffs.

We can show that the highest price offered in the market, $\bar{p}$, equals the highest cutoff, $\max_\theta p_\theta$, because a buyer could otherwise lower its offer to, for example, $\delta\bar{p}+(1-\delta)\max_\theta p_\theta$ and each seller would accept it.

We can also demonstrate that the highest price offered in the market, $\bar{p}$, cannot exceed C_h because, otherwise, a buyer could offer a lower price, $\delta\bar{p} + (1-\delta)C_h \geq \max_\theta p_\theta$, which either seller would still accept.

Altogether, this implies that seller continuation values V_θ cannot exceed $C_h - C_\theta$, obtained by trading at the highest price $\bar{p}$ in the next period. Therefore, the cutoff p_h of a high-quality seller cannot exceed $C_h + \delta(C_h - C_h) = C_h$, which shows that $p_h = C_h$. Similarly, the cutoff p_l of a low-quality seller cannot exceed $C_l + \delta(C_h - C_l)$, which gives $p_l \leq (1-\delta)C_l + \delta C_h < p_h = C_h$. This demonstrates that $p_l < p_h$.

Step 2. Optimal price offers.

Three things can happen in a meeting between a buyer and a seller: i. a buyer offers a price $p \geq p_h$ that both sellers accept, ii. a buyer offers a price $p \in [p_l, p_h)$ that a high-quality seller rejects but a low-quality seller accepts, and iii. a buyer offers a price $p < p_l$ that both sellers reject. As a result, since any price exceeding a cutoff is accepted by a seller, it is optimal for a buyer to reduce any price $p > p_h$ to p_h and any price $p \in (p_l, p_h)$ to p_l. As no seller accepts a price $p < p_l$ its optimal level remains indeterminate.

Step 3. Optimal high offers.

The highest price offer p_h equals the highest payoff C_h. Because p_h is the highest price offered in the market, the seller continuation value cannot exceed $\delta(p_h - C_h)$. This is what the seller would obtain by trading at the highest price p_h in the next time period. A seller will thus accept any price

$$p - C_h \geq \delta(p_h - C_h)p \geq \delta p_h + (1-\delta)C_h.$$

If $p_h > C_h$, it is optimal for a buyer to offer the seller $\delta p_h + (1-\delta)C_h$ and not p_h. Because a seller obtain a lower payoff by trading later at p_h, a buyer can offer a price lower than p_h in the current meeting. This beneficial deviation arises as long as $p_h > C_h$. The optimal highest price offer is therefore $p_h = C_h$.

Step 4. Optimal low offers.

Buyer knows that a low price offer p_l is accepted by low-quality sellers and rejected by high-quality sellers. If a buyer trades with a low-quality seller, the benefit to the buyer is

$$U_l - p,$$

while the benefit to the low-quality seller is

$$p - C_l.$$

As a result, the buyer is better of trading with a low-quality seller rather than returning to the market if and only if $U_l - p \geq \delta V_b$ while the low-quality seller is better of trading with a buyer rather than returning to the market if and only if $p - C_l \geq \delta V_l$.

Thus, the upper bound for acceptable prices is $U_l - \delta V_b$ for a buyer whereas the lower bound for acceptable prices is $C_l + \delta V_l$ for the low-quality seller. There are prices to which both can agree if and only if

$$C_l + \delta V_l \leq U_l - \delta V_b \Longleftrightarrow \Delta_l \geq \delta(V_l + V_b).$$

As a result, presuming that a buyer does not make a high price offer, p_h, buyer will offer p_l if $p_l = C_l + \delta V_l < U_l - \delta V_b$ but p_0 if $p_l = C_l + \delta V_l > U_l - \delta V_b$. In case of indifference, a buyer may offer either p_l or p_0.

Step 5. Cutoff for offers.

We denote by q the probability that the seller has a high-quality asset.

The payoff to a buyer for making a high price offer $p_h = C_h$ is

$$q(U_h - C_h) + (1-q)(U_l - C_h) = q\Delta_h + (1-q)(-\Delta_g)$$

The payoff to a buyer for making a low price offer $p_h = C_l + \delta V_l$ is

$$(1-q)(U_l - p_l) + q\delta V_b,$$

and the payoff to a buyer for making a low price offer $p_0 < p_l$ is δV_b.

Therefore, a buyer prefers offering p_h if

$$q\Delta_h + (1-q)(-\Delta_g) \geq \max\{q\delta V_b + (1-q)(U_l - p_l), \delta V_b\}.$$

but otherwise offers either p_l or p_0

Case 1: $U_l - p_l > V_b$. Buyer prefers p_l over p_0.

Buyer prefers p_h over p_l if

$$\begin{aligned} q\Delta_h + (1-q)(-\Delta_g) &> q\delta V_b + (1-q)(U_l - p_l), \\ q &> \frac{\Delta_g + (U_l - p_l)}{\Delta_g + (U_l - p_l) + \Delta_h - \delta V_b}, \end{aligned}$$

where $q < 1$ for $\Delta_h > \delta V_b$ and $q \geq 1$ for $\Delta_h \leq \delta V_b$.

Case 2: $U_l - p_l \leq V_b$. Buyer prefers p_0 over p_l.

Buyer prefers p_h over p_0 if

$$\begin{aligned} q\Delta_h + (1-q)(-\Delta_g) &> \delta V_b, \\ q &> \frac{\Delta_g + \delta V_b}{\Delta_g + \Delta_h}, \end{aligned}$$

where $q < 1$ for $\Delta_h > \delta V_b$ and $q \geq 1$ for $\Delta_h \leq \delta V_b$.

This shows that there is a cutoff belief $q(y)$ above which a buyer prefers offering p_h over p_l or p_0, whichever gives a higher buyer payoff.

As $q_c(s)$ in (5) is increasing in s, the cutoff belief $q(y)$ defines a cutoff signal y above which a buyer prefers offering p_h over p_l or p_0.

We allow in Lemma 2 for the possibility that the cutoff belief and the cutoff signal can equal unity, which just means that a buyer prefers making a low price offer for all beliefs and for all signals. □

Proof of Lemma 3

Lemma 3 is a corollary of Lemmata 1 and 2.

Step 1. Seller trade probabilities.

Lemma 2 shows that a buyer offers p_h for $s \geq y$. Both high and low-quality assets thus trade for these signals.

Lemma 2 shows that a buyer offers p_1 for $s < y$ if $\Delta_l \geq \delta(V_l + V_b)$. Only low quality thus trades for these signals.

Lemma 2 also shows that a buyer offers p_0 for $s < y$ if $\Delta_l \leq \delta(V_l + V_b)$. Neither seller thus trades for these signals.

Step 2. Unconditional buyer beliefs.

Suppose that $\Delta_l > \delta(V_l + V_b)$. Because high quality trades at high signals ($s \geq y$), with probability $1 - F_h(y)$, but low quality trades for all signals, with probability one, trade dynamics are standard and unconditional buyer beliefs (4) given by

$$q_u = \frac{1}{1 + (1 - F_h(y))} > 1/2.$$

Suppose that $\Delta_l < \delta(V_l + V_b)$. Because both qualities are traded only at high signals ($s \geq y$), which high-quality assets generate more often than low ($1 - F_h(y) > 1 - F_l(y)$), trade dynamics are reversed and unconditional buyer beliefs (4) given by

$$q_u = \frac{1}{1 + \frac{1 - F_h(y)}{1 - F_l(y)}} < 1/2.$$

Step 3. Knife-edge trade dynamics.

If $\Delta_l = \delta(V_l + V_b)$, a buyer offers p_h for high signals ($s \geq y$) but is indifferent between offers p_0 and p_l for low signals ($s < y$). A buyer is hence willing to either mix between the offers, or offer p_0 for some low signals and offer p_1 for other low signals. We show next that these alternatives are payoff-equivalent if trade probabilities are unchanged.

To define a mixed equilibrium, we suppose that a buyer mixes between p_0 and p_l in proportions $r_0 >$ and $r_l = 1 - r_0 > 0$ for $s < y$, resulting in payoffs that satisfy $\Delta_l = \delta(V_l + V_b)$. Now, we can define a simple pure equilibrium with the same payoffs as follows. First, we find a new low cutoff signal $z < y$ such that

$$r_0 = \frac{F_l(z)}{F_l(y)} \text{ and } r_1 = \frac{F_l(y) - F_l(z)}{F_l(y)}.$$

Second, we suppose that a buyer offers p_0 for all $s \in [0, z)$ and offers p_l for all $s \in [z, y)$. Similarly as with the original mixed strategy, a low quality seller thus trades at p_0 with probability r_0 and at p_l with probability r_l when the signal is low ($s < y$) with this pure strategy. This results in the unconditional buyer beliefs being

$$q_u = \frac{1}{1 + \frac{1-F_h(y)}{1-F_l(z)}} \lesseqgtr 1/2. \quad \square$$

Proof of Lemma 5

We show that $y > 0$ by contradiction, by assuming to the contrary that $y = 0$. Lemma 2 shows that $y = 0$ implies that buyers offer $p_h = C_h$ for all signals $s \geq 0$ which all sellers accept by Lemmata 1–2. Thus, as all sellers trade at all signals, unconditional buyer beliefs (4) are

$$q_u = \frac{1}{1 + \frac{1}{1}} = 1/2$$

and conditional buyer beliefs (5) are

$$q_c(s) = \frac{1}{1 + \frac{f_l(s)}{f_h(s)}}.$$

According to Assumption 1, conditional buyer beliefs (5) are continuous in s, and $q_c(s) \to 0$ as $\frac{f_h(s)}{f_l(s)} \to 0$ for $s \to 0$.

This continuity entails that for any number $\epsilon > 0$ there exists a number $\delta(\epsilon) > 0$ such that $q_c(s) < \epsilon$ for all $s < \delta$.

In particular, if we choose the number $\epsilon = \frac{\Delta_g}{\Delta_h + \Delta_g} > 0$, then conditional buyer beliefs $q_c(s)$ are so low for all signals $s < \delta(\epsilon)$ that a buyer's expected benefit from offering a high price is negative,

$$E(U|s) - C_h < \epsilon\Delta_h - (1-\epsilon)\Delta_g < \frac{\Delta_g}{\Delta_h + \Delta_g}\Delta_h - \frac{\Delta_h}{\Delta_h + \Delta_g}\Delta_g = 0.$$

But this shows that a buyer strictly prefers offering p_0 or p_l to offering $p_h = C_h$ for $s < \delta(\epsilon)$, that are observed with the positive probability of $\frac{F_h(\delta(\epsilon)) + F_l(\delta(\epsilon))}{2}$. This contradicts the assumption that $y = 0$. $\square$

Derivation of value functions

Step 1. Seller value functions.

The payoff of a seller of quality θ who obtains the price offer p is given by the Bellman equation (1),

$$V_\theta(p) = \max_{a_\theta} \quad a_\theta(p - C_\theta) + (1 - a_\theta)\delta V_\theta,$$

Since the price p follows the distribution of offers G_θ, the expected valuation of this seller type θ is

$$V_\theta = \int V_\theta(p) dG_\theta(p).$$

Lemmata 1–2 show that a seller of quality θ obtains a high price offer, p_h, with probability $1 - F_l(y)$ and obtains a low price offer, either p_l or p_0, with probability $F_l(y)$. Price p_h gives a seller the payoff of $(C_h - C_\theta)$ as all sellers accept this high price. Prices p_l and p_0 give a seller equal payoff δV_θ as a high-quality seller will never accept them whereas a low-quality seller is indifferent between accepting and rejecting them. Thus, we can express the seller value function as

$$\begin{aligned} V_\theta &= (1 - F_\theta(y))(C_h - C_\theta) + F_\theta(y)\delta V_\theta, \\ (1 - \delta F_\theta(y))V_\theta &= (1 - F_\theta(y))(C_h - C_\theta), \\ V_\theta &= \frac{1 - F_\theta(y)}{1 - \delta F_\theta(y)}(C_h - C_\theta). \end{aligned}$$

This shows that

$$\begin{aligned} V_h &= \frac{C_h - C_h}{\frac{1-\delta F_h(y)}{1-F_h(y)}} = \frac{0}{1+\nu_h} = 0, \\ V_l &= \frac{C_h - C_l}{\frac{1-\delta F_l(y)}{1-F_l(y)}} = \frac{\Delta_g + \Delta_l}{1+\nu_l} > 0, \end{aligned}$$

where $\nu_\theta := \frac{1-\delta F_\theta(y)}{1-F_\theta(y)} - 1 = \frac{1-F_\theta(y)+F_\theta(y)+\delta F_\theta(y)}{1-F_\theta(y)} - 1 = 1 + \frac{F_\theta(y)+\delta F_\theta(y)}{1-F_\theta(y)} - 1 = (1-\delta)\frac{F_\theta(y)}{1-F_\theta(y)}$. This gives Eq. (8).

Step 2. Buyer value functions.

The payoff of a buyer who obtains a quality signal s is given by the Bellman equation (2).

$$\begin{aligned} V_b(s) = \max_p \quad & q(s)a_h(p)(U_h - p) + (1 - q(s))a_l(p)(U_l - p) + \\ & (q(s)(1 - a_h(p)) + (1 - q(s))(1 - a_h(p)))\delta V_b, \end{aligned}$$

Lemma 2 shows that a buyer makes a high price offer p_h for $s \geq y$ and makes a low price offer p_l or p_0 for $s < y$. The unconditional probability of meeting a high-quality sellers is q_u and that of meeting a low-quality seller is $1 - q_u$. The expected buyer valuation can thus be decomposed into the following four components

$$\begin{aligned} V_b =& q_u \int_y^1 \delta V_b dF_h(s) + q_u \int_0^y (U_h - p_h) dF_h(s) + \\ & (1 - q_u) \int_y^1 \max\{U_l - p_l, \delta V_b\} dF_l(s) + (1 - q_u) \int_0^y (U_l - p_h) dF_l(s), \end{aligned}$$

denoting the buyer value of offering high and low prices to high- and low-quality sellers, respectively. By Assumption 1, the probability of observing a signal above the cutoff y is $1 - F_h(y)$ with a high-quality seller and $1 - F_l(y)$ with a low-quality seller. We can therefore simplify the expression of buyer payoffs as follows

$$\begin{aligned} V_b =& q_u F_h(y)\delta V_b + q_u(1 - F_h(y))(U_h - p_h) + \\ & (1 - q_u)F_l(y)\max\{U_l - p_l, \delta V_b\} + (1 - q_u)(1 - F_l(y))(U_l - p_h) = \\ & q_u F_h(y)\delta V_b + q_u(1 - F_h(y))\Delta_h + \\ & (1 - q_u)F_l(y)\max\{\Delta_l - \delta V_l, \delta V_b\} + (1 - q_u)(1 - F_l(y))(-\Delta_g) \end{aligned}$$

Case 1. $\Delta_l \geq \delta(V_l + V_b)$. By Lemma 3, this case results in standard dynamics. Lemma 3 also shows that, under standard trade dynamics, a buyer expects to trade high and low quality with the same probability,

$$q_u(1 - F_h(y)) = \frac{1 - F_h(y)}{1 + 1 - F_h(y)} = 1 - q_u.$$

The buyer value can be obtained by solving for V_b in

$$\begin{aligned} V_b =& q_u F_h(y)\delta V_b + q_u(1 - F_h(y))\Delta_h + \\ & (1 - q_u)F_l(y)(\Delta_l - \delta V_l) + (1 - q_u)(1 - F_l(y))(-\Delta_g) \\ (1 - \delta q_u F_h(y))V_b =& (1 - q_u)\Delta_h + \\ & (1 - q_u)F_l(y)(\Delta_l - \delta V_l) + (1 - q_u)(1 - F_l(y))(-\Delta_g) \\ V_b =& \frac{\Delta_h + F_l(y)(\Delta_l - \delta V_l) + (1 - F_l(y))(-\Delta_g)}{\frac{1 - \delta q_u F_h(y)}{1 - q_u}} \\ V_b =& \frac{\Delta_h + F_l(y)(\Delta_l - \delta V_l) + (1 - F_l(y))(-\Delta_g)}{2 + \nu_h} \end{aligned}$$

where $\frac{1-\delta q_u F_h(y)}{1-q_u} = \frac{1-q_u+q_u-\delta q_u F_h(y)}{1-q_u} = 1 + \frac{q_u(1-\delta F_h(y))}{1-q_u} = 1 + \frac{1-\delta F_h(y)}{1-F_h(y)} = 2 + \nu_h$. This gives Eq. (6).

Case 2. $\Delta_l \leq \delta(V_l + V_b)$. By Lemma 3, this case results in reversed dynamics. Lemma 3 also shows that, under reversed trade dynamics, a buyer expects to trade high and low quality with the same probability,

$$q_u(1 - F_h(y)) = \frac{(1 - F_h(y))(1 - F_l(y))}{1 - F_h(y) + 1 - F_l(y)} = (1 - q_u)(1 - F_l(y)).$$

The buyer value can be obtained by solving for V_b in

$$\begin{aligned} V_b =& q_u F_h(y)\delta V_b + q_u(1-F_h(y))\Delta_h + \\ & (1-q_u)F_l(y)\delta V_b + (1-q_u)(1-F_l(y))(-\Delta_g) \\ (1-\delta q_u F_h(y) - \delta(1-q_u)F_l(y))V_b =& q_u(1-F_h(y))\Delta_h + q_u(1-F_h(y))(-\Delta_g) \\ V_b =& \frac{\Delta_h - \Delta_g}{\frac{1-\delta q_u F_h(y)-\delta(1-q_u)F_l(y)}{q_u(1-F_h(y))}} \\ V_b =& \frac{\Delta_h - \Delta_g}{2+\nu_h+\nu_l}, \end{aligned}$$

where

$$\begin{aligned} &\frac{1-\delta q_u F_h(y) - \delta(1-q_u)F_l(y)}{q_u(1-F_h(y))} = \\ &\frac{q_u + (1-q_u) - \delta q_u F_h(y) - \delta(1-q_u)F_l(y)}{q_u(1-F_h(y))} = \\ &\frac{q_u(1-\delta F_h(y)) + (1-q_u)(1-\delta F_l(y))}{q_u(1-F_h(y))} = \\ &\frac{1-\delta F_h(y)}{1-F_h(y)} + \frac{(1-q_u)(1-\delta F_l(y))}{q_u(1-F_h(y))} = \\ &\frac{1-\delta F_h(y)}{1-F_h(y)} + \frac{(1-F_h(y))(1-\delta F_l(y))}{(1-F_l(y))(1-F_h(y))} = \\ &\frac{1-\delta F_h(y)}{1-F_h(y)} + \frac{1-\delta F_l(y)}{1-F_l(y)} = 2+\nu_h+\nu_l. \end{aligned}$$

This gives Eq. (7).

Proof of Lemma 6

By Lemma 3, trade dynamics can be either standard, reversed, or knife-edge. We show that $y \to 1$ as $\delta \to 1$ separately for these cases, covering standard and knife-edge dynamics in Step 1 and reversed dynamics in Step 2. In each case, the proof is by contradiction, derived with too low screening.

Step 1. Consider any cutoff y satisfying (3) as

$$q_c(y)\Delta_h + (1-q_c(y))(-\Delta_g) = q_c(y)\delta V_b + (1-q_c(y))(\Delta_l - \delta V_l).$$

This case requires that $\Delta_l \le \delta V_l \le \delta(V_l + V_b)$. By applying the previously derived expression (8) for V_l, this condition can be rewritten as

$$\Delta_l \leq \delta V_l = \delta \frac{\Delta_g + \Delta_l}{1+\nu_l} = \delta \frac{1-F_l(y)}{1-\delta F_l(y)}(\Delta_g + \Delta_l).$$

The rhs approaches $\Delta_g + \Delta_l$ as $\delta \to 1$ if $y < 1$, invalidating the inequality. This shows that the condition cannot be satisfied unless $y \to 1$ as $\delta \to 1$.

Step 2. Consider any cutoff y satisfying (3) as

$$q_c(y)\Delta_h + (1 - q_c(y))(-\Delta_g) = \delta V_b.$$

This case requires that $\Delta_l \geq \delta(V_l + V_b)$, which is consistent with knife-edge dynamics, already covered in Step 1, and with reversed dynamics, which we cover in Step 2. Under reversed dynamics, $q_c(y) = \frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}$, as shown by Lemma 3, and V_b is given by the previously derived expression (7). The condition defining the cutoff thus becomes

$$\frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\Delta_h + \left(1 - \frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\right)(-\Delta_g) = \delta\frac{\Delta_h - \Delta_g}{2+\nu_h+\nu_l}, \tag{12}$$

Now, Assumption 1 introduces MLRP, which implies that $\frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}} < 1/2$. Also, if $y < 1$, then

$$\nu_\theta = (1-\delta)\frac{F_\theta(y)}{1-F_\theta(y)} \to 0,$$

as $\delta \to 1$. As a result, we can see that the rhs of (12) exceeds the lhs of (12) unless $y \to 1$ as $\delta \to 1$. □

We follow by additional analysis regarding the cutoff under standard dynamics.

Step 3. Again, consider any cutoff y satisfying (3) as

$$q_c(y)\Delta_h + (1 - q_c(y))(-\Delta_g) = \delta V_b.$$

As discussed previously, this case requires that $\Delta_l \geq \delta(V_l + V_b)$, but now we assume that this cutoff arises with average market quality derived from standard dynamics. Under standard dynamics, $q_c(y) = \frac{1}{1+(1-F_h(y))\frac{f_l(y)}{f_h(y)}}$, as shown by Lemma 3, and V_b is given by the expression derived beneath in Step 4. The condition defining the cutoff thus becomes

$$\frac{1}{1+(1-F_h(y))\frac{f_l(y)}{f_h(y)}}\Delta_h + \left(1 - \frac{1}{1+(1-F_h(y))\frac{f_l(y)}{f_h(y)}}\right)(-\Delta_g) = \delta\frac{\Delta_h - (1-F_l(y))\Delta_g}{2+\nu_h+(1-F_l(y))\nu_l}.$$

As before, if $y < 1$, then

$$\nu_\theta = (1-\delta)\frac{F_\theta(y)}{1-F_\theta(y)} \to 0,$$

as $\delta \to 1$, which gives

$$\frac{1}{1+(1-F_h(y))\frac{f_l(y)}{f_h(y)}}\Delta_h + \frac{(1-F_h(y))\frac{f_l(y)}{f_h(y)}}{1+(1-F_h(y))\frac{f_l(y)}{f_h(y)}}(-\Delta_g) = \frac{\Delta_h - (1-F_l(y))\Delta_g}{2}.$$

By the MLRP, $(1-F_h(y))\frac{f_l(y)}{f_h(y)} > 1 - F_l(y)$. As a result, we can see that the rhs of (12) exceeds the lhs of (12) as $\delta \to 1$ unless y exceeds the value for which $(1-F_h(y))\frac{f_l(y)}{f_h(y)} = 1$. Thus, conditional buyer beliefs $q_c(c)$ at the cutoff must exceed 1/2 in this case.

Step 4. The buyer value can be obtained by solving for V_b assuming $q_u = \frac{1}{1+(1-F_h(y))}$, which gives

$$\begin{aligned}
V_b =& q_u F_h(y)\delta V_b + q_u(1-F_h(y))\Delta_h + \\
& (1-q_u)F_l(y)\delta V_b + (1-q_u)(1-F_l(y))(-\Delta_g) \\
(1-\delta q_u F_h(y) - \delta(1-q_u)F_l(y))V_b =& q_u(1-F_h(y))\Delta_h + q_u(1-F_h(y))(-\Delta_g) \\
V_b =& \frac{\Delta_h - (1-F_l(y))\Delta_g}{\frac{1-\delta q_u F_h(y) - \delta(1-q_u)F_l(y)}{q_u(1-F_h(y))}} \\
V_b =& \frac{\Delta_h - (1-F_l(y))\Delta_g}{2+\nu_h+(1-F_l(y))\nu_l},
\end{aligned}$$

where

$$\begin{aligned}
&\frac{1-\delta q_u F_h(y) - \delta(1-q_u)F_l(y)}{q_u(1-F_h(y))} = \\
&\frac{q_u + (1-q_u) - \delta q_u F_h(y) - \delta(1-q_u)F_l(y)}{q_u(1-F_h(y))} = \\
&\frac{q_u(1-\delta F_h(y)) + (1-q_u)(1-\delta F_l(y))}{q_u(1-F_h(y))} = \\
&\frac{1-\delta F_h(y)}{1-F_h(y)} + \frac{(1-q_u)(1-\delta F_l(y))}{q_u(1-F_h(y))} = \\
&\frac{1-\delta F_h(y)}{1-F_h(y)} + (1-F_l(y))\frac{1-\delta F_l(y)}{1-F_l(y)} = 2+\nu_h+(1-F_l(y))\nu_l.
\end{aligned}$$

Proof of Lemma 7

Step 1. Relative asset screening.

We start by rewriting ν_l/ν_h as

$$\frac{\nu_l(y,\delta)}{\nu_h(y,\delta)} = \frac{1-\delta}{1-\delta}\frac{F_l(y)}{F_h(y)}\frac{1-F_h(y)}{1-F_l(y)},$$

and study its limit as $y \to 1$. Since $F_\theta(y) \to 1$ as $y \to 1$, L'Hopital's rule dictates that

$$\frac{1 - F_h(y)}{1 - F_l(y)} \to \frac{f_h(y)}{f_l(y)} \to \infty,$$

as $y \to 1$. This equals saying that for any $M > 1$ there exists a signal $s^\star < 1$ such that

$$\frac{\nu_l(y, \delta)}{\nu_h(y, \delta)} \geq \frac{1}{M^2} \tag{13}$$

for all $y \geq s^\star$, for any $\delta < 1$.

Step 2. Asset-specific screening.

We continue by studying

$$\nu_\theta(y, \delta) = (1 - \delta)\frac{F_\theta(y)}{1 - F_\theta(y)},$$

which is increasing in y, approaches zero as $y \to 0$, and approaches infinity as $y \to 1$. We can thus show that for any $M > 1$ and the associated $s^\star$ such that $\nu_l(s^\star)/\nu_h(s^\star) \geq 1/M^2$, there exist $\delta \geq 1 - 1/M$ and $s_0 < s_l < s_h$ such that

$$\nu_h(s_l, \delta) = (1 - \delta)\frac{F_h(s_l)}{1 - F_h(s_l)} = 1/M, \tag{14}$$

$$\nu_l(s_0, \delta) = (1 - \delta)\frac{F_l(s_0)}{1 - F_l(s_0)} = 1/M. \tag{15}$$

and

$$\nu_h(s_h, \delta) = (1 - \delta)\frac{F_h(s_h)}{1 - F_h(s_h)} = \nu_l(s_l, \delta) = (1 - \delta)\frac{F_l(s_l)}{1 - F_l(s_l)}. \tag{16}$$

If $\frac{F_h(s^\star)}{1 - F_h(s^\star)} \geq 1$, we set $s_l = s^\star$. Otherwise, we set $s_l = s > s^\star$ where s is the lowest signal satisfying $\frac{F_h(s)}{1 - F_h(s)} = 1$. Because $\nu_l(y, \delta) > \nu_h(y, \delta)$, s_0 satisfying (15) is smaller than s_l satisfying (14), whereas s_l smaller than s_h satisfying (16).

The result follows from (13), (14), (15), and (16), which show that

$$\nu_l(s_0, \delta) = \nu_h(s_l, \delta) = 1/M < M \leq \nu_l(s_l, \delta) = \nu_h(s_h, \delta).$$

As $\nu_l/\nu_h \to \frac{f_h(y)}{f_l(y)} \to \infty$ as $y \to 1$, we can see from (13) that $s^\star \to 1$ as $M \to \infty$. Signals s_0, s_l, s_h increase as signal $s^\star \to 1$ increases. The convergence of $s^\star$ to unity as $M \to \infty$ implies that also s_0, s_l, and s_h converge to one as $M \to \infty$. □

Proof of Proposition 1

Under reversed trade dynamics, the cutoff signal y satisfies (FP-r) and (IC-r). The fixed point condition $FP = rhs - lhs = 0$ in (FP-r) is equivalent to $rhs = lhs$ in

$$\frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\Delta_h + \left(1 - \frac{1}{1+\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)}}\right)(-\Delta_g) = \delta\frac{\Delta_h - \Delta_g}{2+\nu_h+\nu_l}. \tag{17}$$

Step 1. Cutoff for positive payoffs.

By Lemma 6, we know that $y \to 1$ as $\delta \to 1$. Applying L'Hopital's rule, we thus obtain that $\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)} \to 1/2$ as $y \to 1$ whereas $\frac{1-F_h(y)}{1-F_l(y)}\frac{f_l(y)}{f_h(y)} \to \infty$ as $y \to 0$. Because $\Delta_h > \Delta_g$, there is hence a cutoff signal $y_0 < 1$ such that

$$\frac{1}{1+\frac{1-F_h(y_0)}{1-F_l(y_0)}\frac{f_l(y_0)}{f_h(y_0)}}\Delta_h + \left(1 - \frac{1}{1+\frac{1-F_h(y_0)}{1-F_l(y_0)}\frac{f_l(y_0)}{f_h(y_0)}}\right)(-\Delta_g) = 0.$$

Lemma 7 shows that there exists s_0, s_l and δ such that $\nu_l(s_0,\delta) \to 0$ and $\nu_h(s_0,\delta) \to 0$ as $(s_0,\delta) \to (1,1)$ and $\nu_l(s_l,\delta) \to \infty$ and $\nu_h(s_l,\delta) \to 0$ as $(s_l,\delta) \to (1,1)$. This implies that s_0 and s_l exceed the cutoff $y_0 < 1$ as frictions disappear.

Step 2. $FP < 0$ for $(y_0,\delta) \to (y_0,1)$ where $\nu_l \to 0$.

Consider first a sequence $(y_0,\delta) \to (y_0,1)$ where $y_0 \in (0,s_0)$ such that $\nu_l(y_0,\delta) \to 0$ and $\nu_h(y_0,\delta) \to 0$; Lemma 7 shows that such a sequence exists. For this sequence, the lhs of (17) approaches 0, while the rhs of (17) approaches $\frac{\Delta_h-\Delta_g}{2}$.

Step 3. $FP > 0$ for $(y,\delta) \to (1,1)$ where $\nu_l \to \epsilon > 0$.

Consider next a sequence $(y_1,\delta) \to (1,1)$ where $y_1 \in (s_0,s_l)$ such that $\nu_l(y_1,\delta) \to \epsilon$ and $\nu_h(y_1,\delta) \to 0$; Lemma 7 shows that such a sequence exists. For this sequence, the lhs of (17) approaches $\frac{\Delta_h-\Delta_g}{2}$, while the rhs of (17) approaches $\frac{\Delta_h-\Delta_g}{2+\epsilon}$.

Step 4. Equilibrium existence.

By the continuity of the lhs and rhs of (17) with respect to arguments (y,δ), an equilibrium sequence satisfying (FP-r) hence exists where $\nu_l \to n_l < \epsilon$ for any $\epsilon > 0$ as $\delta \to 1$. This shows that $\nu_l \to 1$ as $\delta \to 1$, allowing to satisfy (IC-r).

Step 5. Equilibrium payoffs.

The previous analysis demonstrates that $\nu_h \leq \nu_l \to 0$ as $(y,\delta) \to (1,1)$. By (7) and (8), limit payoffs hence approach

$$V_l = \frac{\Delta_l + \Delta_g}{1+\nu_l} \to \Delta_l + \Delta_g,$$
$$V_b = \frac{\Delta_h - \Delta_g}{2+\nu_l+\nu_h} \to \frac{\Delta_h - \Delta_g}{2},$$

resulting in efficient limit payoffs

$$W = V_b + \frac{1}{2}V_l = \frac{\Delta_h - \Delta_g}{2} + \frac{\Delta_l + \Delta_g}{2} = \frac{\Delta_h + \Delta_l}{2},$$

that equal the first best payoffs. □

Proof of Proposition 2

Under reversed trade dynamics, the cutoff signal y satisfies (FP-s) and (IC-s). The fixed point condition $FP = rhs - lhs = 0$ in (FP-s) is equivalent to $rhs = lhs$ in

$$\frac{1}{1 + (1 - F_h(y))\frac{f_l(y)}{f_h(y)}}\Delta_h + \left(1 - \frac{1}{1 + (1 - F_h(y))\frac{f_l(y)}{f_h(y)}}\right)(-\Delta_g) = \delta\frac{\Delta_h + F_l(y)(\Delta_l - \delta V_l) + (1 - F_l(y))(-\Delta_g)}{2 + \nu_h}. \tag{18}$$

Case 1. Limit-efficient equilibrium.

Step 1. A lower bound on ν_l to satisfy (IC-s)

As $(y, \delta) \to (1, 1)$, buyer payoffs given by (6) approach $\frac{\Delta_h}{2}$ when buyers obtain no rents from trade with low-quality sellers. A lower bound for screening ν_l required to satisfy $\delta(V_l + V_b) \leq \Delta_l$ for standard dynamics is hence given by

$$\begin{aligned} V_b + V_l &\leq \Delta_l \\ \frac{\Delta_h}{2} + \frac{1}{1+\nu_l}(\Delta_g + \Delta_l) &\leq \Delta_l \\ 2\Delta_g + 2\Delta_l &\leq (1+\nu_l)(2\Delta_l - \Delta_h) \\ 2\Delta_g + \Delta_h &\leq \nu_l(2\Delta_l - \Delta_h) \\ \frac{\Delta_g + \Delta_h/2}{\Delta_l - \Delta_h/2} &\leq \nu_l \end{aligned} \tag{19}$$

Lemma 7 shows that there exists s_0, s_l, s_h, and δ such that $\nu_l(s_0, \delta) \to 0$ and $\nu_h(s_0, \delta) \to 0$ as $(s_0, \delta) \to (1, 1)$, and $\nu_l(s_l, \delta) \to \infty$ and $\nu_h(s_l, \delta) \to 0$ as $(s_l, \delta) \to (1, 1)$. The lowest signal y_2 satisfying (19) thus lies between s_0 and s_l.

Step 2. $FP > 0$ for $(y_2, \delta) \to (1, 1)$ where $\nu_l \to \frac{2\Delta_g + \Delta_h}{2\Delta_l - \Delta_h}$.

Consider first a sequence $(y_2, \delta) \to (1, 1)$ such that $\nu_l(y_2, \delta) \to \frac{2\Delta_g + \Delta_h}{2\Delta_l - \Delta_h}$ and $\nu_h(y_2, \delta) \to 0$; Lemma 7 shows that such a sequence exists. For this sequence, the lhs of (18) approaches Δ_h, while the rhs of (18) approaches $\frac{\Delta_h}{2}$.

Step 3. $FP < 0$ for $(s_l, \delta) \to (1, 1)$ where $\nu_l \to \infty > 0$.

Consider next a sequence $(s_l, \delta) \to (1, 1)$ such that $\nu_l(s_l, \delta) \to \infty$ and $\nu_h(s_l, \delta) \to 0$; Lemma 7 shows that such a sequence exists. For this sequence, the lhs of (18) approaches

Δ_h, while the rhs of (18) approaches $\frac{\Delta_h+\Delta_l}{2}(> \Delta_h)$.

Step 4. Equilibrium existence.

By the continuity of the lhs and rhs of (18) with respect to arguments (y, δ), an equilibrium sequence satisfying (FP-s) hence exists where $\nu_h \to 0$ and $\nu_l \to n_l \in (\frac{2\Delta_g+\Delta_h}{2\Delta_l-\Delta_h}, \infty)$, allowing to satisfy (IC-s).

Step 5. Equilibrium payoffs.

Limit payoffs satisfy (18), (7), and (8) for $\nu_h \to 0$, which gives

$$\Delta_h = \frac{\Delta_h + \Delta_l - \frac{\Delta_g+\Delta_l}{1+\nu_l}}{2} \Longrightarrow \nu_l = \frac{\Delta_g + \Delta_h}{\Delta_l - \Delta_h} > \frac{\Delta_g + \Delta_h/2}{2\Delta_l - \Delta_h/2} > 0,$$

and

$$V_l = \frac{\Delta_l + \Delta_g}{1 + \nu_l} = \Delta_l - \Delta_h,$$
$$V_b = \frac{\Delta_h + \Delta_l - (\Delta_l - \Delta_h)}{2} = \Delta_h,$$

resulting in efficient limit payoffs

$$W = V_b + \frac{1}{2}V_l = \Delta_h + \frac{\Delta_l - \Delta_h}{2} = \frac{\Delta_h + \Delta_l}{2},$$

that equal the first best payoffs.

Case 2. Limit-inefficient equilibrium.

Lemma 7 shows that there exists s_l, s_h, and δ such that $\nu_l(s_l, \delta) \to \infty$ and $\nu_h(s_l, \delta) \to 0$ as $(s_l, \delta) \to (1, 1)$, and $\nu_l(s_h, \delta) \to \infty$ and $\nu_h(s_h, \delta) \to \infty$ as $(s_h, \delta) \to (1, 1)$.

Step 1. $FP < 0$ for $(s_l, \delta) \to (1, 1)$ where $\nu_h \to 0$ and $\nu_l \to \infty$.

Consider first a sequence $(s_l, \delta) \to (1, 1)$ such that $\nu_l(s_l, \delta) \to \infty$ and $\nu_h(s_l, \delta) \to 0$; Lemma 7 shows that such a sequence exists. For this sequence, the lhs of (18) approaches Δ_h, while the rhs of (18) approaches $\frac{\Delta_h+\Delta_l}{2}(> \Delta_h)$.

Step 3. $FP > 0$ for $(s_l, \delta) \to (1, 1)$ where $\nu_l \to \infty > 0$.

Consider next a sequence $(s_h, \delta) \to (1, 1)$ such that $\nu_l(s_h, \delta) \to \infty$ and $\nu_h(s_h, \delta) \to \infty$; Lemma 7 shows that such a sequence exists. For this sequence, the lhs of (18) approaches Δ_h, while the rhs of (18) approaches $0(< \Delta_h)$.

Step 4. Equilibrium existence.

By the continuity of the lhs and rhs of (18) with respect to arguments (y, δ), an equilibrium sequence satisfying (FP-s) hence exists where $\nu_h \to n_h > 0$ and $\nu_l \to \infty$, allowing to satisfy (IC-s).

Step 5. Equilibrium payoffs.

Limit payoffs satisfy (18), (7), and (8) for $\nu_l \to \infty$, which gives

$$\Delta_h = \frac{\Delta_h + \Delta_l}{2 + \nu_h} \Longrightarrow \nu_h = 0,$$

and

$$V_l = \frac{\Delta_l + \Delta_g}{1 + \nu_l} = 0,$$
$$V_b = \frac{\Delta_h + \Delta_l}{2 + \nu_h} = \Delta_h,$$

resulting in inefficient limit payoffs

$$W = V_b + \frac{1}{2} V_l = \Delta_h,$$

that lie below the first best payoffs.

Proof of Proposition 3

By Lemma 4, we know that an equilibrium cannot feature reversed dynamics if $\Delta_g > \Delta_h$. An equilibrium must thus feature either standard dynamics or knife-edge dynamics.

Case 1. Standard dynamics.

Under standard trade dynamics, the fixed point conditions for the cutoff signal is given by (18). As $(y, \delta) \to (1, 1)$, the rhs of (18) approaches Δ_h while the lhs of (18) approaches V_b. As $V_b \to \Delta_h$, it becomes impossible to satisfy the incentive condition $\Delta_l \geq \delta(V_l + V_b)$.

Case 2. Knife-edge dynamics.

The proof is by contradiction.

Let us try to construct a knife-edge equilibrium where a buyer is indifferent between offering p_0 and p_l for $s < y$, and offers p_0 for $s \in [0, z)$ and p_l for $s \in [z, y)$. The probability of trade for low-quality assets is thus $1 - F_l(z)$.

We introduce new notation $\nu_0 = (1 - \delta)\frac{F_l(z)}{1 - F_l(z)} < \nu_l = (1 - \delta)\frac{F_l(y)}{1 - F_l(y)}$, where ν_0 denotes the difficulty of trading low-quality and ν_l the difficulty of trading low-quality for high price. We can express the valuation of buyers as

$$\begin{aligned} V_b &= \frac{q_u(y,z)(1 - F_h(y))\Delta_h - (1 - q_u(y,z))(1 - F_l(y))\Delta_g}{1 - \delta q_u(y,z)F_h(y) - \delta(1 - q_u(y,z))F_l(y)} \\ &= \frac{\Delta_h - \frac{\nu_0(z)}{\nu_l(y)}\Delta_g}{1 + \frac{\nu_0(z)}{\nu_l(y)} + \nu_0(z) + \nu_h(y)} \end{aligned} \tag{20}$$

where

$$q_u(y,z) = \frac{1}{1 + \frac{1-F_h(y)}{1-F_l(z)}}.$$

Now, y satisfies both the fixed point condition, which can be expressed in the limit as

$$V_b = \frac{\Delta_h - \frac{\nu_0(z)}{\nu_l(y)} m(y) \Delta_g}{1 + \frac{\nu_0(z)}{\nu_l(y)} m(y)}, \tag{21}$$

where

$$m(y) = \frac{f_l(y)}{f_h(y)} \frac{1 - F_h(y)}{1 - F_l(y)} \to 1 \text{ as } y \to 1,$$

and y satisfies also the incentive condition, which can be written in the limit as

$$V_b = \frac{\Delta_l - \frac{1}{\nu_l(y)} \Delta_g}{1 + \frac{1}{\nu_l(y)}}, \tag{22}$$

To equate (20) with (21) as $(y, \delta) \to (1, 1)$, we need $\nu_0 \to 0$ and $\nu_h \to 0$, implying $V_b \to \Delta_h$. But this will violate (22) for $\Delta_h > \Delta_l$. $\square$