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Welfare Effects of R&D Support Policies

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Welfare Effects of R&D Support Policies*

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Abstract

We conduct a welfare analysis of R&D subsidies and tax credits using a model of innovation policy incorporating externalities, limited R&D participation and financial market imperfections. We estimate the model using R&D project level data from Finland. The intensive, not the extensive R&D margin is important. Financial frictions do not matter much. Tax credits and subsidies do not reach first best but increase R&D 30-50% compared to laissez-faire. Once the subsidy application costs are accounted for, tax credits increase welfare by 1% and subsidies slightly reduce welfare. In terms of fiscal cost, tax credits are 90% more expensive than R&D subsidies.

KEY WORDS: R&D subsidies, R&D tax credits, extensive and intensive margin, financial market imperfections, welfare, counterfactual, economic growth.

JEL codes: O30, O38, H25

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1 Introduction

R&D subsidies and tax credits are widely used to encourage private sector R&D: E.g., OECD countries spend in excess of $50 billion on them annually.[1] In this paper we develop and apply a framework to compare the welfare impacts of R&D subsidies, R&D tax credits, financial market imperfections, laissez-faire policy of no government support, and first and second best. We compare the impacts of the policies both at the extensive and intensive margins of R&D.

The well-known justifications for public support to private R&D are appropriability problems and financial market imperfections (e.g., Bloom et al., 2019). Government innovation policy officials often add the objective to entice non-R&D-performing firms to start R&D. We build a dynamic model of the subsidy application and allocation process that incorporates all three rationals for public support to private R&D. Using revealed preference, we identify the structural parameters by estimating key decisions: The firm's project level R&D investment yields information on the marginal profitability of R&D and the cost of external finance; the decision to invest in R&D allows us to identify the fixed costs of R&D; the decision to apply for subsidies is informative about the costs of application; and finally, the government agency's decision of what fraction of R&D costs to reimburse allows us to identify the parameters of the government utility function.

We take the model to detailed R&D project-level data from Finland where the ratio of R&D to GDP is among the highest. In the early 1980s a government agency (Tekes) was established to provide R&D subsidies to firms, and other public financial support to R&D were abolished.[2] We use the large variation in government subsidy decisions - Figure 1 displays the distribution of the project-level fraction of R&D cost covered by the government among all applicants - that most papers ignore.

[Figure 1. Distribution of the subsidy rate]

[1] We arrive at this figure by multiplying business enterprise R&D measured in 2010 PPP US$ by the percentage of that R&D financed by government (OECD Main Science and Technology Indicators www-site, accessed December 29, 2023).

[2] Finland's R&D subsidy regime is comparable to those of e.g. Belgium, Germany, the Netherlands and to the US SBIR programs, and is highly regarded (van der Veen et al., 2012, p. 29).

In our welfare analysis, we displace R&D subsidies with an optimally calculated R&D tax credit to contrast the two main government financial support policies. A key trade-off between these policies is that R&D subsidies can be tailored but only for applied projects, whereas R&D tax credits can reach a much larger share of the firm population at the cost of being "one-size-fits-all". To provide benchmarks, we consider a laissez-faire economy with no government support and the first and second best regimes where the government can directly determine the level of private R&D investments. In a final benchmark we remove financial market imperfections.

The calculation of optimal R&D subsidies and R&D tax credits becomes become complex when the extensive margin of R&D is introduced: The effect of financial market imperfections on the level of optimal support delicately depends on the margin at which the support operates. In our counterfactuals, close to 40% of Finnish firms do not invest in R&D, nor should they, as their R&D ideas are neither privately nor socially profitable. Subsequently, the R&D subsidy and tax credit policies have on average little impact at the extensive margin. Conditional on investing, the R&D support policies increase R&D investments by 30-50%, and the first best regime by over 100% compared to laissez-faire.

We estimate the value of spillovers to be 58 cents per euro of R&D. While the differences in spillovers across the policy regimes are of the same order of magnitude as differences in the R&D investments, differences in profits are small. As profits turn out to be the main element of welfare, we find only small differences in welfare across the regimes. An explanation for spillovers being low relative to profits is that a significant fraction of spillovers generated by the Finnish R&D are likely flowing outside Finland, and should be ignored by a Finnish agency.

The optimal tax credit rate is 34%. In terms of fiscal costs, tax credits are over 90% more expensive than R&D subsidies, but once all benefits and costs are accounted for, tax credits yield higher welfare. Estimated financial market imperfections are small and hardly affect welfare. Our results thus suggest that spillovers should be emphasized in the design of optimal R&D subsidy and tax credit policies over financial market imperfections.

We believe to be the first to build and estimate a microeconomic model of innovation policy in which R&D externalities, financial market imperfections and fixed cost of R&D affect government support, R&D investment levels, and R&D participation. The extensive empirical literature on the effects of R&D support policies has focused on the causal effect of a policy on some outcome variable (e.g., on private R&D) rather than welfare.[3] Nor do the existing models provide a solid foundation for a welfare analysis: E.g., the model in Takalo et al. (2013, hereafter TTT) assumes perfect financial markets and that all firms invest in R&D despite evidence to the contrary and they do not consider R&D tax credits.

Our paper asks similar questions as Acemoğlu et al. (2018), Akcigit et al. (2021), and Akcigit et al. (2022). We differ from this more macro-oriented literature in terms of data and modeling, but our welfare results and estimate of the optimal R&D tax credit are quite close to those in Acemoğlu et al. (2018) and Akcigit et al. (2021). Our approach to identifying spillovers and social returns complements the one by Bloom et al. (2013), and our result on the intensive margin being more important than the extensive margin is reminiscent of Garcia-Macia et al. (2019).

Our precursors in the literature estimating structural models of innovation include, besides TTT (2013a), González et al. (2005) who focus on R&D subsidies, Doraszelski and Jaumandreu (2013) who study R&D and productivity, and Peters et al. (2017) who use a dynamic empirical model to uncover the fixed and sunk costs of R&D. Matcham and Schankerman (2023) develop and estimate a dynamic model of the patent application and screening process. Also relevant are Arqué-Castells and Mohnen (2015) who study the impact of fixed and sunk costs of R&D on the effectiveness of R&D subsidies, Boller et al. (2015) who study the link between R&D, imports and exports, and Chen and Xu (2022), who estimates an industry equilibrium model with R&D spillovers. Kireyev (2020), Bhattacharya (2021), and Lemus and Marshall (2021) study innovation contests, with Bhattacharya's application being on government support.

Next, we outline the Finnish institutional environment for R&D and our data. We explain our model in section 3 and its estimation in section 4. Sections 5 and 6 contain estimation results and the counterfactual

[3]See surveys by Garciá-Quevedo (2004), Cerulli (2010), and Zúñica-Vicente et al. (2014). More recent contributions include Bronzini and Iachini (2014), Einiö (2014), Howell (2017), Hünermund and Czarnitzki (2019) and Santoleri et al. (2022) on R&D subsidies, and Dechezleprêtre et al. (2023) on R&D tax credits.

experiments, respectively. Section 7 concludes.

2 Institutional Environment and Data

2.1 Institutional Environment

Finland rapidly transformed from a resource- to an innovation and knowledge-based economy at the end of the millennium (Trajtenberg, 2001). The R&D/GDP ratio in Finland doubled over the last two decades of the 20^{th} century and overtook that of the US (see Appendix A). The Finnish innovation policy hinges on direct R&D subsidies. During our observation period 2000-2008 there were no R&D tax credits. Tekes,[4] where our subsidy data comes from, is the main public organization providing funding (grants and loans) for private R&D. Some other public organizations provide limited finance for innovative firms, but their funding is not generally for R&D investments nor consist of subsidies.

During our observation period Tekes' mission was to promote "*the development of industry and services by means of technology and innovations. This helps to renew industries, increase value added and productivity, improve the quality of working life, as well as boost exports and generate employment and well being.*" (Tekes, 2008, 2011). While boosting exports is a part of Tekes' objective, Tekes' strives to increase domestic welfare. For example, during our data period some supported companies were sold abroad, which caused concern that foreigners appropriate the benefits of Tekes' funding. A technology director of Tekes reassured the public that "*Our goal is that economic benefits [of our funding] remain in Finland.*" (Flink, 2005). Access to finance was not emphasized by Tekes, and it was hardly a problem for most of the Finnish firms in the boom years preceding the global financial crisis (Hyytinen and Pajarinen, 2003; Hyytinen, 2013).

In 2012 Tekes' funding was circa 600M€, up from circa 400M€ in 2004 (see Appendix A). In its funding decisions, Tekes emphasizes small and medium sized enterprises (SMEs), but large companies may also obtain funding from Tekes. Tekes' funding decisions are based on "*the novelty of the project, market distance, and the size of the company*" (Tekes, 2011).

[4]Tekes became part of a larger government organization, Business Finland, in 2018.

To acquaint ourselves with Tekes' decision making in detail, one of us spent 11 months in Tekes. After receiving an application, a team of Tekes' experts reviews the application and grades the proposal in several dimensions. The technological challenge and commercial *risk* are the two most important grading dimensions; thus we focus on them. The expert team then makes a proposal for a funding committee which decides the subsidy rate. The minimum subsidy rate is zero, and the maximum depends on the firm's SME status, and is either 0.5, 0.6 or 0.7. Tekes has several safeguards against misuse (e.g., subsidies are paid against receipts–see TTT, 2013a). The danger of misreporting should thus be much smaller than in some other institutional environments (cf. Boeing and Peters, 2021).

2.2 Data

Our data comes from two main sources: From Tekes, we obtain detailed data on all project level R&D subsidy applications for 2000-2008. These data include the applied amount of funding, internal screening outcomes, final funding decisions, realized project expenses and reimbursements, and information on other sources of funding. We match these data to the R&D survey and balance-sheet data from Statistics Finland. We end up with 22 504 firm-year observations for 6 077 firms (see Appendix B for details).[5] Compared with TTT (2013a), our data cover a considerably longer time period and is richer, containing information, e.g., on the actual (in addition to planned) R&D expenditure and reimbursements at the project level for successful applicants, on firm level R&D also for firms not receiving subsidies and on funding from other sources.

Descriptive statistics in Table 1 show that applicant and non-applicant firms in our data are 14 and 17 years on average; their average number of employees are 121 and 101, and their average sales per employee 19 000€ and 22 000€ (normalized to 2005 euros). Of the applicant and non-

[5]We follow TTT (2013a) and randomly choose one application for those firms with more than one application in a given year. This choice follows from our model in which each firm only receives one R&D idea per year. Relaxing this assumption provides a challenging task for future research. We also follow TTT (2013a) in calculating the subsidy rate as the sum of grants and subsidized loans divided by the planned R&D investment. As a robustness test, we repeat the analysis using only grants (see section 6).

applicant firms, 83% and 86%, respectively, are SMEs, 19% and 13% are located in the regions eligible for EU regional aid, and 83% and 59% invested in R&D in the preceding year; these differences are statistically significant. On average, 62% of the firms invest in R&D and 18% apply for subsidies.[6]

Table 1 also displays descriptive statistics for successful and rejected applicants; here the differences are statistically insignificant, except for the differences in R&D investment and past subsidy application behaviors. The average subsidy rate of successful applicants is 0.35, and their average actual R&D investment over the (max. 3 year) lifetime of a project is 483 000€. As to the Tekes evaluation grades, we convert (see Appendix B) the original Likert scale 0-5 of both technological challenge (*tech*: Ranging from 0 = "no technological challenge" to 5 = "international state-of-the-art") and commercial risk (*risk* : Ranging from 0 = "no identifiable risk " to 5 = "unbearable risk ") to scale 1-3 because of few observations at the tails. Using the modified grades, the average technological challenge and commercial risk are 2.1 and 2.3.

[Table 1]

A key data challenge is to observe firms' funding costs and opportunities at a project level. There is no consensus on how to measure financial constraints at a firm level (Farre-Mensa and Ljunqvist, 2016) and attempts to measure financial constraints at a project level are rare. Evidence (Lian and Ma, 2021), however, suggests that lenders pay particular attention to borrowers' cashflow, and our Tekes-data contains unique information about an applicant's cashflow pledgeable to the proposed project. We measure (lack of) financial market imperfections faced by a firm by the ratio of pledgeable cashflow to the planned R&D project size. The mean ratio for all successful applicants and for those successful applicants with no R&D in the previous year are 1.12 and 1.16. This ratio is less than one for 38% of the successful applicants (for which this measure is observable).

[6]One may wonder why so few firms apply for subsidies. One explanation is that application costs and fixed costs of R&D are non-trivial. Another potential explanation is that firms are unaware of the application possibility. Our understanding is however that Tekes was by the 2000s well-known among firms potentially investing in R&D in Finland.

3 The Model

We extend the model of TTT (2013a,b) by introducing R&D tax credits with corporate taxation, a financial sector with frictions, and the extensive margin of R&D. These features are critical for a welfare evaluation of R&D support policies. We outline the model and discuss the main arguments in the body of the paper, relegating technical details into Appendix C.

3.1 Assumptions and Payoffs

We consider interactions among a public *agency* allocating R&D subsidies, a continuum of *firms* with access to R&D projects, and many competitive private sector *investors* with access to liquid funds. All agents are *risk* neutral and for brevity there is no time preference.

Each firm needs to invest a *fixed cost* $F \in [0, \infty)$ and a *variable cost* $R \in (0, \infty)$ to undertake an R&D project. As in Holmström and Tirole (1997), the firms can choose between two projects. A good project pays

$$\pi(R) = A \ln R, \tag{1}$$

with probability $P \in (0, 1)$ and 0 otherwise. $A \in (0, \infty)$ is a constant shifting the project returns.[7] Firms' project successes are *i.i.d*; thus, there is no aggregate uncertainty. A bad project fails with probability one but yields non-verifiable private benefits for the firm.

We focus on cashflow- rather than collateral-based financial constraints and for brevity assume that firms have no existing liquidable assets of their own. In equilibrium, this lack of liquidable assets does not prevent firms from raising external funding if their pledgeable cashflows are sufficient. To obtain funding, firms may apply for public R&D subsidies. Since the agency pays subsidies as reimbursements, the firms must first raise external funding from investors, who can flexibly raise funds at a constant rate

[7] We employ the logarithmic R&D technology specified by equation (1) to obtain our econometric model. We have also experimented with the functional form $\pi(R) = A(R^{1-\gamma} - 1)/(1 - \gamma)$ in which $\gamma \in [0, \infty)$. This functional form yields logarithmic conditional profits when $\gamma \to 1$. As our data strongly suggests that $\gamma \approx 1$, we impose the logarithmic functional form from the outset for both simplicity and computational gain. A consequence of the logarithmic R&D technology is that in equilibrium the firm, conditional on investing, chooses $R > 1$ – see Lemma A3 in Appendix C.

$r \in [1, \infty)$. As in Holmström and Tirole (1997), an investor can eliminate the bad project from the firm's action set by incurring a *monitoring cost* $c \in [0, \infty)$ per unit of investment. The private benefits are large enough to make the bad project privately attractive to the firm unless the investor monitors (see Assumption A2 in Appendix C.)

To raise funds, a firm promises to repay its investor $\pi^I \in [0, \infty)$ if its project is successful. This *repayment promise* accommodates both debt and equity interpretations.[8] The expected payoff of an investor who chooses to finance and monitor a firm's project when the firm offers a *financing contract* $\left(R, \pi^I\right) \in [0, \infty)^2$ and the agency awards a *subsidy rate* $s \in [0, \overline{s}]$, $\overline{s} < 1$, to the project is given by

$$\Pi^I(s, R, \pi^I) = (1 - \tau)\left[P\pi^I - (r + c)(R + F) + sR\right]. \tag{2}$$

In equation (2), $\tau \in [0, 1]$ is the *corporate tax rate.* We make corporate taxation neutral with respect to R&D investments and subsidy decisions: We assume that each investor is large so that the law of large numbers can be applied to the investor's asset portfolio. As the project successes are *i.i.d.*, we invoke the common assumption that the empirical mean equals the expetation with probability one (see Judd, 1985) and, consequently, a fraction P of the investor's projects will succeed and $1-P$ will fail. Because expenses of both successful and failed projects are tax deductible against the revenues from the successful projects, the investor's net investment cost $((r + c)(R + F) - sR)$ of an individual project is tax deductible even if the project fails. Similar assumptions are common in the banking literature – e.g., models in the tradition of Diamond and Dybvig (1983) apply the law of large numbers to banks' liabilities.

Equation (2) shows how the investor needs to fund the whole investment $R + F$, and to cover the costs of funds r and of monitoring c. A fraction s of the realized variable R&D costs may be reimbursed by the agency. The agency reimburses neither fixed nor external financing costs. In our insti-

[8] In Appendix C we write the repayment promise as a part of a standard debt contract, i.e., as $\min\left\{\pi^I, \max\left\{\pi(R), 0\right\}\right\}$ implying that the investor has seniority if the firm cannot honor its promise. In equilibrium (see Lemma A3 in Appendix C) the firm is able to make the promised repayment unless the project pays no returns and both equity and debt contracts lead to the same outcome.

tutional setting, Tekes has detailed rules on eligible expenses which exclude the costs of external finance. Tekes also primarily reimburses variable R&D expenses since they are easy to allocate to projects.

Since the investor is funding the whole investment, the expected payoff of a firm investing in the good project and offering the contract $\left(R,\pi^I\right) \in (0,\infty)\times[0,\infty)$ may be expressed as

$$\Pi^E(R,\pi^I) = (1-\tau)P(\pi(R)-\pi^I). \tag{3}$$

The payoffs to a firm that makes no investment ($R=0$) and to an investor who chooses not to finance a firm are zero. The investor's no-financing decision results in $R=0$ and, consequently, in zero payoff to the firm, too.

If a firm applies for R&D subsidies, the agency examines the firm's application, learns the *spillover rate* $v \in \mathbb{R}$ per unit of variable R&D to be invested in the project, and decides on the subsidy rate. The agency's expected payoff from awarding a subsidy rate s to a project funded by a monitoring investor is given by

$$U\left(v,s,R,\pi^I\right) = (v-gs)\,R + \frac{1}{1-\tau}[\Pi^E(R,\pi^I)_+ + \Pi^I(s,R,\pi^I)_+] \tag{4}$$

in which the firm's and investor's profits are net of taxes since, for the agency, corporate tax payments are just transfers and cancel out in a welfare calculation. Throughout the paper, for $x \in \mathbb{R}$, we write $x_+ := \max\{x,0\}$. In equation (4)) the max operators capture the private sector's participation constraints. The term vR captures the (agency's evaluation of) total spillovers from the project. A theoretical tradition dating at least back to Ruff (1969) assumes a similar linear relation between total spillovers and R&D investments; see Amir (2000) for justification. For simplicity, we assume that the agency faces a shadow cost of public funds, captured by $g>1$ in equation (4). If $R=0$, the agency's payoff is zero.

Equation (4) is our measure of welfare: Identifying its parameters allows us to compare counterfactual policies to the current policy from the government's point of view without necessarily taking a stand on whether the government is a benevolent social planner or not. A project's spillover rate v can reflect standard positive welfare externalities of R&D investments (e.g. consumer surplus, technological spillovers), but also other social needs such as climate change or national defense. Whether or not the agency takes into account spillovers flowing abroad affects the interpretation of our welfare results (see section 6.2). A spillover rate v can also be negative, e.g., due to duplication of R&D costs, business stealing effects, or negative environmental externalities. As we shall see, in equilibrium the agency rejects the application even if v is positive but sufficiently small.

We assume that v is (partially) unknown to both firms and the agency when the firms contemplate applying. As a consequence, prior to applying, the firms are uncertain about the agency's subsidy rate decisions. This assumption of incomplete but symmetric information ensures, in line with data, equilibrium outcomes with rejected applications without the need to model complexities arising from signaling games. It also seems reasonable that potential applicants do not exactly know ex ante how the agency evaluates the spillovers arising from their projects.

While our assumption of incomplete-but-symmetric information (at the application stage) is common in related settings (e.g, Holmström, 1982; Aghion et al., 2013), it may ignore some features of R&D subsidy programs which build on the firm's type being private information (cf. Takalo and Tanayama, 2010; Lach et al., 2021). On the other hand, the agency might have private information about its own or the firm's type, in line with a strand of literature dating back at least to Rock (1986) assuming that a firm's financiers know more about the firm than the firm itself. It may be also unclear who has an informational advantage. E.g., a firm may know more about its competitive environment than the agency or the agency, upon observing the rivals' applications, knows more.

Applying for a subsidy involves a fixed, non-tax deductible cost, denoted by $K \in [0, \infty)$. Application costs can be thought of mainly consisting of non-deductible effort (Tekes requires a detailed, written application; the application process also involves other communications between the applicant and Tekes – see section 2.1).

The dynamic game describing the interactions of the agency, a firm and an investor proceeds in five stages. We describe the agents' binary action choices by $d_k \in \{0, 1\}$ in which subscript k indicates an action to be chosen, and 1 and 0 indicate choosing and not choosing that action. In stage 1, we thus describe the firm's subsidy application choice by $d_a \in \{0,1\}$. In stage 2 the agency, upon evaluating the application, learns the spillover rate v and decides the subsidy rate. We model the agency's learning of v as a realization of a continuous, real-valued random variable V with a probability density function $\phi(v)$ and a cumulative distribution function $\Phi(v)$.[9] A strategy for the agency can be described as a mapping $s : \mathbb{R} \times \{0,1\} \to [0, \overline{s}]$ in which $s(v, d_a)$ is the agency's subsidy rate upon a realization of V and the firm's application choice d_a.

In stage 3 the firm and investor sign a financing contract: The firm's behavior in the third stage consists of two mappings $R : [0, \overline{s}] \to [0, \infty)$ and $\pi^I : [0, \overline{s}] \to [0, \infty)$ in which $R(s)$ and $\pi^I(s)$ identify the firm's project and repayment proposal, respectively, when the subsidy rate is s. The investor's strategy consists of two mappings $d_k : [0, \bar{s}] \times [0, \infty)^2 \to \{0,1\}, k = f, m$, in which $d_k(s, R, \pi^I), k = f, m$, identify the investor's financing and monitoring choices for each subsidy rate s and contract offer (R, π^I). In stage 4 the firm chooses the project and makes an R&D investment according to the contract. The firm's project choice may be described by a mapping $d_G : [0, \infty)^2 \times \{0,1\}^2 \to \{0,1\}$ in which $d_G(R, \pi^I, d_f, d_m)$ identifies the project for each contract offer, financing and monitoring decisions. In stage 5, subsidies are paid, the project return is realized, and claims are settled according to the contract.[10]

In a pure strategy perfect Bayesian equilibrium the firm has rational prior beliefs about the agency's spillover rate evaluations, and chooses to apply for a subsidy if the expected profits from applying are larger than from not applying and does not apply otherwise, given the agency's and

[9] In our econometric model, v and other parameters of the theoretical model are functions both of observable firm and project characteristics and of realizations of unobserved random variables.

[10] Many timing assumptions are inconsequential: For example, assuming that the financing contract is not written contingent on subsidies would make only inconsequential differences – see Appendix D. Also, whether subsidies are paid before or after project return realizations makes no difference, and the size of the R&D project could equally well be chosen after the investor's decisions. What matters is the timing of the firm's project choice decision after the investor's monitoring decision; this timing avoids the need of considering mixed strategies. Tekes is also legally prohibited from reimbursing expenses that have incurred before the application (see the Government Decree on the Funding for Research, Development and Innovation Activities 1444/2014 § 3).

investor's behavior in the later stages. After making its spillover rate evaluation, the agency chooses a subsidy rate to maximize its payoff, given the firm's and the investor's behavior in later the stages. If the agency receives no application, the agency will give no subsidy: The Finnish law prevents Tekes from granting a subsidy without a formal, written application.[11] The investor finances and monitors the firm whenever the investor's expected payoffs to financing and monitoring are larger than from no-financing and no-monitoring, and neither finances nor monitors otherwise. The firm offers a financing contract to maximize its expected profits given the investor's behavior, and chooses the good project if the investor monitors and the bad one otherwise. Appendix C contains a formal definition of the equilibrium.

3.2 Equilibrium Analysis

Since the agency cannot award a subsidy without an application, in equilibrium $s^*(v,0) = 0$. We simplify exposition and denote by $s(v) := s(v,1)$ the agency's strategy after receiving an application.

Cost of external financing, R&D investment level and R&D participation. (See Lemmas A1-A4 in Appendix C for details.) The payoff to an investor who chooses not to invest is 0, whereas the payoff to an investor who invests but does not monitor is $(1-\tau)\left[-r\left(R+F\right)+sR\right]<0$ – recall that $r \geq 1 > \overline{s} \geq s$. Thus, in equilibrium, the firm is either investing in the good project with funds supplied by a monitoring investor or no R&D investment is made, i.e., either $d_k^*(\cdot) = 1$ for $k = f, m, G$ or $d_f^*(\cdot) = 0$.

Since investors behave competitively we can seek a financing contract $\left(\pi^I, R\right) \in [0,\infty)^2$ that maximizes the firm's expected payoff. Letting the investor's expected payoff from choosing $d_k = 1, k = f, m$, from equation (2) to be equal to 0 and solving the resulting equation for π^I yields

$$\pi^{I*}(s,R) = \frac{(r+c)\left(R+F\right)-sR}{P}. \tag{5}$$

Equation (5) identifies the minimal repayment that makes the investor willing to finance a project of size R. After inserting equations (1) and (5)

[11] The Act on Discretionary Government Transfers 688/2001 § 9 and the Government Decree on the Funding for Research, Development and Innovation Activities 1444/2014 § 3.

into equation (3), we can write the firm's R&D investment problem as

$$\max_{R\in[0,\infty)} \Pi^E(s,R)_+ = (1-\tau)[\alpha \ln R - (r+c-s)R - (r+c)F]_+, \qquad (6)$$

in which $\alpha := AP$ is a constant shifting the expected profitability of the R&D project and $r+c-s$ captures the firm's marginal cost of R&D.

Solving the problem of equation (6) yields the firm's optimal R&D investment decision as

$$R^*(s) = \begin{cases} R^{**}(s) := \arg\max_{R>0} \Pi^E(s,R) = \frac{\alpha}{r+c-s} & \text{if } \Pi^{E**}(s) \geq 0 \\ 0 & \text{if } \Pi^{E**}(s) < 0, \end{cases} \qquad (7)$$

in which the firm's expected profit from a positive equilibrium investment $\left(\Pi^{E**}(s) := \Pi^E(s, R^{**}(s))\right)$ is given by

$$\Pi^{E**}(s) = (1-\tau)\left\{\alpha\left[\ln\left(\frac{\alpha}{r+c-s}\right) - 1\right] - (r+c)F\right\}. \qquad (8)$$

Agency decision. If the agency receives a subsidy application in stage 2, the agency observes a realization v and chooses a subsidy rate s to maximize its payoff. We write the agency's problem as

$$\max_{s\in[0,\bar{s}]} U^*(v,s)_+ = \{(v-gs)R^*(s) + [\alpha \ln R^*(s) - (r+c-s)R^*(s) - (r+c)F]_+\}_+, \qquad (9)$$

in which the agency's expected payoff $U^*(v,s) = U\left(v,s,R^*(s),\pi^{I*}(s)\right)$ follows from insertion of equations (2), (5) and (6) into equation (4).

We solve the agency's problem (equation (9)) in Lemma A5 in Appendix C. From equation (8) we obtain two threshold values of F, $\underline{F}$ and $\bar{F}$ with $\underline{F} < \bar{F}$. If $F > \bar{F}$, $\Pi^{E**}(s) < 0$ for all $s \in [0,\bar{s}]$. Knowing that the firm would not invest even with the maximum subsidy rate, the agency awards no subsidy. In contrast, if $F \leq \underline{F}$, $\Pi^{E**}(s) \geq 0$ for all $s \in [0,\bar{s}]$. In this case the firm will invest even without a subsidy. The agency's behavior can be described by the mapping

$$s^*(v) = \begin{cases} 0 & \text{if } v \leq \underline{v} := (r+c)(g-1) \\ s^{**}(v) := v - (r+c)(g-1) & \text{if } v \in (\underline{v}, \bar{v}) \\ \bar{s} & \text{if } v \geq \overline{v} := \underline{v} + \bar{s}, \end{cases} \tag{10}$$

in which $0 < \underline{v} < \overline{v}$, and $s^{**}(v)$ identifies for each realization $v \in \mathbb{R}$ a unique optimal subsidy rate when the constraints on the feasible subsidy rates ($s \in [0, \overline{s}]$) are ignored. The subsidy rule characterized by equation (10) implies a rejection for sufficiently low spillover rates, the optimal interior subsidy rate $s^{**}(v)$ for intermediate spillover rates, and the maximum subsidy rate $\bar{s}$ for sufficiently high spillover rates.

The case when $F \in \left(\underline{F}, \bar{F}\right]$ is more complex since then the firm will invest only if it receives a subsidy. The agency's optimal subsidy rule is given by the mapping

$$s^*(v) = \begin{cases} 0 & \text{if } v < v^0 \\ \tilde{s} := r + c - \alpha e^{-\left[1 + \frac{(r+c)F}{\alpha}\right]} & \text{if } v \in [v^0, \tilde{v}) \\ s^{**}(v) & \text{if } v \in [\tilde{v}, \bar{v}) \\ \bar{s} & \text{if } v \geq \overline{v} := \underline{v} + \bar{s}, \end{cases} \tag{11}$$

in which $\tilde{s}$ is obtained from equation (8) as the unique subsidy rate satisftying $\Pi^{E**}(\tilde{s}) = 0$, and v^0 and $\tilde{v}$, with $0 \leq v^0 < \tilde{v} \leq \overline{v}$, denote the (unique) values of v that satisfy $U^*(\tilde{s}(v^0))$=0 and $s^{**}(\tilde{v}) = \tilde{s}$, respectively. Compared to the subsidy rule (10), for a project with $v \in (\max\{v^0, \underline{v}\}, \tilde{v})$ the rule (11) prescribes the agency to increase the subsidy rate from the unconstrained rate $s^{**}(v)$ to $\tilde{s}$ so as to satisfy the firm's zero-profit constraint. While $\tilde{v} \geq \max\{v^0, \underline{v}\}$, v^0 may be smaller or greater than $\underline{v}$ (see Remark A1 in Appendix C). However, we also have $\min\{v^0, \underline{v}\} \geq 0$. Thus, a necessary condition for the firm to obtain a subsidy is a positive realization of V for its project.

Application decision. (See Lemma A6 in Appendix C for details.) The firm's subsidy application decision in stage 1 can be written as:

$$\max_{d_a \in \{0,1\}} d_a \left[\int_{-\infty}^{\infty} \Pi^{E**} \left(s\left(v\right)\right)_+ \phi\left(v\right) dv - K \right] + \left(1 - d_a\right) \Pi^{E**} \left(0\right)_+ . \qquad (12)$$

The term in the square-brackets in the maximization problem 12) captures the firm's expected payoff to applying for a subsidy, including the fixed application cost K. The term shows how the firm, when contemplating subsidy applications, takes expectation over all possible spillover rate evaluations and, consequently, all possible subsidy rate decisions of the agency. The firm can then estimate the expected investment levels resulting from those subsidy rates, and, ultimately, the firm's expected profits. The last term captures the expected profits if the firm does not apply for a subsidy and therefore receives no subsidy. The max operators embodied in these expected profit terms in problem (12) reflect the firm's option to invest only if doing so is profitable in expectation.

The solution to the problem (12) is easy when $F > \bar{F}$, as then the firm knows it will get no subsidy in any circumstances. Therefore, the firm does not apply ($d_a^* = 0$). In contrast, if $F < \underline{F}$, the firm will invest even without a subsidy. In this case the firm knows that the agency's subsidy rule $s^*\left(v\right)$ is given by equation (10). Therefore the first term in the square-brackets of the problem (12) can be expressed as

$$\int_{-\infty}^{\infty} \Pi^{E**} \left(s^*(v)\right)_+ \phi\left(v\right) dv$$
$$= \Phi\left(\underline{v}\right) \Pi^{E**}\left(0\right) + \int_{\underline{v}}^{\overline{v}} \Pi^{E**}\left(s^{**}\left(v\right)\right) \phi\left(v\right) dv + \left(1 - \Phi\left(\overline{v}\right)\right) \Pi^{E**}\left(\overline{s}\right).$$

As a result the solution to the problem (12) is $d_a^* = 1$ if and only if

$$\int_{\underline{v}}^{\overline{v}} \Pi^{E**}\left(s^{**}\left(v\right)\right) \phi\left(v\right) dv + \left(1 - \Phi\left(\overline{v}\right)\right) \Pi^{E**}\left(\overline{s}\right) - \left(1 - \Phi\left(\underline{v}\right)\right) \Pi^{E**}\left(0\right) \geq K, \qquad (13)$$

and $d_a^* = 0$ otherwise.

If $F \in \left[\underline{F}, \bar{F}\right]$, the firm will not invest without a subsidy. In this case the agency's subsidy rule is given by (11). Thus, if $v \geq \tilde{v}$, the firm's zero-profit constraint is irrelevant for the agency's decision, and if $v < \tilde{v}$, the firm will either receive no subsidy in which case it will not invest or it will receive subsidy $\tilde{s}$ that just satisfies the firm's zero-profit constraint, which by definition also leads to the zero profits. Therefore the solution to the problem (12) is $d_a^* = 1$ if and only if

$$\int_{\tilde{v}}^{\overline{v}} \Pi^{E**}\left(s^{**}\left(v\right)\right)\phi\left(v\right)dv + \left(1 - \Phi\left(\overline{v}\right)\right)\Pi^{E**}\left(\overline{s}\right) \geq K, \tag{14}$$

and $d_a^* = 0$ otherwise.

In Proposition A1 in Appendix C we show that the equilibrium is a well defined mapping on the set of fixed R&D costs $F \in [0, \infty)$. This equilibrium admits a number of comparative static results. We focus on the effects of financial market imperfections. In Proposition A2 in Appendix C we establish that an increase in c naturally worsens firms' financial constraints: Both $\underline{F}$ and $\bar{F}$ are decreasing in c. If a firm remains unconstrained despite the increase in c, the higher c reduces the firm's R&D investment $R^{**}(s^{**})$, and calls for smaller subsidies: The optimal unconstrained subsidy rate $s^{**}(v)$ is decreasing in c and the agency's propensity to reject the application (as measured by $\underline{v}$) is increasing in c. From the agency's perspective, higher c means less efficient R&D technology. An increase in c calls for larger subsidies only if the agency wants to help a firm to overcome its financial constraint: The optimal constrained subsidy rate $\tilde{s}$ is increasing in c. The firm's constrained R&D investment level $R^{**}(\tilde{s})$ is also increasing in c.

4 Econometric Implementation

We next describe how to estimate the agents' four key decisions of the theoretical model: The firm's decision whether to launch an R&D project and the optimal R&D investment level conditional on launching, the firm's decision to apply for a subsidy, and the agency's subsidy rate decision. We provide details of the estimation process, including the order of estimation equations, and a discussion of auxiliary estimations, in Appendix B.[12]

We denote by $\mathbf{X}_{it}^l$ a vector of observable firm and project characteristics, and by β^l the associated vector of parameters, in which subscript i denotes a project (and a firm), subscript t denotes the year and superscript $l \in \{F, K, R, s\}$ refers to the variable of the interest. The $\mathbf{X}_{it}^l$ vector contains at least the following variables: A 2^{nd} order polynomial in firm (log) age, (log) number of employees, sales per employee, and dummies for a calendar year, an industry, an R&D investment in the previous year, and a dummy for eligibility for EU regional aid. All explanatory variables are lagged by one year. We bootstrap the whole estimation procedure to obtain standard errors.

R&D investment level and cost of external financing. We define the constant shifting the expected profitability of an R&D project (see equation (6)) as

$$\alpha_{it} := e^{\mathbf{X}_{it}^R \beta^R + \varepsilon_{it}}. \tag{15}$$

in which ε_{it} is a random shock affecting the expected profitability of project i in year t. This profitability shock is observed by all three agents of the model but unobserved by the econometrician.

From equation (7) we obtain an empirical counterpart for the size of the firm's R&D project as $R_{it}(s_{it}) = \alpha_{it}/\left(r_t + c_{it} - s_{it}\right)$. Substituting equation (15) for α_{it} and taking logs of both sides yield

$$\ln R_{it}(s_{it}) = \mathbf{X}_{it}^R \beta^R - \ln(r_t - s_{it} + c_{it}) + \varepsilon_{it}. \tag{16}$$

Equation (16) is our estimation equation for the level of R&D investment, conditional on the firm launching a project. The coefficient of the term $\ln(r_t - s_{it} + c_{it})$ is unity. By this stage, s_{it} is known, and we use the one year Euribor rate to measure r_t, the investors' cost of raising funds (which should not be specific to funded projects).

Identifying the project-specific cost of monitoring cf_{it} is less straightforward. We assume that

[12] Our approach necessitates a number of auxiliary estimations since we only observe project level R&D for successful applicants and cashflow and Tekes grades for submitted applications.

$$c_{it} = \begin{cases} e^{(\ln cf_{99} - \ln cf_{it})\beta^c} & \text{if } cf_{it} < cf_{99} \\ 0 & \text{if } cf_{it} \geq cf_{99}, \end{cases} \tag{17}$$

in which cf_{99}/cf_{it} measures a *cashflow gap* of project i in year t. Here cf_{it} is the ratio of the project's pledgeable casflow to its size and cf_{99} is the 99^{th} percentile of the distribution of $\ln cf_{it}$. We thus assume the cost of monitoring c_{it} to be an increasing function of the cashflow gap. If there is no gap ($cf_{99}/cf_{it} \leq 1$), $c_{it} = 0$. The idea is, in line with Holmström and Tirole (1997) and Lian and Ma (2021), that a project in which a firm has more skin in the game requires less monitoring. We consider this approach to identify the cost of financial market imperfections at a project level worthwhile given the lack of a consensus on how to measure financial constraints at a firm level (Farre-Mensa and Ljunqvist, 2016). As a robustness test, we use an estimated cost of external funding at a firm level from balance sheet data as an alternative measure of c_{it}.

With $\mathbf{X}_{it}^R$, r_t, s_{it}, cf_{99} and cf_{it}, being observed, estimation of equation (16) (with equation (17) substituted in) using maximum likelihood yields $\hat{\beta}^R$, $\hat{\beta}^c$, and the variance of ε_{it}. Since we only observe the project level realized R&D investments of those firms that receive a subsidy, we use sample selection methods. For identification, we exploit the agency's goal of prioritizing SMEs in its subsidy allocation decisions: The maximum subsidy is 10 percentage points higher for SMEs. Since the criteria for qualifying as an SME are decided at the EU level (see Recommendation 2003/361EC), they can be taken as exogenous. This non-linearity of the agency decision rule means that an SME is more likely to apply for a subsidy but its SME status *per se* should have no impact on its R&D investment level. Our exclusion restriction is based on the SME status of a firm. The first stage dependent variable is a dummy taking value one if firm i obtained a subsidy in year t, and zero otherwise. We execute the first stage by estimating the model separately for SMEs and non-SMEs.[13]

R&D participation. The fixed cost of launching project i is

[13] An LR-test leads us to reject the Null hypothesis that the two sets of firms have similar coefficient vectors, lending credence to our exclusion restriction.

$$F_{it} := e^{\mathbf{X}_{it}^F \beta^F + \zeta_{it}}. \tag{18}$$

Using equations (7) and (8) we may express an empirical counterpart of the firm's participation constraint as $\alpha_{it} \left[\ln\left(\alpha_{it}/\left(r_t + c_{it} - s_{it}\right)\right) - 1\right] \geq \left(r_t + c_{it}\right) F_{it}$. After substitution of equations (15) and (18) into this inequality, taking logs, and rearranging, we may rewrite the firm's decision of whether or not to launch an R&D project as an indicator function

$$\begin{aligned}\mathbf{1}_{[0,\infty)}(\ln \hat{\alpha_{it}} + \ln \left[\ln\left(\frac{e^{\mathbf{X}_{it}^R \hat{\beta}^R + \varepsilon_{it}}}{r_t + \hat{c}_{it} - s_{it}}\right) - 1\right] \\ - \ln\left(r_t + \hat{c}_{it}\right) - \mathbf{X}_{it}^F \beta^F - \zeta_{it}),\end{aligned} \tag{19}$$

in which $\mathbf{X}_{it}^R$, $\mathbf{X}_{it}^F$, r_t and s_{it} are observed, and $\hat{c}_{it}$ and $\hat{\beta}^R$ are obtained from the estimation of equation (16). The vector of parameters to be estimated is thus β^F. We have identifying variation because the first three terms have a coefficient of unity and because the fixed cost is independent of the subsidy rate s_{it}. We use simulated (quasi-) maximum likelihood (SML) to take into account that ε_{it} needs to be simulated (see Appendix B).

Agency decision. An estimate $\hat{\tilde{s}}_{it}$ for the subsidy rate satisfying the firm's zero-profit constraint is directly obtained by plugging equation (18) and the parameters $\hat{\alpha}_{it}$, r_t, $\hat{c}_{it}$ and $\hat{\beta}^F$ into $\tilde{s}$ of equation (11). To derive an estimable equation for the agency's unconstrained optimal subsidy rate $s^{**}(v)$ specified in equation (10) we define

$$v_{it} := \mathbf{X}_{it}^s \beta^s + \eta_{it}, \tag{20}$$

in which η_{it} is a random shock to the spillover rate of project i in year t. It is observed by the agency evaluating an application in stage 2 of the game, but it is unobserved by the econometrician and by the private sector in stage 1. Inserting equation (20) together with the parameters r_t and $\hat{c}_{it}$ into $s^{**}(v)$ of equation (10) gives

$$s_{it}^{**} = \mathbf{X}_{it}^s \beta^s - (r_t + \hat{c}_{it})(g - 1) + \eta_{it}, \tag{21}$$

To estimate the agency decision rule of equation (21), we use value 1.2

for the shadow cost of public funds g, and use only those observed positive subsidy rates with $s_{it} > \hat{\tilde{s}}_{it}$ because, according to our model, $s_{it}^{**} > \hat{\tilde{s}}_{it+}$. Estimation of equation (21) by generalized two-limit Tobit provides us $\hat{\beta}^s$. The vector of observable firm and project characteristics $\mathbf{X}_{it}^s$ includes the SME-dummy to accommodate the agency's priorities, and the agency's grades for each project.

Equations (4), (16), and (20) show how spillovers generated by project i, $v_{it}R_{it}$, are a function of both η_{it} and ε_{it}. While spillovers and profits are thus correlated, the shock to the *spillover rate* v_{it} (i.e., spillovers per euro of R&D) and the shock to the profitability of R&D (ε_{it}) are assumed to be uncorrelated. As a result the agency decision rule is not subject to selection on unobservables.

Application decision. We specify the application costs as

$$K_{it} := e^{\mathbf{X}_{it}^K \beta^K + \mu_{it}}, \tag{22}$$

in which $\mu_{it} := \xi\varepsilon_{it} + \mu_{0it}$ is a random shock to the application costs, observed by the firm but unobserved by the econometrician. We thus allow the application cost shock μ_{it} and the profitability shock ε_{it} to be correlated, with ξ being a measure of their covariation. The sign of parameter ξ provides information on whether or not firms with higher profitability shocks have systematically different application costs than otherwise similar firms.

We estimate the firm's application decision by SML. For each simulation draw, we numerically integrate the expected discounted profits from applying for subsidies (the expression in square brackets in equation (12) with equation (22) substituted for the costs of applying). We use all the parameters estimated in the prior stages of the estimation process. To calculate the expected benefits from applying for a subsidy, we take into account the agency's grading of each subsidy application (see Appendix B). Identifying variation comes from several sources: First, the subsidy rate is a function of the SME status of a firm. Second, the R&D investment is a function of the subsidy rate. Neither of these variables ought to have a direct effect on the application cost. Third, we allow the firm's past application behavior to affect the application costs but assume it has no direct impact on the fixed cost of R&D nor on the subsidy rate. Finally, the correlation coefficient of the profitability and application cost shocks ξ is identified in SML through variation in the simulated profitability shocks and because theory dictates that the coefficient of the impact of expected profitability on the decision to apply decision is unity.

Statistical assumptions. The unobservables ε_{it}, ζ_{it}, η_{it}, and μ_{it} are assumed to be normally distributed with mean zero and with variances that we estimate, and uncorrelated with observed applicant characteristics. We also assume that a) μ_{0it} (in $\mu_{it} = \xi\varepsilon_{it} + \mu_{0it}$) is a random shock whose variance is normalized to unity; b) $\eta_{it}, \zeta_{it} \perp \varepsilon_{it}$; c) $\eta_{it}, \zeta_{it} \perp \mu_{0it}$ and d) $\eta_{it} \perp \zeta_{it}$.

The economic interpretation of assumption b) is that the shock to spillovers per euro of R&D investment are uncorrelated with the shock to the private value of the idea. Privately more lucrative projects thus create larger spillovers in absolute but not relative terms. Assumptions c) and d) mean that the spillover rate shock η_{it} and the fixed cost shock ζ_{it} are uncorrelated with the application cost shock μ_{it} and with each other. These assumptions are made for convenience: The assumptions rule out a selection problem for the subsidy rate equation (21), make the subsidy rate s_{it} independent of the profitability shock ε_{it}, and render the observability of μ_{it} inconsequential for the agency. Note that assumptions b) and c) also imply that $\varepsilon_{it} \perp \mu_{0i}$. However, these assumptions introduce the selection problem for the R&D investment equation (16) discussed above. Under these assumptions, we can identify all the structural parameters of our model, including those governing the distribution of the shocks.

5 Estimation Results

We collect into Table 2 the coefficients from all main estimation equations and relegate the results of the auxiliary estimations into Appendix B.

[Table 2]

R&D investment level and cost of external financing. Column 1 of Table 2 displays the estimated coefficients of the intensive margin R&D equation (16). These coefficients measure how firm characteristics affect the marginal profitability of R&D. Firm age, size and productivity (measured by sales per employee) affect R&D nonlinearly. Firms in less-developed regions invest significantly less and firms that invested in the previous year significantly more in R&D. The negative coefficient of the Mills ratio indicates negative selection, i.e., firms with more profitable projects are less likely to appear in our R&D investment sample and, thus, to apply for subsidies, ceteris paribus. The unreported coefficient estimates of industry dummies indicate significant heterogeneity in marginal profitability of R&D across industries and those of year dummies suggest that Finnish firms invested less in the base year 2005 than earlier or later.

The coefficient for $\ln cashflowgap$ (0.95) essentially implies a one-to-one relation between the monitoring cost and the gap: The lower is the firm's cashflow-to-investment ratio, the higher its cost of external finance. The estimated mean cost of external finance ($r_t + c_{it} - 1$) is 0.04 (p-value 0.00), supporting the evidence suggesting that access to finance was not a major problem for Finnish firms in our data period.

R&D participation. In column 2 we report the coefficients from the estimation of the extensive margin R&D equation (19). The results provide information about the determinants of the fixed costs of R&D, helping to understand the selection into R&D in terms of observable firm characteristics. The fixed costs of R&D are a nonlinear function of the number of employees and productivity. Exporters and firms in the less-developed regions have a lower fixed cost. In line with Arqué-Castells and Mohnen (2015) and Peters et al. (2017), past R&D reduces the fixed R&D cost. The omitted results regarding year and industry dummies suggest that fixed costs are higher in the first two years and vary over industries.

Agency decision. Column 3 shows the estimated coefficients of the agency decision equation (21). We find sales per employee to have a nonlinear impact on the subsidy rate. Firms with no R&D in the previous year get a 1.6 percentage points higher subsidy rate (significant at 10% level). Our results suggest that SMEs obtain no higher subsidy rates, despite the higher maximum subsidy rate allowed for SMEs. Tekes' internal grading variables only appear to play a minor role: A one point increase in the estimated commercial *risk* of the project increases the subsidy rate by one percentage point. According to the unreported coefficients, the awarded subsidy rates were lower in the early years of the millennium. We find no evidence that Tekes targeted subsidies to any particular industry. The estimated mean spillover per euro of R&D is 0.58 (s.e. 0.01).

Application decision. In column 4 we report results from estimating the application decision. Firm size affects positively and productivity non-linearly the cost of application. Exporters and past applicants face lower application costs, as do firms investing in R&D in the previous year and firms in less developed regions. The shock to application costs is positively correlated with the profitability shock, though the parameter estimate is insignificant. The unreported results suggest higher application cost in the early years of our sample and considerable heterogeneity over industries.

Implications of the estimated coefficients. Table 3 shows the simulated fixed costs of R&D (F_{it}) and application costs (K_{it}). As is the case with discrete choice models, these costs are estimated more accurately for those firms that invest or apply for subsidies than for those that do not. While the simulated mean fixed R&D cost is 1.2M€, the median is only 105 000€. Almost 40% of firms do not invest in R&D and the model explains these non-investments by fixed costs, resulting in the relative high mean. Fixed cost are lower than 16 000€ for the firms in the decile with the lowest fixed costs. The mean application cost may also seem high at 112 000€, but is similarly explained by the long right tail: In the data, only 18% of firms apply. Some 10% of firms have application costs lower than 1 800€.

[Table 3]

6 Counterfactual Welfare Analysis

6.1 Policies

As an alternative to the actual R&D subsidy policy, we consider an optimal R&D tax credit policy. As benchmarks, we consider a laissez-faire economy without government interventions in firms' R&D investments; the first-best policy where the social planner forces the firms to invest the desired amount in each project; the second-best (Ramsey) policy where the social planner is constrained by the firm's zero profit condition; and a laissez-faire economy without financial market imperfections (present in all other policies).

Optimal R&D tax credit. To analyze an optimal R&D tax credit policy, we make two modifications: First, we set the subsidy rate s to zero. Second, we introduce an R&D tax credit rate $\tilde{\tau}_R \in [0,1]$. The R&D tax credit means that a firm investing R euros in R&D is reimbursed for $\tilde{\tau}_R R$ euros. It is more convenient to work with $\tau_R := \tilde{\tau}_R/(1-\tau)$, a tax credit rate adjusted to the corporate tax level.

Our modeling of the R&D tax credit policy is motivated by the tax credit regime in several countries (Belgium and the UK among others) where even loss-making firms can claim tax credit: In the case of insufficient corporate tax liability, the firm is assumed to receive a full refund of unused tax credits. We assume that only variable R&D costs are subject to the tax credit. This assumption facilitates the comparison of the tax credit policy with the subsidy policy. We also assume that all R&D performing firms claim the tax credit. This assumption may bias the counterfactual results as evidence (e.g.,Verhoeven et al., 2012; Busom et al., 2014) shows that some eligible firms fail to claim the tax credit.

Under these assumptions, the firm's optimal R&D investment rule with an R&D tax credit is equivalent to the one given by equations (7) and (8) with τ_R replacing s (see Appendix D). The agency's project-specific expected payoff with an R&D tax credit can be obtained by replacing s by τ_R in $U^*(v,s)$ specified in equation (9). After substituting the empirical counterparts for the other variables in $U^*(v,\tau_R)$, we write the agency's R&D tax credit problem as

$$\max_{\tau_R\in[0,1]} \sum_{i=1}^{N} \iiint U^*(\epsilon_i,\zeta_i,\eta_i,\tau_R)\phi(\epsilon_i,\zeta_i,\eta_i)d\epsilon_i d\zeta_i d\eta_i, \tag{23}$$

in which N is the total number of potential R&D projects in the economy and $\phi(\varepsilon_i,\zeta_i,\eta_i)$ is the joint normal distribution of the profit, fixed cost, and spillover rate shocks to project i. To determine the optimal R&D tax credit τ_R^*, we perform a grid search over the region $\tau_R \in [0,1]$ with a step size of 0.01, and choose τ_R^* as the value that yields the highest agency welfare. We simulate the shocks 100 times from their estimated distributions.

While subsidies and tax credits have identical marginal impacts on the firms' R&D investment decisions, they have major welfare differences. The maximization problems (9) and (23) illustrate the main welfare advantage of subsidies over tax credits: The marginal effect of tax credit on R&D is invariant across projects whereas a subsidy policy enables project-specific treatment. However, the subsidy application and examination processes hinder access to the treatment whereas all firms investing in R&D have access to R&D tax credits: The aggregate realized welfare under the optimal tax credit policy is $\sum_{i=1}^{N} U^*(\epsilon_i, \zeta_i, \eta_i, \tau_R^*)$ whereas the aggregate realized welfare under the optimal subsidy policy is $\sum_{i=1}^{N_A}[U^*(\epsilon_i, \zeta_i, \eta_i, s_i^*)_+ - K_i] + \sum_{i=N_A+1}^{N} U^*(\epsilon_i, \zeta_i, \eta_i, 0)$ in which $N_A \subseteq N$ is the number of applications. If N_A is small relative to N, as is the case in our data, the subsidy policy can hardly generate large economy-wide effects.

Benchmarks. In laissez-faire, there are neither R&D subsidies nor tax credits. In the first best scenario the perfectly informed agency chooses R&D investment for each project. We assume that R&D is financed at the same cost as private funding is provided. As the first best investment level may lead to negative profits for a firm, we also consider the second best policy where the agency chooses the optimal level of each R&D investment subject to the firms' zero profit constraints. Finally, to study the effects of financial market imperfections, we set the monitoring cost c_{it} to zero for all projects in the laissez-faire regime. As a result, the firms' cost of external funding is equal to the funding cost of the investor.

6.2 Results

We compare R&D participation, R&D investment levels, spillovers, profits and welfare across the different policy regimes. The reported means and medians are calculated over all firms and simulation draws (see Appendix E). We also report the ratio of a mean outcome of a policy regime to the mean outcome in the laissez-faire scenario.

R&D participation. In Table 4 we report the firms' propensity to conduct R&D in various policy regimes. Under the laissez-faire scenario, 62% of firms invests in R&D in a given year and the median investment probability over all firms is 77%. The policy interventions have no major effects: The first best policy and R&D tax credits increase R&D participation by 2% from laissez-faire. Neither subsidies nor financial market imperfections have marked effects. These results are in line with Peters et al. (2017) and Dechezleprêtre et al. (2023) who find little effects of R&D tax credits at the extensive margin. However, Table 4 masks some differences across the regimes in R&D participation: E.g., the first best includes projects with positive spillovers but negative profits which are excluded from the laissez-faire scenario, and vice versa for the projects with positive profits but negative spillovers.

[Table 4]

R&D investment level. Table 5 shows large differences across policy regimes at the intensive margin, again in line with Peters et al. (2017) and Dechezleprêtre et al. (2023). The mean R&D investment under laissez-faire, conditional on investing (left panel), is 197 000€ per project but almost two and a half times higher under the first and second best policies. R&D tax credit and subsidy policies induce roughly 30-50% higher average R&D investments than laissez-faire. The R&D tax credit regime generates a somewhat higher mean investment than the subsidy regime (289 000€ versus 253 000€). However, the mean R&D investment of successful applicants (last row, left panel) is substantially higher than investments under R&D tax credits and close to the first best level, emphasizing the project-specificity of the subsidy policy. Financial market imperfections hardly affect R&D investments.

To compare the R&D intensities in different scenarios taking both the extensive and intensive margins into account, we report the unconditional means in the right panel. Given the small differences across policies in the probability to invest in R&D (Table 4), the rankings and ratios in the right panel are close to those in the left panel. R&D tax credits have a larger relative effect than subsidies when we account for the extensive margin.

The R&D distribution is right-skewed: We plot the distribution from one simulation round of the counterfactual analysis across policy regimes in Figure 2. The first and second best, and R&D support policies shift the R&D distribution to the right.[14]

[14] The differences between some policy regimes are increasing in project size. E.g., the mean 50th percentile for the subsidy regime is 69 000€ and for laissez-faire 55 000€, a difference of 25%, whereas the difference at the 90th percentile is 36%. The differences between laissez-faire and first and second best are also increasing in project size. In contrast, for the R&D tax credit the difference to laissez-faire is 41-44% irrespective of the measurement point.

[Table 5][Figure 2. Distribution of R&D conditional on policy].

Profits. The left panel of Table 6 displays the profit estimates. Profit differences across policy regimes are much smaller than those in R&D investment because some 40% of the firms invest in R&D in none of the regimes. The mean expected discounted profits are slightly higher under the two support regimes than under laissez-faire. Because financial market imperfections have little impact on R&D, they do not affect profits much. Profits in the first and second best regimes are lower than in laissez-faire by some 5%: The firms generating positive spillovers invest in these regimes more than the profit-maximizing level and the firms generating negative spillovers invest less.

[Table 6]

Spillovers. Estimates reported in the middle panel of Table 6 suggest that spillovers are much lower than firm profits in all regimes, ranging from 56 000€ (5% of the profits) under laissez-faire to 138 000€ (12% of the profits) under first best. Spillovers in the R&D tax credit regime are somewhat higher than in the R&D subsidy regime on average, but for the actually subsidized firms, spillovers relative to profits are higher. While R&D subsidy and tax credit policies significantly increase spillovers (by 28 and 49%) compared to laissez-faire, the first and second best regimes generate even larger spillovers.

Welfare. The ultimate measure of the effectiveness of different R&D support policies is their impact on welfare. We find (right panel of Table 6) that the first and second best regimes improve welfare by 2% compared to laissez-faire. There is thus no significant room to increase welfare: The optimal R&D tax credit increases welfare by 1%. These results are comparable to Acemoğlu et al. (2018) who find that a first best innovation policy increases welfare by 4% and the optimal uniform R&D subsidy by 1%, and to Akcigit et al. (2021), in which the optimal uniform R&D subsidy increases welfare by 1%, too. In Acemoğlu et al. (2018) and Akcigit et al. (2021), the uniform subsidy applies equally for all R&D investing firms and is hence similar to our optimal R&D tax credit.

Thus, while the two R&D support policies increase R&D investments and spillovers, they do not improve welfare much once the shadow costs of public funds are taken into account. If anything, the R&D subsidy regime generates lower welfare than laissez-faire. The reason for this adverse net welfare effect comes from application costs: Since the agency commits to no subsidy rate rule, it does not internalize the effects of its policy on the number and costs of applications. If application costs are ignored, the subsidy regime creates a small welfare improvement. As financial market imperfections have little effect on investments, they cannot have notable welfare effects either.

Our estimates of the welfare of the R&D support policies do not capture some relevant considerations. On the one hand, our welfare estimates are likely to be upward biased: Although we take into account the firms' application costs, we ignore the agency's administrative costs which are ca. 50 million euro (Tekes, 2010) a year (i.e., some 2 000 euros per firm). On the other hand, global welfare effects are likely understated because a large part of consumer surplus and technological spillovers generated by the Finnish R&D projects is captured abroad but that part is not necessarily included in the Finnish agency's objective function. We also ignore firms' international R&D location decisions, which may lead us to underestimate the benefits of support policies at a national level.

We assume that all eligible firms use the R&D tax credit, which is likely to create an upward bias in both benefits and costs of the R&D tax credit policy. On the other hand, we allow no relabeling of corporate expenditures, making our counterfactual R&D tax credit policy less costly for the society than it would probably be in practice (Chen et al., 2021 report significant relabeling in a different environment). Our welfare estimations also ignore the agency's possible budget constraint, which is likely to create a downward bias in the estimates if the constraint is binding and an upward bias if unused budget leads to a wasteful end-of-year spending (see, e.g., Liebman and Mahoney, 2017).

Policy parameters. As Table 7 reports, on average 15% of firms apply for a subsidy and the mean subsidy rate, conditional on getting one, is 39%. Both figures are close to those in the data (18% and 35%). We find the optimal tax credit rate τ_R^* to be approximately 34% (0.34, with a bootstrapped standard error of 0.01).[15] In calculating the optimal tax

[15]Since $\tau_R := \tilde{\tau}_R / (1 - \tau)$, with the Finnish corporate tax rate τ of 0.26 prevailing in

credit rate the agency recognizes that some projects should get a larger tax subsidy than the maximum subsidy rate $\bar{s}$ but that some projects should be taxed because of negative spillovers. Acemoğlu et al. (2018) find the optimal uniform subsidy rate to be 39% whereas it is 54% but rapidly decreasing with trade openness in Akcigit et al. (2021).

We find that the mean subsidy, conditional on getting one, has a fiscal cost of 59 000€, whereas the mean tax credit conditional on investing in R&D has a fiscal cost of 98 000€. The unconditional fiscal costs of a mean subsidy and a mean tax credit are 27 000€ and 51 000€.

[Table 7]

Robustness. We report the results of robustness analyses in Appendix E: First, we estimate the cost of external finance using balance sheet data on interest rates; second, we use only subsidies, not subsidies and subsidized loans, in calculating the subsidy rate; and third, we exclude the three largest firms. We find that the alternative measure yields a somewhat higher estimate of cost of external finance and, thus, lower estimates of R&D investment, profits and welfare; using subsidies only yields results close to those in the main text; and excluding the top three firms yields somewhat higher R&D investment, profits and welfare. When comparing the other policy regimes to laissez-faire, we obtain similar R&D ratios with one exception: Removing financial market imperfections increases R&D by 9% when using the alternative cost of external finance. The other ratios deviate at most by one percentage point. As a fourth (unreported) robustness test, we introduce 3^{rd} order terms into our polynomials, and an expanded set of industry dummies. This counterfactual produces results that are similar to our main results.

7 Conclusions

We build and estimate a dynamic model of an innovation policy which incorporates the main policy motivations, and conduct a counterfactual analysis of different R&D support policies. We employ self-reported project-level cashflow data of subsidy applicants to measure financial market im-

our data period, the corresponding socially optimal $\tilde{\tau}_R^*$ is 0.25 ($\approx 0.34 \times (1 - 0.26)$).

perfections. In a departure from most existing work, we use the variation in government R&D subsidy rate decisions to identify the parameters of the government's utility function.

We show theoretically that financial market imperfections should decrease the optimal level of support at the intensive margin but increase it at the extensive margin. Quantitatively, we find only small effects of financial market imperfections on R&D and on the effectiveness of R&D support policies. An explanation might be that our data period consists of the boom years before the global financial crisis when access to finance was not an issue for the Finnish firms. We find that larger and more productive firms invest more and have higher fixed costs of R&D. The agency takes firm characteristics into account in deciding the subsidy rate. Costs of applying for subsidies are heterogeneous and affect the effectiveness of R&D subsidy policy.

In the counterfactual analysis, the actual R&D subsidy policy with applied-but-tailored support is compared with the one-size-fits-all R&D tax credit policy. We find that these R&D support policies substantially increase R&D investment levels, but do not increase R&D participation much. In contrast to R&D tax credits, R&D subsidies achieve close to first best investments but only reach a modest fraction of firms. The optimal R&D tax credit rate is 34%, which increases fiscal costs more than 90% compared with R&D subsidies but also ultimately yields higher welfare. First and second best double R&D levels from laissez-faire. The same effects apply to spillovers, but profits are roughly constant over policies. We find profits to be considerably larger than spillovers, perhaps because the Finnish agency internalizes profits fully but only cares about domestic spillovers. First and second best increase welfare by 2% and R&D tax credits by 1%. R&D subsidies reduce welfare slightly despite increasing R&D and spillovers by 25% or more.

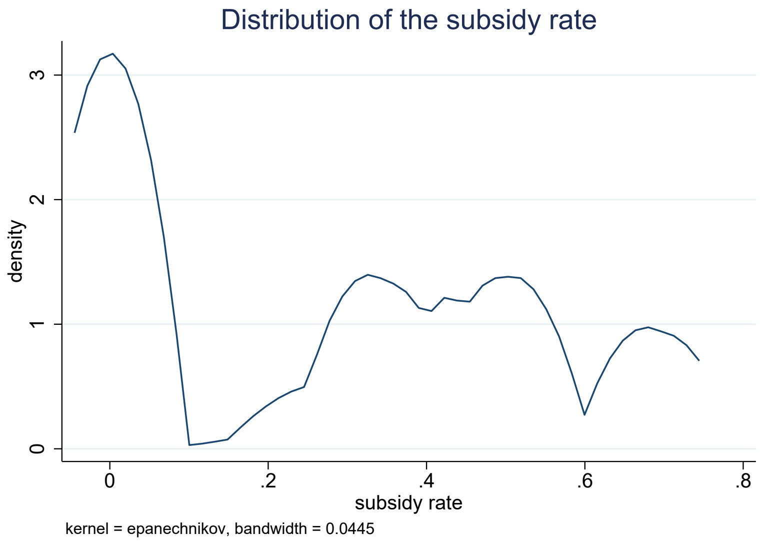

Figure 1. Distribution of the subsidy rate

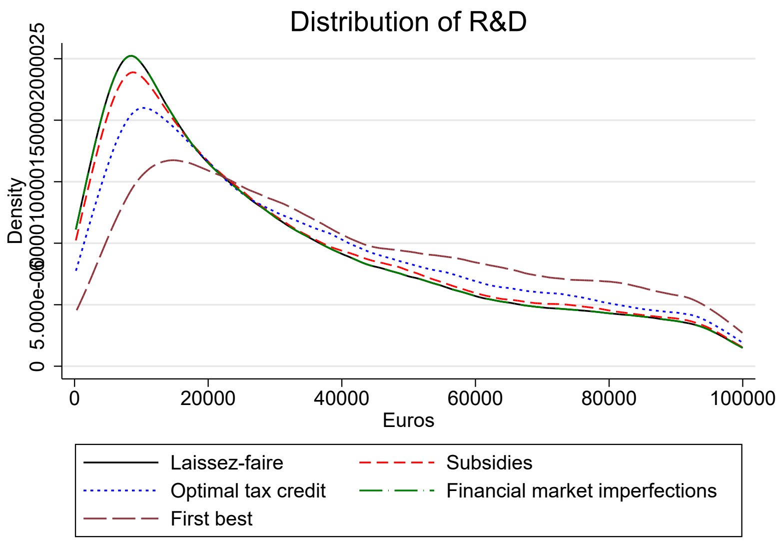

Figure 2. Distribution of counterfactual R&D investment (truncated at 100 000€)

Table 1. Descriptive statistics

	Non-applicants			Applicants			Rejected applicants			Successful applicants		
	mean	s.d.	p50	mean	s.d.	p50	mean	s.d.	p50	mean	s.d.	p50
subsidy rate	0.00	0.00	0.00	0.27	0.26	0.34	0.00	0.00	0.00	0.35	0.24	0.35
$R\&D_{actual}$	.	.	.	.	.	.	.	.	.	483 352	871 969	181 60
$1[R\&D]_t$	0.55	0.50	1.00	0.95	0.21	1.00	0.78	0.41	1.00	1.00	0.05	1.00
tech	.	.	.	2.08	0.79	2.00	1.71	0.79	2.00	2.17	0.76	2.00
risk	.	.	.	2.31	0.83	2.00	2.26	0.93	2.00	2.32	0.80	2.00
prev applicant	0.13	0.34	0.00	0.22	0.42	0.00	0.18	0.39	0.00	0.24	0.42	0.00
$1[R\&D]_{t-1}$	0.59	0.49	1.00	0.83	0.38	1.00	0.81	0.39	1.00	0.83	0.37	1.00
SME	0.86	0.35	1.00	0.83	0.38	1.00	0.83	0.37	1.00	0.83	0.38	1.00
age	17.39	13.20	14.00	14.11	11.12	11.00	13.92	11.07	11.00	14.16	11.13	11.00
#empl.	100.55	187.85	35.70	120.75	238.70	26.00	115.06	226.28	26.70	122.28	241.95	25.95
sales/empl.	0.22	0.31	0.13	0.19	0.29	0.11	0.20	0.34	0.10	0.19	0.28	0.11
region	0.13	0.33	0.00	0.19	0.39	0.00	0.17	0.38	0.00	0.19	0.39	0.00
cfratio	.	.	.	.	.	.	.	.	.	1.12	0.71	1.00
$cfratio \mid 1[R\&D]_{t-1} = 0$	.	.	.	.	.	.	.	.	.	1.16	0.71	1.00
#Observations	18 538			3 966			840			3 126		

NOTES: Monetary values are in year 2005 euros. Observations are at firm-year level.
subsidy rate is the fraction of the R&D investment in the project reimbursed by the agency.
$R\&D_{actual}$ is the realized R&D investment in the project. $1[R\&D]_t$ takes value 1 if the firm invested in R&D in year $t$and 0 otherwise.
tech and *risk* are the technological challenge and commerrcial *risk* of the project as evaluated by the agency, on an 1-3 Likert scale.
#Observations for *tech* and *risk* : 369 and 367 unsuccessful applicants; 1 634 and 1634 successful applicants.
prev applicant takes value 1 if the firm applied for a subsidy in year $t-1$ and 0 otherwise.
$1[R\&D]_{t-1}$ takes value 1 if the firm invested in R&D in year $t-1$and 0 otherwise.
SME takes value 1 if the firm in year t is an SME according to the EU guidelines and 0 otherwise. *age* is the age of the firm in year t in years.
sales/employee is in 100 000 euros. *region* takes value 1 if the firm is located in a region eligible for EU regional aid and 0 otherwise.
cfratio is the available cashflow for the project divided by planned R&D investments. #Observations for *cfratio* :1 952 (1 620 for which $1[R\&D]_{t-1}$
The difference between the sample averages of *cfratio* and $cfratio \mid 1[R\&D]_{t-1} = 0$ is significant at 1% level.
All differences between the sample averages of non-applicants and applicants are significant at 1% level.
The differences in the sample averages of $1[R\&D]_t$ and *prev applicant* between successful and rejected applicants are significant.

Table 2. Coefficient estimates

	R&D investment	R&D participation	subsidy rate	application
ln age	-0.5300**	-0.4224	-0.0076	-0.2652
	(0.2621)	(0.3135)	(0.0287)	(0.2968)
ln $age2$	0.0833*	0.0739	0.0024	0.0715
	(0.04923)	(0.0584)	(0.0058)	(0.0558)
ln emp	0.0536	0.1945***	-0.0082	0.0174
	(0.0569)	(0.0660)	(0.0075)	(0.0613)
ln $emp2$	0.0364***	0.0171*	-0.0015	0.0277***
	(0.0076)	(0.0091)	(0.0012)	(0.0083)
$sales/emp$	2.3152***	2.0867***	-0.1414***	2.7154***
	(0.4382)	(0.4982)	(0.0371)	(0.4763)
$sales/emp2$	-1.1956***	-1.0176***	0.1006***	-1.3645***
	(0.2621)	(0.3001)	(0.0239)	(0.2844)
$exporter$	-0.0170	-1.3884***	0.0084	-0.2366*
	(0.0871)	(0.1000)	(0.0077)	(0.1344)
$region$	-0.2710***	-0.9290***	-0.0045	-0.5353***
	(0.0838)	(0.0990)	(0.0088)	(0.1068)
RD_{t-1}	0.4204***	-2.6989***	0.0156*	0.0217
	(0.1107)	(0.1245)	(0.0088)	(0.4329)
$Mills$	-0.5214***	-	-	-
	(0.1768)			
#Obs.	2 289	22 504	1 123	22 504

NOTES: Standard errors (in parentheses) are bootstrapped (399 rounds) .*** p<0.01, ** p<0.05, * p<0.1.

Table 2. Coefficient estimates

	R&D investment	R&D participation	subsidy rate	application
SME	-	-	-0.0045	
			(0.0125)	
$risk$	-	-	0.0104***	
			(0.0037)	
$tech$	-	-	0.0062	
			(0.0045)	
$prev\ applicant$	-	-	-	-0.3271***
				(0.0488)
$\ln cashflowgap$	0.9540***	-	-	-
	(0.2129)			
σ_ε	0.4541***	-	-	-
	(0.0239)			
σ_η	-	-	0.0981***	-
			(0.002)	
ξ	-	-	-	0.9659
				(1.067)
#Obs.	2 289	22 504	1 123	22 504
Year dummies	YES	YES	YES	YES
Industry dummies	YES	YES	YES	YES

NOTES: Standard errors (in parentheses) are bootstrapped (399 rounds). *** p<0.01, ** p<0.05, * p<0.1

Table 3. Fixed cost of R&D and cost of subsidy application

	mean	s.d.	p10	p25	median	p75
Fixed cost	1 204 784	5 027 150	16 115	32 967	104 704	685 460
Application cost	111 791	57 266	1 823	71 233	100 204	138 530

NOTES: The cost figures are from the counterfactual simulations.
Percentiles are calculated over firm averages.

Table 4. R&D participation

Regime	mean	median	ratio
Benchmark regimes			
Laissez-faire	0.62	0.77	1.00
1^{st} best	0.63	0.78	1.02
2^{nd} best	0.62	0.77	1.00
No financial market imperfections	0.62	0.77	1.00
Policies of interest			
Tax credits	0.63	0.77	1.02
Subsidies	0.62	0.77	1.00

NOTES: The figures are calculated over all simulation rounds and firms.
ratio = the mean for the regime in question divided by the laissez-faire mean.

Table 5. R&D investment

Simulation rounds conditional on $R > 0$				All simulation rounds		
Regime	mean	median	ratio	mean	median	ratio
Benchmark regimes						
Laissez-faire	196 558	108 138	1.00	101 408	55 502	1.00
1^{st} best	475 656	265 085	2.42	234 547	146 044	2.31
2^{nd} best	464 407	267 730	2.36	230 597	142 983	2.27
No financial market imperfections	196 574	108 150	1.00	101 418	55 509	1.00
Policies of interest						
Tax credits	289 381	159 588	1.47	151 072	82 963	1.49
Subsidies	253 481	122 356	1.29	127 075	64 656	1.25
$s\|s > 0$	484 652	194 497	2.47			

NOTES: The figures are calculated over over simulation rounds and firms with $R > 0$ (left panel) or all simulation rounds and firms (right panel).
ratio = the mean for the regime in question divided by the laissez-faire mean.
$s|s > 0$ shows the average R&D investment from the subsidy regime conditional on a firm receiving a subsidy.

Table 6. Profit, spillovers and welfare

	Profit			Spillovers			Welfare		
Regime	mean	median	ratio	mean	median	ratio	mean	median	ratio
Benchmark regimes									
Laissez-faire	1 170 343	559 085	1.00	55 760	33643	1.00	1 226 103	593 595	1.00
1^{st} best	1 115 867	517 719	0.95	137 743	91619	2.47	1 253 610	611 885	1.02
2^{nd} best	1 118 462	519 485	0.96	134 530	89760	2.41	1 252 992	611 257	1.02
No financial market imperfections	1 170 365	559 101	1.00	55 766	33648	1.00	1 226 131	593 643	1.00
Policies of interest									
Tax credits	1 212 153	582 458	1.04	83 089	50259	1.49	1 233 604	599 178	1.01
Subsidies	1 178 357	561 307	1.01	71 287	39396	1.28	1 217 671	590 015	0.99

NOTES: The figures are calculated over all simulation rounds and firms.
ratio = the mean for the regime in question divided by the laissez-faire mean.

Table 7. Counterfactual estimates

variable	mean
$\Pr[apply]$	0.15
$subsidy\ rate \mid s > 0$	0.39
τ_R	0.34
$\tilde{\tau}_R = \tau_R(1 - \tau)$	0.25
Government cost, $s \mid s > 0\ \&\ R\&D > 0$	59 410
Government cost, $\tau_R \mid R\&D > 0$	98 389
Government cost, s	26 644
Government cost, τ_R	51 365

NOTES: The figures are calculated over all simulation rounds and firms unless stated otherwise.
$\Pr[apply]$ is the average probability to apply for a subsidy. $subsidy\ rate \mid s > 0$ is the average subsidy rate conditional on it being strictly positive. τ_R is the optimal tax credit.
Government cost $s \mid s > 0\ \&\ R\&D > 0$ is the average cost to the government from those projects it subsidizes in euros.
Government cost $\tau_R \mid R\&D > 0$ is the average cost to the government from those projects that claim tax credits in euros.
Government cost s and government cost, τ_R is the average cost of subsidies and tax credits, respectively, in euros.

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Appendix: For online publication

Appendix A: Figures

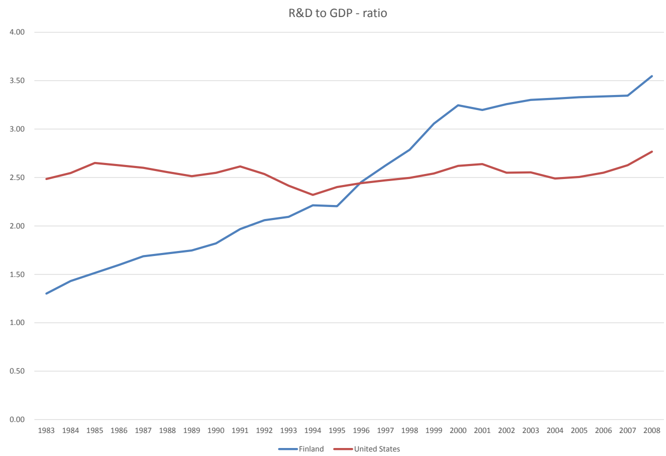
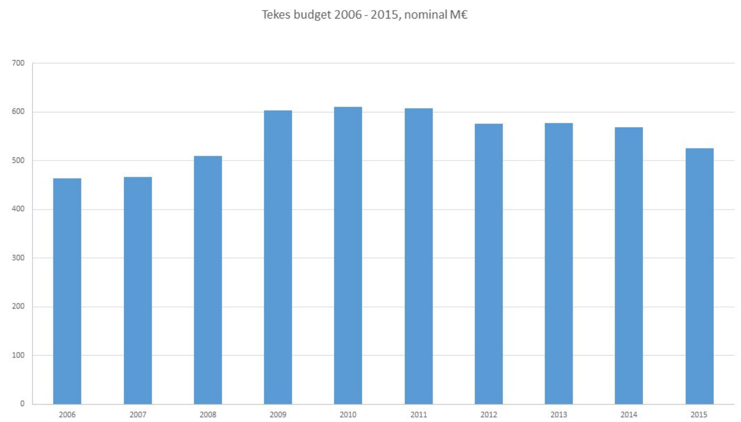


Figure A1. R&D/GDP-ratio, Finland and the US. Source: OECD Main Science and Technology Indicators.



Source: https://www.tekes.fi/globalassets/global/tekes/.../tekesin_organisaatio.pptx

Figure A2. Tekes budget 2006 - 2015.

Appendix B: Descriptive statistics and estimation details

Estimation sample

We first drop those observations where sales are negative (7 observations). We then exclude those firms for which we fail to observe age at any point (17 241 obs.): In case employment is observed in adjacent years but not in the year in question, we substitute primarily the employment level in the previous, and secondarily the employment level in the following year. We exclude from the estimations outliers as follows: We first exclude all observations in the top 1% of the size (#employees) distribution (265 obs.); second, we drop any remaining observations in the top 1% of the age distribution (223 obs.); third, we drop those observations in the top 1% of the sales/employee-ratio distribution (179 obs.); fourth, we drop those remaining firms whose mean employment is above the 99^{th} percentile (22 obs.); the same regarding age (145 obs.); and the same regarding sales/employee (183 obs.). Finally, we drop all those remaining 2 597 firm-year observations for which we observe no R&D expenditures; these observations come from firms not included in the R&D survey of Statistics Finland.

According to the Statistics Finland www-site,[16] statistics on research and development are based on the European Union's Regulations (Decision No 1608/2003/EC of the European Parliament and of the Council and Commission Implementing Regulation No 995/2012). The inquiry includes enterprises in different fields having reported R&D activities in the previous inquiry, enterprises having received product development funding from the Finnish Funding Agency for Technology and Innovation Tekes and the Finnish Innovation Fund Sitra, and all enterprises with more than 100 employees and a sample of enterprises with 10 to 99 employees. We experimented with using weights that correct for the sampling frame. As these had no material impact on the estimations but increased the computation time significantly, we do not use weights in the reported estimations.

Number of observations per firm

Table B1 shows the distribution of the number of observations per firm in our estimation sample.

[16] See http://tilastokeskus.fi/keruu/yrtk/index_en.html, accessed June 17, 2017.

Table B1. Distribution of #obs / firm

#obs	#firm-year obs.	%	cum. %
1	1 143	5.08	
2	2 564	11.30	16.47
3	3 048	13.54	30.02
4	2 896	12.87	42.89
5	2 985	13.26	56.15
6	2 256	10.02	66.17
7	2 009	8.93	75.10
8	2 120	9.42	84.52
9	3 483	15.48	100
Total	22 504		

Descriptive statistics on number of applications

Table B2 reports the distribution of the number of applications by firm across our estimation sample. Table B3 shows the distribution of the number of applications in a given year.

Table B2. Distribution of #applications / firm

#applications	#firms	%	cum. %
0	3 979	65.48	
1	1 142	18.79	8427
2	493	8.11	92.38
3	224	3.69	96.07
4	123	2.02	98.09
5	65	1.07	99.16
6	22	0.36	99.52
7	17	0.28	99.80
>7	12	0.19	100
Total #firms	6 077	100	

Table B3. Distribution of #applications/ year

year	#applications
2000	454
2001	455
2002	413
2003	432
2004	472
2005	453
2006	445
2007	416
2008	426
Total # applications	3 966

Flow of estimations

We have compiled the different estimation equations into Table B4 in the order they are estimated. We delineate the estimation process below in more detail. The first equation to be estimated is a probit model where the dependent variable takes value 1 if we observe the cashflow prediction of firm i in year t and is 0 otherwise. This equation is used to generate a Mills ratio to project (in estimation equation number 2) the (log) cashflow of firm i in year t onto firm characteristics. These estimations are needed to generate a predicted cashflow prediction for those firm -year observations for which we do not observe it (mostly firms that did not apply for a subsidy in a given year). The third estimation equation is again a probit which we use to generate a Mills ratio for the fourth and fifth estimation equations, i.e., ordered probit - grading equations where the dependent variables are the *tech* and *risk* grades that a project of firm i in year t achieved when Tekes evaluated it. The dependent variable for the probit generating this Mills ratio takes value 1 if firm i in year t applies for a subsidy and is zero otherwise.

The same Mills ratio (from equation 3) is used to correct for sample selection bias in first structural estimation where the dependent variable is the log of actual R&D investment of firm i in year t (estimation equation 6). The remaining equations do not need a sample selection correction. Estimation equation number 7 has as its dependent variable a dummy taking value 1 if firm i invests in R&D in year t and value 0 otherwise. Estimation equation 8 is the agency's decision rule where the dependent variable is the subsidy rate. The final estimation equation is the firm's application decision: The dependent variable takes value 1 if firm i applies for a subsidy in year t and value 0 otherwise.

Finally, we scale the estimates to match the predicted mean R&D investment with the realized mean (for the firm-year observations for which the R&D investment is observed)

Table B3. Estimation equations

Estimation eqn. number	Dependent variable	Econometric model	Sample selection correction	Use
Auxiliary equations				
1	1[observe cashflow]	probit	NO	generate Mills ratio for cash flow estimations
2	lncashflow	OLS	eqn 1	generate predicted cashflow for those observations where it is missing
3	1[apply]	probit, separately for SMEs and non-SMES	NO	generate Mills ratio for *risk* , *tech* and lnR&D estimation
4	*risk*	ordered probit	eqn 3	predicted grades for agency decision rule
5	Tech	ordered probit	eqn 3	predicted grades for agency decision rule
Structural equations				
6	lnR&D	structural OLS	eqn 3	parameters of marginal profitability of R&D
7	1[R&D]	structural probit	NO	parameters of fixed cost of investment
8	subsidy rate	two-limit Tobit	NO	parameters of gov.-specific utility
9	1[apply]	SML structural probit	NO	parameters of the cost-of-applying function

NOTES: 1[x] indicates a dummy variable taking value 1 if x observed and 0 otherwise.

Estimating the cashflow for the project

We use the information submitted by the applicants on their cashflow. We estimate a sample selection model in which the first stage dependent variable is a dummy taking value one for those observations for which we observe the cashflow. The second stage dependent variable is the log of the reported cash flow. The explanatory variables are the same as in the main equations. The exclusion restriction is having applied earlier; we know from TTT (2013a) that past application behavior is highly correlated with current application behavior and hence also with observing the cashflow. The identifying assumption is that past application behavior is not correlated with the cashflow firms report to be pledgeable for the project. Using the results from this regression we predict the log cashflow for those firms for which we do not observe it, correcting for the sample selection bias. We assume that the errors in these equations are normally distributed, possibly correlated with each other, and that the second stage error is uncorrelated with the shocks in the structural model (ε_{it}, ζ_{it}, η_{it}, μ_{0it}). We present the results of the above probit in the first column and those of the log cashflow equation in column two of Table B5.

Agency's grading and grading equations

Upon receiving an application the agency grades it in two dimensions, technological challenge and commercial *risk* , by using a 5-point Likert scale. The agency has six grades but uses only five of them in practice. A loose translation of the six grades of technological challenge is 0 = "no technological challenge", 1 = "technological novelty only for the applicant", 2 = "technological novelty for the network or the region", 3 = "national state-of-the-art", 4 = "demanding international level", and 5 = "international state-of-the-art". For commercial risk, it is 0 = "no identifiable risk", 1 = "small risk", 2 = "considerable risk", 3 = "big risk", 4 = "very big risk", and 5 = "unbearable risk". As explained in the main text, we group some grades as follows: Grades 0 and 1 on the one hand, and grades 3, 4 and 5 on the other hand. Table B5 displays the original and the augmented grades' distribution.

Building on the process described in TTT (2013a) – see in particular equation (9) – we estimate the two grading rules by using ordered probits. In contrast to TTT (2013a), we correct for sample selection in these estimations. The first stage dependent variable is a dummy variable taking value one if we observe the grading outcome in question. The second stage dependent variables are the grades. The first and second stage explanatory variables are the same as in the cashflow estimation. We assume that the unobservables of the two grading equations are normally distributed and uncorrelated with each other, and with the four unobservables (ε_{it}, ζ_{it}, η_{it}, μ_{0it}) of the main equations. This estimation provides us with two vectors of parameters that are used to generate a firm's prediction on how the agency would grade its application in the two grading dimensions, if the firm applied for a subsidy. Estimation is by maximum likelihood. The results are presented in Table B6. We use the thus generated probabilities for calculating the expected discounted profits from applying for a subsidy (see below for more detail).

Table B5. Distribution of agency grades, %

	tech		*risk*	
grade	original	augmented	original	augmented
0	0.86		0.81	
1	30.52	31.38	20.42	21.22
2	32.29	32.29	26.89	26.89
3	35.11	36.33	49	49
4	1.22		2.85	2.89
5			0.04	
#Obs.	2 546		2 596	

NOTES: The numbers in the "original" and "augmented" columns are % of observations.

The results presented in Table B6 (Table B6 is split into two panels for space reasons) are: Those from the probit regression where the dependent variable is a dummy taking value one if we observe the cashflow available for the R&D project of the firm (column 1); the log cashflow equation (column 2); the probit models for the sample selection for non-SMEs (column 3) and SMEs (column 4) which are used to generate the Mills' ratio for the Tekes grades technological challenge (column 5) and commercial *risk* (column 6), as well as the structural equations presented in Table 2.

Table B6, panel A. Cashflow and Tekes grading rule estimation: First stages of the sample selection models

	(1)	(2)	(3)	(4)	(5)	(6)
	$1[observe\ cashflow]$	$\ln cashflow$	application non-SME	application SME	$tech$	$risk$
$\ln age$	-0.1471	0.0885	-0.2035*	-0.6725***	0.0386	-0.2089
	(0.1077)	(0.1542)	(0.0941)	(0.1956)	(0.1923)	(0.1942)
$\ln age2$	0.0006	-0.0113	0.0032	0.1045***	-0.0011	0.0287
	(0.0205)	(0.0284)	(0.0184)	(0.0356)	(0.0374)	(0.0368)
$\ln emp$	0.0504	0.0934***	0.1391***	-0.2163***	0.0736	-0.0730
	(0.0206)	(0.0311)	(0.0202)	(0.0608)	(0.0435)	(0.0492)
$\ln emp2$	-0.0042	0.0097**	-0.0257***	0.0385***	-0.0020	0.0000
	(0.0031)	(0.0041)	(0.0037)	(0.0067)	(0.0058)	(0.0066)
$sales/emp$	-0.1201	1.5145***	-1.2927***	0.0547	-0.2744	-1.6070***
	(0.1531)	(0.2158)	(0.1536)	(0.2388)	(0.3008)	(0.3059)
$sales/emp2$	-0.0281	-0.9066***	0.7290***	0.1069	0.2262	0.9942***
	(0.0876)	(0.1356)	(0.0888)	(0.1318)	(0.1728)	(0.1729)
$exporter$	0.2849	-0.0447	0.2720***	0.2384***	0.1115	-0.0437
	(0.0300)	(0.0438)	(0.0249)	(0.0635)	(0.0646)	(0.0610)

Table B6, panel B. Cashflow and Tekes grading rule estimation: first stages of the sample selection models

	(1)	(2)	(3)	(4)	(5)	(6)
	$1[observe\ cashflow]$	$\ln cashflow$	application non-SME	application SME	tech	$risk$
$region$	0.1881	-0.0704	0.2513***	0.1423*	0.0730	0.0017
	(0.0328)	(0.0395)	(0.0273)	(0.0751)	(0.0646)	(0.0656)
RD_{t-1}	0.3876	0.0401	0.4205***	0.4962***	0.2669***	0.0640
	(0.0288)	(0.0447)	(0.0229)	(0.0728)	(0.0804)	(0.0830)
$prev\ applicant$	0.3226	-	0.2851***	0.4086***	-	-
	(0.0332)		(0.0303)	(0.0604)		
$Mills$	-	-1.3620*	-	-	0.1017	-0.0579
		(0.6096)			(0.1355)	(0.1394)
$Constant$	-1.6470	0.0284	-1.0192	-1.2140	-	-
	(0.1355)	(0.5442)	(0.1207)	(0.3397)		
Observations	22 504	1 952	19 232	3 272	2 003	2 001
Year dummies	YES	YES	YES	YES	YES	YES
Industry dummies	YES	YES	YES	YES	YES	YES

NOTES: Bootstrapped standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Simulation for estimation

We use the simulation estimator for discrete choice introduced by McFadden (1989) – see also Stern (1997). We simulate the profitability shock of the firm (ε_{it}) both for the R&D participation and the subsidy application decisions. We use 40 simulation rounds and draw the shocks using Halton sequences. The draws are the same for all estimation equations.

Expected profits from applying for subsidies

To estimate the firm's application decision, we need to deal with both agency grading and the stochastic component of agency utility, η_{it}, which are unknown to the firm contemplating application. We assume that the firm knows the probabilities of obtaining particular grades for *tech* and *risk*, and the distribution of η_{it}. We therefore calculate for each firm and each simulation draw the expected discounted profits from obtaining a particular grade combination, integrating over the distribution of η_{it}. These profits are then weighted by the probability of getting a particular grade combination; we obtain these probabilities from the ancillary (ordered probit) grading equations. For numerical integration we use Simpson's method. The integration is repeated separately for each simulation round and each iteration.

Bootstrap

We bootstrap the whole estimation process and the generation of the optimal tax credit. We use 399 bootstrap rounds. To speed up computation, we limit the number of Newton-Raphson iterations to 5 for the R&D investment, R&D participation and application equations, while using the estimated coefficients as starting values. We restrict the number of iterations to 150 for the agency decision rule. We further restrict the number of simulation rounds for the calculation of the optimal tax credit to 50 (100 in the estimation), and restrict the support of the grid search to be [20,50] (in the estimation [0,100]). The grid step is kept at 1 (percentage point). For the calculation of the optimal tax credit, we restrict the number of simulation rounds to 50 (we use 100 rounds in the estimation).

Appendix C: Details and proofs of the theoretical model

For $F \in [0,\infty)$ we denote by $\Gamma(F)$ the dynamic game among the agency, a firm and an investor. Let us first write the firm' expected payoff fully as

$$\tilde{\Pi}^E(R,\pi^I,d_f,d_m,d_G) = \begin{cases} \Pi^E(R,\pi^I)_+ & \text{if } d_f = d_G = 1 \\ \Pi^E_B(R) & \text{if } d_f = 1 \text{ and } d_m = d_G = 0 \\ 0 & \text{if } d_f = 0. \end{cases} \tag{A1}$$

in which $\Pi^E(R,\pi^I)$ is the firm's expected payoff from the good project of equation (3), and $\Pi^j_B(R)$ denotes the firm's expected payoff from the bad project (as will be specified below in Assumption A2). As a reminder, the subscripts *f, m, G* refer to financing, monitoring and project choice, respectively. Similarly, let us write the investor's expected payoff as

$$\tilde{\Pi}^I(F,s,R,\pi^I,d_f,d_m,d_G) = \begin{cases} \Pi^I(F,s,R,\pi^I,d_m,d_G) & \text{if } d_f = 1 \\ 0 & \text{if } d_f = 0, \end{cases} \tag{A2}$$

in which $\Pi^I(F,s,R,\pi^I) := \Pi^I(F,s,R,\pi^I,1,1)$ is given by equation (2).

To shorten notation we also define the firm's and the agency's, respectively, expected payoffs to an equilibrium financing contract $(R^*(F,s),\pi^{I*}(F,s))$ as

$$\begin{aligned}\tilde{\Pi}^{E*}(F,s) &:= \tilde{\Pi}^E(R^*(F,s),\pi^{I*}(F,s),\\ &\quad d^*_k(F,s,R^*(F,s),\pi^{I*}(F,s)), d^*_G(R^*(F,s),\pi^{I*}(F,s),d^*_k(F,s,R^*(F,s),\pi^{I*}(F,s)))),\\ &\qquad\qquad k = f,m,\end{aligned}$$

and

$$\begin{aligned}U^*(F,v,s) &:= U(F,v,s,R^*(F,s),\pi^{I*}(F,s),\\ &\quad d^*_k(F,s,R^*(F,s),\pi^{I*}(F,s),d^*_G(R^*(F,s),\pi^{I*}(F,s),d^*_k(F,s,R^*(F,s),\pi^{I*}(F,s)))),\\ &\qquad\qquad k = f,m,\end{aligned}$$

in which we recall the firm's project choice $d_G(R,\pi^I,d_f,d_m)$ as a function of its R&D investment and repayment promise, and the investor's financing and monitoring choice,s respectively.

Definition A1. A profile

$$(d^*_a(F), s^*(F,\cdot), R^*(F,\cdot), \pi^{I*}(F,\cdot), d^*_f(F,\cdot), d^*_m(F,\cdot), d^*_G(F,\cdot))$$

is a pure-strategy perfect Bayesian equilibrium of $\Gamma(F)$ if it satisfies;

(i) For all $(R,\pi^I) \in [0,\infty)^2$ and for $d_f = 1$, $d^*_G(R,\pi^I,1,1) = 1$, and for $d_m = 0$, $\Pi^E(R,\pi^I)_+ > \Pi^E_B(R)$ implies $d^*_G(R,\pi^I,1,0) = 1$ and $\Pi^E(R,\pi^I)_+ < \Pi^E_B(R)$ implies

$d_G^*(R, \pi^I, 1, 0) = 0$;

(ii) For $\left(s, R, \pi^I, d_m\right) \in [0, \bar{s}] \times [0, \infty)^2 \times \{0, 1\}$ and for $k = f, m$, $\Pi^I(F, s, R, \pi^I, d_m, d_G^*(R, \pi^I, d_k(F, s, R, \pi^I))) > 0$ implies $d_f^* \left(F, s, R, \pi^I\right) = 1$, and $\Pi^I(F, \cdot) < 0$ implies $d_f^* \left(F, s, R, \pi^I\right) = 0$. For $\left(s, R, \pi^I\right) \in [0, \bar{s}] \times [0, \infty)^2$, $\Pi^I(F, s, R, \pi^I, 1, d_G^*(R, \pi^I, d_f(F, s, R, \pi^I), 1))_+ > \Pi^I(F, s, R, \pi^I, 0, d_G^*(R, \pi^I, d_f(F, s, R, \pi^I), 0))$ implies $d_m^* \left(F, s, R, \pi^I\right) = 1$ and, $\Pi^I(F, \cdot, 1, d_G^*(\cdot, 1)) < \Pi^I(F, \cdot, 0, d_G^*(\cdot, 0))$ implies $d_m^* \left(F, s, R, \pi^I\right) = 0$.

(iii) For all $s \in [0, \bar{s}]$,

$$\left(R^*(F, s), \pi^{I*}(s)\right) \in \underset{(R,\pi^I)\in[0,\infty)^2}{\arg\max} \tilde{\Pi}^E \left(F, R, \pi^I, d_k^*(F, s, R, \pi^I), d_G^*(R, \pi^I, d_k^*(F, s, R, \pi^I))\right), k = f, m;$$

(iv) For $d_a = 1$,

$$s^*(F, v) := s^*(F, v, 1) \in \underset{s\in[0,\bar{s}]}{\arg\max}\, U^*(F, v, s)_+ .$$

For $d_a = 0$, $s^*(F, v, 0) = 0$;

(v)

$$d_a^*(F) \in \underset{d_a\in\{0,1\}}{\arg\max}\, d_a \left[\int_{-\infty}^{\infty} \tilde{\Pi}^{E*}(F, s^*(F, v))\, \phi(v)\, dv - K\right] + (1 - d_a)\tilde{\Pi}^*(F, 0). \quad \text{(A3)}$$

Condition (i) warrants that the firm's project choice is rational if the investor does not monitor; the firm chooses the project that yields a higher expected payoff. If the investor monitors, the bad project is eliminated from the firm's action set and the firm must choose the good project. Condition (ii) warrants that the investor's behavior is rational anticipating the firm's project choice; the investor finances the project only if financing yields a positive expected payoff, monitors only if monitoring yields a positive expected payoff and is more profitable than no-monitoring, and neither finances nor monitors otherwise. Condition (iii) warrants that the firm's R&D investment and repayment promise maximize its expected payoff anticipating both its own project choice and the investor's behavior.

Condition (iv) warrants that, conditional on receiving a subsidy application, the agency's subsidy rate choice maximizes its expected payoff anticipating the firm's investment, repayment and project choices, and the investor's behavior. As mentioned in the main text, if the agency receives no application, it cannot give a subsidy. Condition (v) warrants that the firm's subsidy application decision maximizes its expected payoff anticipating the agency's, investor's and its own behaviors in the subsequent stages.

As is customary, we assume tie-breaking rules in favor of equilibrium. Some of these rules are institutionalized. For example, Tekes's internal funding rules prohibit awarding subsidizes if Tekes's "funding would have no effect on the realisation of the project" or if "the project has only a small impact on the company's business". These rules break the agency's indifference to whether or not to award a subsidy in favor of equilibrium, e.g, when the agency knows that no project would be implemented even if it awarded a subsidy.

Let us next formalize the mild restrictions on the project return functions and the form of financing contract, discussed in the main text.

Assumption A1. $\ln(\alpha/(r+c)) \geq 1$;

Assumption A2. $\Pi_B^E(R) = bR$ in which $b \in [\alpha, \infty)$ is non-verifiable;

Assumption A3. The firm offers to the investor a debt contract that pays $\min\left\{\pi^I, \pi(R)\right\}_+$ in the case of success.

Assumption A1 ensures that the productivity of the good project is sufficiently high so make the firm's R&D investment profitable if the fixed costs of R&D are ignored. While models of R&D typically must invoke similar assumptions to make the models meaningful, we will also, after Proposition A1 at the end of Appendix C, characterize equilibria when Assumption A1 is relaxed. Assumption A2 means that the bad project yields non-verifiable return $b \in [\alpha, \infty)$ per unit of investment.

As discussed in the main text, the financing contract of our model accommodates both debt and equity interpretations. For brevity, we only consider the standard debt contract interpretation formally as stipulated by Assumption A3. Here the min operator captures the seniority of the investor's claims if the firm cannot honor its promise. As a result, we rewrite the investor's and firm's expected payoffs of equations (2) and (3) as

$$\Pi^I(F, s, R, \pi^I) = (1-\tau)\left[P\min\{\pi^I, \pi(R)\}_+ - (r+c)R + F) + sR\right]1 \tag{A4}$$

and

$$\Pi^E(R, \pi^I) = (1-\tau)P(\pi(R) - \min\{\pi^I, \pi(R)\}_+). \tag{A5}$$

Using a series of lemmas, we show that the game $\Gamma(F)$ has a unique equilibrium on $[0, \infty)$.

Lemma A1. Let $(R, \pi^I) \in [0, \infty)^2$ and $d_f = 1$. Then

$$d_G^*(R, \pi^I, 1, d_m) = \begin{cases} 1 & \text{if } d_m = 1 \\ 0 & \text{if } d_m = 0. \end{cases}$$

Proof. Consider the subgame in which the investor provides funding ($d_f = 1$) for some project $R > 0$ but does not monitor ($d_m = 0$). Then, for all $(R, \pi^I) \in [0, \infty)^2$, $\Pi_B^E(R) = bR$, whereas $\Pi^E(R, \pi^I)$ is at most $(1-\tau)P\pi(R)$ (see equation (A5)). Therefore, the firm chooses the bad project if $bR \geq (1-\tau)P\pi(R) = (1-\tau)\alpha \ln R$ in which the last step uses equation (1). Since $R - \ln R > 0$ for all $R \in [0, \infty)$, a sufficient condition for choosing the bad project is $b \geq (1-\tau)\alpha$, which holds by Assumption A2. If the investor monitors ($d_m = 1$), the bad project is eliminated from the firm's action set by assumption. ■

Lemma A2 identifies the investor's equilibrium behavior and marginal costs of funds.

Lemma A2. Let $(F, s, R) \in [0, \bar{s}] \times [0, \infty)^2$.

(i) For all $\pi^I \in [0, \infty)$, $d_f^* \left(F, s, R, \pi^I\right) = 1$ only if $d_m = 1$. Otherwise, $d_f^* \left(F, s, R, \pi^I\right) = 0$. Therefore, if $d_f^* \left(F, s, R, \pi^I\right) = 1$, $d_m^* \left(F, s, R, \pi^I\right) = 1$, and the investor's marginal cost cost of funds is $r + c$.

(ii) If $\pi(R) \geq [(r + c)(R + F) - sR]/P > 0$, then $d_f^* \left(F, s, R, \pi^I\right) = 0$ for $\pi^I < [(r + c)(R + F) - sR]/P$, and $d_f^* \left(F, s, R, \pi^I\right) = 1$ for $\pi^I \geq [(r + c)(R + F) - sR/P$. Otherwise, $d_f^* \left(F, s, R, \pi^I\right) = 0$ for all $\pi^I \in [0, \infty)$.

Proof. Recall first from equation (A2) that for all $\pi^I \in [0, \infty)$, the investor's payoff to $d_f = 0$ is 0 and note that the investor's (non-subsidized) marginal cost of funds may be written as $rd_f + cd_m$.

(i) Lemma A1 implies that, for all $\pi^I \in [0, \infty)$, $d_G^*(R, \pi^I, 1, 0) = 0$. Hence, if $d_f = 1$ but $d_m = 0$, the investor's payoff to funding the project is $\Pi^I(F, s, R, \pi^I, 0, 0) = (1 - \tau)[-r(R + F) + sR] < 0$ in which the inequality follows from $r \geq 1 > \bar{s} \geq s$. Therefore, in equilibrium either $d_f^* \left(F, s, R, \pi^I\right) = 0$ or if $d_f^* \left(F, s, R, \pi^I\right) = 1$, then $d_m^* \left(F, s, R, \pi^I\right) = 1$. If $d_k^* \left(F, s, R, \pi^I\right) = 1, k = f, m$, the investor's marginal cost funds is given by $r + c$.

(ii) Assume $d_m = 1$ which, by Lemma A1 implies that $d_G = 1$. Assume also that $\pi(R) \geq [(r + c)(R + F) - sR]/P$ in which $[(r + c)(R + F) - sR]/P > 0$ because $r + c \geq 1 > \bar{s} \geq s$. Let $\pi_1^I, \pi_2^I \in [0, \infty)$ be such that $\pi_1^I < [(r + c)(R + F) - sR]/P \leq \pi_2^I$. Then $\min\left\{\pi_1^I, \pi(R)\right\} = \pi_1^I$ and therefore

$$\Pi^I(F, s, R, \pi_1^I) = (1 - \tau)\left[P\pi_1^I - (r + c)(R + F) + sR\right] < 0,$$

and $d_f^* \left(F, s, R, \pi_1^I\right) = 0$. Similarly, $\min\left\{\pi_2^I, \pi(R)\right\} \geq [(r + c)(R + F) - sR]/P$ and therefore

$$\Pi^I(F, s, R, \pi_2^I) = (1 - \tau)\left[P\min\left\{\pi_2^I, \pi(R)\right\} - (r + c)(R + F) + sR\right] \geq 0.$$

Hence, $d_f^* \left(F, s, R, \pi_2^I\right) = 1$.

Assume next that $\pi(R) < [(r + c)(R + F) - sR]/P$. Then for all $\pi^I \in [0, \infty)$, $\min\left\{\pi^I, \pi(R)\right\} < [(r + c)(R + F) - sR]/P$, and

$$\Pi^I(F, s, R, \pi^I) = (1 - \tau)\left[P\min\left\{\pi^I, \pi(R)\right\} - (r + c)(R + F) + sR\right] < 0$$

and, hence, $d_f^* \left(F, s, R, \pi^I\right) = 0$. ■

Part (i) of Lemma A2 proves that in equilibrium, either a project is funded by a monitoring investor or no project is launched. Therefore, if the project is launched, its investor's marginal cost also include monitoring costs. Part (ii) identifies the repayments that are sufficiently high to attract the investor to finance the project.

Lemma A3 identifies the repayment promises that may arise in equilibrium and shows that, in an equilibrium in which the project is launched, the firm will be able to make the promised repayment unless the project fails to pay return.

Lemma A3. For all $(F, s) \in [0, \infty) \times [0, \bar{s}]$, the equilibrium repayment is $\pi^{I*}(F, s) = [(r + c)(R^*(F, s) + F) - sR^*(F, s)]/P > 0$. Moreover, if $d_f^*\left(F, s, R^*(F, s), \pi^{I*}(F, s)\right) = 1$, then $\pi(R^*(F, s)) > \pi^{I*}(F, s)$ and $R^*(F, s) > 1$.

Proof. Assume that $\pi(R^*(F, s)) > [(r + c)(R^*(F, s) + F) - sR^*(F, s)]/P$ in which $[(r + c)(R^*(F, s) + F) - sR^*(F, s)]/P > 0$ because $r + c \geq 1 > \bar{s} \geq s$. Note from equation (1) that $\pi(R^*(F, s)) > 0$ only if $R^*(F, s) > 1$. Then, by offering a contract $\left(R^*(F, s), \pi^{I*}(F, s)\right)$ in which $\pi^{I*}(F, s) = [(r + c)(R^*(F, s) + F) - sR^*(F, s)]/P$ the firm can secure a positive expected payoff, since Lemma A2 implies that $d_k^*\left(F, s, R^*(F, s), \pi^{I*}(F, s)\right) = 1$, $k = f, m$, and then, $\Pi^E\left(R^*(F, s), \pi^{I*}(F, s)\right)$ $= P\left[\pi(R^*(F, s)) - \pi^{I*}(F, s)\right] > 0$. Higher repayment promises are strictly dominated since $\Pi^E\left(R^*(F, s), \pi^{I*}(F, s)\right) > \Pi^E\left(R^*(F, s), \pi^I\right) \geq 0$ for $\pi^I \in ([(r + c)(R^*(F, s) + F) - sR^*(F, s)]/P, \pi(R^*(F, s))$, whereas for smaller repayment promises $\pi^I < [(r + c)(R^*(F, s) + F) - sR^*(F, s)]/P$, Lemma A2 implies that $d_f^*\left(F, s, R^*(F, s), \pi^I\right) = 0$ and, hence, the firm's payoff is zero (recall equation (A1).

Moreover, if $\pi(R^*(F, s)) < [(r + c)(R^*(F, s) + F) - sR^*(F, s)]/P$, then Lemma A2 implies that for $\pi^I \in [0, \infty)$, $d_f^*\left(F, s, R^*(F, s), \pi^I\right) = 0$, and the firm's payoff is zero (equation (A1)).■

Lemma A4 identifies the firm's equilibrium R&D investment behavior.

Lemma A4. There are two values of $F \in [0, \infty)$, $\underline{F}$ and $\bar{F}$, $\underline{F} < \bar{F}$, such that for all $s \in [0, \bar{s}]$, $R^*(F, s) = R^{**}(s) > 1$ for $F \in [0, \underline{F})$ and $R^*(F, s) = 0$ for $F \in \left(\bar{F}, \infty\right)$. There is also a strictly increasing function $\tilde{s} : \left[\underline{F}, \bar{F}\right] \to [0, \bar{s}]$ such that if $s \in [0, \tilde{s}(F))$, then $R^*(F, s) = 0$ and if $s \in [\tilde{s}(F), \bar{s}]$ then $R^*(F, s) = R^{**}(s) > 1$.

Proof: Note first from equations (7) and Lemma A3 that in equilibrium either $R^*(F, s) = R^{**}(s) > 1$ or $R^*(F, s) = 0$ depending on whether $\Pi^{E**}(F, s) \geq 0$ or not. For $s = 0$, we observe from equation (8) that $\Pi^{E**}(F, 0) > 0$ when

$$F < \underline{F} := \frac{\alpha}{r + c}\left[\ln\left(\frac{\alpha}{r + c}\right) - 1\right]. \tag{A6}$$

Since equation (8) also implies that $\partial\Pi^{E**}(F, s)/\partial s > 0$ on $[0, \bar{s}]$ (recall that $r + c \geq 1 > \bar{s}$), $\Pi^{E**}(F, s) > 0$ for all $s \in [0, \bar{s}]$ if the inequality (A6) holds. Thus, $R^*(F, s) = R^{**}(s)$ for $F < \underline{F}$ and $s \in [0, \bar{s}]$.

Similarly, letting $s = \bar{s}$ in equation (8) implies that $\Pi^{E**}(F, \bar{s}) < 0$ when

$$F > \bar{F} := \frac{\alpha}{r + c}\left[\ln\left(\frac{\alpha}{r + c - \bar{s}}\right) - 1\right]. \tag{A7}$$

Since $\partial\Pi^{E**}(F, s)/\partial s > 0$ on $[0, \bar{s}]$, $\Pi^{E**}(F, s) < 0$ for all $s \in [0, \bar{s}]$ under the condition (A7). Therefore, $R^*(F, s) = 0$ for $F > \bar{F}$ and $s \in [0, \bar{s}]$. Assumption A1 and equations (A6) and (A7) imply that $0 \leq \underline{F} < \bar{F}$.

Finally, letting $\Pi^{E**}(F,s)$ from equation (8) to be equal to zero and solving the equality for s yields

$$\tilde{s}(F) = r + c - \alpha e^{-\left(\frac{\alpha+(r+c)F}{\alpha}\right)}, \tag{A8}$$

which is the subsidy rate familiar from equation (11) of the main text. Note next that $\partial\tilde{s}/\partial F > 0$, $\tilde{s}(\underline{F}) = 0$ and $\tilde{s}(\bar{F}) = \bar{s}$, and recall that $\partial\Pi^{E**}(F,s)/\partial s > 0$ on $[0,\bar{s}]$. Therefore, if $F \in \left[\underline{F},\bar{F}\right]$, $\Pi^{E**}(F,s) < 0$ and hence $R^*(F,s) = 0$ for $s \in [0,\tilde{s}(F))$, and $\Pi^{E**}(F,s) \geq 0$ and hence $R^*(F,s) = R^{**}(s)$ for $s \in [\tilde{s}(F),\bar{s}]$. ■

In more words, Lemma A4 identifies two threshold values for fixed R&D costs. If F is below the lower threshold $\underline{F}$ (if equation (A6) holds), the fixed costs are so low that the firm will invest even without a subsidy. In contrast, if F is above the higher threshold $\bar{F}$ (equation (A7) holds), the fixed costs are so high that they prevent the firm's investment even with a maximum subsidy rate $\bar{s}$. If $F \in \left[\underline{F},\bar{F}\right]$, the firm will invest only if it receives a subsidy rate that is at least as large as $\tilde{s}(F)$ as identified by equation (A8), and does not invest otherwise.

Lemma A5 identifies the agency's equilibrium behavior.

Lemma A5. Let $d_a = 1$. (i) For $F \in [0,\underline{F})$,

$$s^*(F,v) = \begin{cases} 0 & \text{if } v < \underline{v} := \rho(g-1) \\ s^{**}(v) & \text{if } v \in [\underline{v},\overline{v}] \\ \bar{s} & \text{if } v > \overline{v} := \underline{v} + \bar{s}, \end{cases}$$

in which $0 < \underline{v} < \bar{v}$;
(ii) For $F \in \left[\underline{F},\bar{F}\right]$,

$$s^*(F,v) = \begin{cases} 0 & \text{if } v < v^0(F) \\ \tilde{s}(F) & \text{if } v \in \left[v^0(F),\tilde{v}(F)\right) \\ s^{**}(v) & \text{if } v \in [\tilde{v}(F),\overline{v}(F)] \\ \bar{s} & \text{if } v > \overline{v} := \underline{v} + \bar{s}, \end{cases}$$

in which $v^0(F)$ and $\tilde{v}(F)$, with $0 \leq v^0(F) < \tilde{v}(F) \leq \overline{v}$, denote the (unique) values of v that satisfy $U^*(F,v^0,\tilde{s}(F))$=0 and $s^{**}(\tilde{v}) = \tilde{s}(F)$, respectively;

(iii) For $F \in \left(\bar{F},\infty\right)$, $s^*(F,v) = 0$ for all $v \in \mathbb{R}$.

Proof: Conditional on $d_a = 1$, the agency's problem is given in equation (9)) in which $R^*(F,s)$ is given by (7). We first solve the agency's problem by ignoring the non-negativity constrains on the firm's and agency's expected payoffs in equations (6) and (9), respectively. Equation (7) implies that in this case, $R^*(F,s)$ =$R^{**}(s) = \alpha/(r+c-s)$. Using this equation and the envelope theorem to differentiate the agency's expected payoff $U^*(F,v,s)$ from equation (9) then yields

$$\frac{dU^*\left(F,v,s\right)}{ds}=\frac{\alpha}{\left(r+c-s\right)^2}\left[v-s-(r+c)(g-1)\right]. \tag{A9}$$

Clearly, the unique interior solution, if it exists, to the problem $\max_{s\in[0,\overline{s}]}U^*(F,s,v)$ can be expressed as

$$s^{**}\left(v\right)=v-(r+c)(g-1), \tag{A10}$$

which is the subsidy rate familiar from equation (10) of the main text. (Note that $s\rightarrow \Sigma\!\!\!\!+c$ may also maximize $U^*(F,s,v)$ but it violates the feasibility constraint $s\in[0,\overline{s}]$ (as $r+c\geq 1>\overline{s}$).)

According to Lemma A4, the firm's zero-profit constraint does not bind if equation (A6) holds. Therefore, for $F\in[0,\underline{F})$,equations (A9) and (A10) imply that the optimal subsidy policy is given by $s^*(F,v)=0$ if $v<\underline{v}$ in which

$$\underline{v}:=(r+c)(g-1)>0, \tag{A11}$$

$s^*(F,v)=\overline{s}$ if $v>\bar{v}:=\underline{v}+\overline{s}$, and $s^*(F,v)=s^{**}(v)$ if $v\in\left[\underline{v},\bar{v}\right]$. The claim in part (i) of Lemma A5 follows.

Proving part (ii) of Lemma A5 involves an additional complexity since, when $F\in\left[\underline{F},\bar{F}\right]$, the firm will invest only if it receives a subsidy (see Lemma A4). This complexity matters if $s^{**}(v)<\overline{s}$ but $\Pi^{E**}\left(F,s^{**}(v)\right)<0$. In such circumstances the agency may consider the subsidy rate $\tilde{s}(F)$ identified by Lemma A4. Note that if $s^{**}(v)<\overline{s}$ and $\Pi^{E**}\left(F,s^{**}(v)\right)<0$ then $\tilde{s}(F)>s^{**}(v)$, since $\tilde{s}(F)\in[0,\overline{s}]$ and $\partial\Pi^{E**}\left(F,s\right)/\partial s>0$ on $[0,\overline{s}]$. Also, since $s^{**}(v)$ is the unique interior solution to the problem $\max_{s\in[0,\overline{s}]}U^*(F,v,s)$, awarding any higher subsidy $s'\in(\tilde{s}(F),\overline{s}]$ would imply $U^*(F,v,s')<U^*(F,v,\tilde{s}(F))$. On the other hand, awarding any lower subsidy $s'\in[0,\tilde{s}(F))$ would imply $R^*(F,s')=0$ and therefore $U^*(F,v,s')=0$ for all $s'\in[0,\tilde{s}(F))$. Thus, if $\Pi^{E**}\left(F,s^{**}(v)\right)<0$), the agency needs to decide between $\tilde{s}(F)$ and $s^*(F,v)=0$. As $R^*(F,0)=0$, and therefore $U^*(F,v,0)=0$, awarding $\tilde{s}(F)$ maybe optimal if $U^*(F,v,\tilde{s}(F))\geq U^*(F,v,0)=0$. To summarize, awarding $\tilde{s}(F)$ is optimal for the agency if $s^{**}(v)<\overline{s}$, $\Pi^{E**}\left(F,s^{**}(v)\right)<0$, and $U^*(F,v,\tilde{s}(F))\geq 0$.

Since $\Pi^{E**}\left(F,s^{**}(v)\right)<0$ if and only if $s^{**}(v)<\tilde{s}(F)$ we first characterize the circumstances in which $s^{**}(v)<\tilde{s}(F)$. Because $\tilde{s}(F)$ is independent of v but $s^{**}\left(v\right)$ is strictly increasing in v (see equations (A8) and (A10)), there exists a unique value of v, denoted by $\tilde{v}(F)$, such that $s^{**}\left(\tilde{v}(F)\right)=\tilde{s}(F)$. Equations (A8) and (A10) then yield

$$\tilde{v}(F):=g(r+c)-\alpha e^{-\left(\frac{\alpha+(r+c)F}{\alpha}\right)}. \tag{A12}$$

Because $s^{**}\left(v\right)$ is strictly increasing, $s^{**}(v)<\tilde{s}(F)$ for $v<\tilde{v}(F)$. Thus, only if $v<v\tilde{(F)}$, the agency may award subsidy $\tilde{s}(F)>s^{**}(v)$ that just satisfies the firm's zero-profit constraint $\Pi^{E**}\left(F,\tilde{s}(F)\right)=0$.

We next characterize the conditions in which the agency's participation constraint $U^*(F,v,\tilde{s}(F))\geq 0$ holds. Since both the investor's and firm's zero-profit constraints are binding at $s = \tilde{s}(F)$ by definition, we observe from equation (4) that $U^*(F, v, \tilde{s}(F)) = (v - g\tilde{s}(F))R^{**}(\tilde{s}(F))$. As a result, $U^*(F, v, \tilde{s}(F), v) \geq 0$ if $v - g\tilde{s}(F) \geq 0$. Inserting $\tilde{s}(F)$ from equation (A8) into $v - g\tilde{s}(F) \geq 0$ yields $v \geq v^0(F)$ in which

$$v^0(F) := g\left[r + c - \alpha e^{-\left(\frac{\alpha+(r+c)F}{\alpha}\right)}\right] = \tilde{v}(F) - (g-1)\,\alpha e^{-\left(\frac{\alpha+(r+c)F}{\alpha}\right)}, \tag{A13}$$

in which the latter equality uses equation (A12). Since $g > 1$, $v^0(F) < \tilde{v}(F)$. As a result, $s^*(F, v) = \tilde{s}(F)$ constitutes the optimal agency decision for $v \in \left[v^0(F), \tilde{v}(F)\right)$. If $v < v^0(F)$, the agency's and the private sector's participation constraints cannot be simultaneously satisfied for any positive subsidy rate, implying $s^*(F, v) = 0$.

Next, note from equations (A8) , (A11), and(A12) that we may write $\tilde{v}(F) = \underline{v} + \tilde{s}(F)$. Since $\tilde{s}(F) \in [0, \bar{s}]$ by Lemma A5, $\tilde{v}(F) \in [\underline{v}, \bar{v}]$ (recall that $\bar{v} := \underline{v} + \bar{s}$). Therefore, we can summarize the agency's optimal decision rule for $F \in \left[\underline{F}, \bar{F}\right]$ as follows: $s^*(F, v) = 0$ for $v < v^0(F)$, $s^*(F, v) = \tilde{s}(F)$ for $v \in \left[v^0(F), \tilde{v}(F)\right)$, $s^*(F, v) = s^{**}(v)$ for $v \in [\tilde{v}(F), \bar{v}]$, and $s^*(F, v) = \bar{s}$ for $v > \bar{v}$. Note also from equations (A8) and (A13) that we may write $v^0(F) = g\tilde{s}(F)$. Since $g > 1$and $\tilde{s}(F) \in [0, \bar{s}]$ by Lemma A5, $v^0(F) \geq 0$.

To prove part (iii), note that if equation (A7) holds, Lemma A4 implies that the firm makes no investments even with a maximum subsidy rate $\bar{s}$. Thus, $R^*(F, s) = 0$, and $U^*(F, s, v) = 0$ for $(F, s, v) \in (\bar{F}, \infty) \times [0, \bar{s}] \times \mathbb{R}$, implying $s^*(F, v) = 0$ for $(F, v) \in \left[\bar{F}, \infty\right) \times \mathbb{R}$. ■

According to Lemma A5, if $F < \underline{F}$, the fixed R&D costs are so small that they affect neither the private sector's nor the agency's decisions. In contrast, if $F > \bar{F}$,the fixed costs are so high that the firm would not invest even if it received the maximum subsidy $\bar{s}$. Therefore, the agency awards no subsidy for such a firm. If $F \in \left[\underline{F}, \bar{F}\right]$, the firm will be able to invest only if it receives a subsidy. Now awarding $\tilde{s}(F)$ of equation (A8) is an option to the agency. Awarding $\tilde{s}(F)$ is optimal for the agency for "intermediate" spillover evaluations, which are not so high to make the unconstrained rate optimal for the agency but are high enough to satisfy the agency's participation constraint.

Lemma A5 also proves that $0 \leq \min\left\{v^0(F), \underline{v}\right\}$ and $\max\left\{v^0(F), \underline{v}\right\} \leq \tilde{v}(F) \leq \bar{v}$, implying that a necessary condition for the firm to obtain a subsidy is that a realization of V for its project is positive. However, $v^0(F)$ and $\underline{v}$ cannot be unambiguously ranked. From equations (A11) and (A13) we obtain the following result:

Remark A1. $\underline{v} \lesseqqgtr v^0(F)$ if and only if $g \lesseqqgtr \frac{(r+c)}{\alpha} e^{\left(\frac{\alpha+(r+c)F}{\alpha}\right)}$.

Since $g > 1$ and $e\rho/\alpha \leq 1$ by assumption, for sufficiently small F or for sufficiently large g or $\alpha/(r+c)$, we have $\underline{v} > v^0(F)$. Intuitively, for $v \in \left[v^0(F), \underline{v}\right]$, if the firm invested without a subsidy the agency would prefer not to give a subsidy since a realization of V relative to shadow cost of public funds g is so small. However, because the firm does not invest at all without the subsidy, the agency prefers to grant the subsidy rate $\tilde{s}(F)$ over the firm's no-investment.

Finally, Lemma A6 identifies the firm's equilibrium application behavior.

Lemma A6. (i) For $F \in [0, \underline{F})$,

$$d_a^*(F) = \begin{cases} 1 & \text{if } \int_{\underline{v}}^{\overline{v}} \Pi^{E**}\left(s^{**}\left(v\right)\right)\phi\left(v\right)dv + \left(1-\Phi\left(\overline{v}\right)\right)\Pi^{E**}\left(\overline{s}\right) - \left(1-\Phi\left(\underline{v}\right)\right)\Pi^{E**}\left(0\right) \geq K, \\ 0 & \text{otherwise;} \end{cases}$$

(ii) For $F \in \left[\underline{F}, \bar{F}\right]$,

$$d_a^*(F) = \begin{cases} 1 & \text{if } \int_{\tilde{v}}^{\overline{v}} \Pi^{E**}\left(s^{**}\left(v\right)\right)\phi\left(v\right)dv + \left(1-\Phi\left(\overline{v}\right)\right)\Pi^{E**}\left(\overline{s}\right) \geq K, \\ 0 & \text{otherwise;} \end{cases}$$

(iii) For $F \in \left(\bar{F}, \infty\right)$, $d_a^*(F) = 0$.

Proof. Differentiating the objective function in the firm's application problem (A3) with respect to d_a suggests that $d_a^*(F) = 1$ if and only if

$$\int_{-\infty}^{\infty} \tilde{\Pi}^{E*}\left(F, s^*\left(F, v\right)\right)\phi\left(v\right)dv - K - \tilde{\Pi}^{E*}\left(F, 0\right) \geq 0, \tag{A14}$$

and $d_a^*(F) = 0$ otherwise.

(i) If $F < \underline{F}$, Lemma A4 implies that $R^*(F, s) = R^{**}(s) > 0$ for all $s \in [0, \bar{s}]$ and the agency's subsidy rule $s^*\left(F, v\right)$ is given by part (i) of Lemma A5. Therefore, recalling equation (A1) and Lemmas A1-A2 implying $d_f^*(\cdot) = d_m^*(\cdot) = d_G^*(\cdot) = 1$, and noting that $\Pi^{E**}(F, s) := \Pi^E(R^{**}(s), \pi^{I*}(F, s)) > 0$, the first term in the left-hand side of equation (A14) can be written as

$$\int_{-\infty}^{\infty} \tilde{\Pi}^{E*}\left(F, s^*(F, v)\right)\phi\left(v\right)dv = \Phi\left(\underline{v}\right)\Pi^{E**}\left(F, 0\right)$$

$$+ \int_{\underline{v}}^{\overline{v}} \Pi^{E**}\left(F, s^{**}\left(v\right)\right)\phi\left(v\right)dv + \left(1-\Phi\left(\overline{v}\right)\right)\Pi^{E**}\left(F, \overline{s}\right).$$

As a result, equation (A14) can be rewritten as

$$\int_{\underline{v}}^{\overline{v}} \Pi^{E**}\left(F, s^{**}\left(v\right)\right)\phi\left(v\right)dv + \left(1-\Phi\left(\overline{v}\right)\right)\Pi^{E**}\left(F, \overline{s}\right) \tag{A15}$$

$$- \left(1-\Phi\left(\underline{v}\right)\right)\Pi^{E**}\left(F, 0\right) \geq K.$$

Thus, as claimed in part (i) of Lemma A6, for $F < \underline{F}$, $d_a^*(F) = 1$ if and only if the condition (A15) holds and $d_a^*(F) = 0$ otherwise.

(ii) If $F \in \left[\underline{F}, \bar{F}\right]$, Lemma A4 implies that if $s \in [0, \tilde{s}(F))$, then $R^*\left(F, s\right) = 0$ and if $s \in [\tilde{s}(F), \bar{s}]$ then $R^*\left(F, s\right) = R^{**}(s) > 0$. Therefore, in equation (A14), $\tilde{\Pi}^{E*}\left(F, 0\right) = 0$. The agency's subsidy rule is given by part (ii) of Lemma A5. Thus the firm contemplating a subsidy application knows that if and only if $v \geq \tilde{v}(F)$, the agency will award a sufficiently high subsidy rate $s \in (\tilde{s}(F), \bar{s}]$ to make $\tilde{\Pi}^{E*}(F, s) = \Pi^{E**}(F, s) > 0$ and that if $v < \tilde{v}(F)$, the firm will either receive no subsidy in which case the firm makes no investment nor profits, or it will receive subsidy $\tilde{s}(F)$ that just satisfies the firm's zero-profit constraint, which by definition also leads to zero profits. Therefore the application constraint (A14) can be rewritten as

$$\int_{\tilde{v}}^{\overline{v}} \Pi^{E**}\left(F, s^{**}\left(v\right)\right)\phi\left(v\right)dv + \left(1 - \Phi\left(\overline{v}\right)\right)\Pi^{E**}\left(F, \overline{s}\right) \geq K. \tag{A16}$$

The claim in part (ii) of Lemma A6 follows: For $F \in \left[\underline{F}, \bar{F}\right]$, $d_a^*(F) = 1$ if and only if the condition (A16) holds and $d_a^*(F) = 0$ otherwise.

(iii) If $F > \bar{F}$,Lemmas A4 and A5 stipulate that the agency awards no subsidy and the firm does not invest (even if it received a maximum subsidy). Therefore, the firm makes no profits, and equation (A14) becomes $-K \geq 0$ which does not hold. As a result, for $F > \bar{F}$, $d_a^*(F) = 0$. ■

Before establishing Propostion A1, we further shorten notation and write

$$R^*(F) := R^*(F, s^*(F, v, d_a^*(F)))$$

and

$$\pi^{I*}\left(F\right) := \pi^{I*}(F, s^*(F, v, d_a^*(F)))$$

as the firm's equilibrium R&D investment repayment promise, respectively. Using this notation we may also define

$$d_k^*(F) := d_k^*(F, s^*(F, v, d_a^*(F)), R^*(F), \pi^{I*}(F)), k = f, m,$$

and

$$d_G^*(F) := d_G^*(R^*(F), \pi^{I*}(F), d_k^*(F)), k = f, m,$$

as the investor's project funding and monitoring decisions, and the firm's project choice and choice decision, respectively. Recall also that $s^*(F, v) := s^*(F, v, 1)$. Proposition A1 summarizes Lemmas A1-A6.

Proposition A1. In the unique equilibrium of $\Gamma(F)$,
$\pi^{I*}\left(F\right) = \left[\left(r + c\right)\left(R^*(F) + F\right) - s^*(F, v, d_a^*(F))R^*(F)\right]/P$ and $s^*(F, v, 0) = 0$. Moreover, there are $\underline{F}$ and $\bar{F}$ with $0 \leq \underline{F} < \bar{F}$ such that

(i) for $F \in [0, \underline{F})$, $d_a^*(F) = 1$ if and only if

$$\int_{\underline{v}}^{\overline{v}} \Pi^{E**}\left(s^{**}\left(v\right)\right)\phi\left(v\right)dv + \left(1 - \Phi\left(\overline{v}\right)\right)\Pi^{E**}\left(\overline{s}\right) - \left(1 - \Phi\left(\underline{v}\right)\right)\Pi^{E**}\left(0\right) \geq K$$

and $d_a^*(F)$ =0 otherwise, $s^*\left(F, v\right) = 0$ for $v < \underline{v}$, $s^*\left(F, v\right) = s^{**}(v)$ for $[\underline{v}, \bar{v}]$, and $s^*\left(F, v\right) = \bar{s}$ for $v > \overline{v}$, $R^*(F) = R^{**}(s^*(F, v, d_a^*(F)))$, and $d_k^*\left(F\right) = 1$,$k = f, m, G$;

(ii) for $F \in \left[\underline{F}, \bar{F}\right]$, $d_a^*(F) = 1$ if and only if

$$\int_{\tilde{v}}^{\overline{v}} \Pi^{E**}\left(s^{**}\left(v\right)\right)\phi\left(v\right)dv + \left(1 - \Phi\left(\overline{v}\right)\right)\Pi^{E**}\left(\overline{s}\right) \geq K.$$

In that case $s^*(F,v) = R^*(F) = d_f^*(F) = 0$ for $v < v^0(F)$ whereas for $v \geq v^0$, $R^*(F) = R^{**}(s^*(F,v)), d_k^*(F) = 1, k = f, m, G$ and $s^*(F,v) = \tilde{s}(F)$ for $v \in [v^0(F), \tilde{v}(F))$,$s^*(F,v) = s^{**}(v)$ for $v \in [\tilde{v}(F), \bar{v}(F)]$ and $s^*(F,v) = \bar{s}$ for $v > \bar{v}$.

If $\int_{\tilde{v}}^{\overline{v}} \Pi^{E**}(s^{**}(v))\phi(v)\,dv + (1 - \Phi(\overline{v}))\Pi^{E**}(\overline{s}) < K$, then $d_a^*(F) = R^*(F) = d_f^*(F) = 0$;

(iii) for $F \in (\bar{F}, \infty)$, $d_a^*(F) = R^*(F) = d_f^*(F) = 0$.

Let us now discuss the consequences of Assumption A1. As shown by the proof of Lemma A4, the key role of Assumption A1 is to ensure that $\underline{F} \geq 0$. Suppose that Assumption A1 fails to hold so but a less stringent condition $\alpha/(r + c - \bar{s}) \geq e$ holds. Then we have $\underline{F} < 0 \leq \bar{F}$. In this case the firm invests only if it receives a subsidy. Part (i) of Proposition A1 no longer exists, but parts (ii) and (iii) are unchanged except that part (ii) exists now for $F \in [0, \bar{F}]$. If $\alpha/(r + c - \bar{s}) < 0$, then $\bar{F} < 0$, and part (iii) of Proposition A1 prevails for all $F \in [0, \infty)$.

Finally, we establish some comparative statics of the equilibrium with respect to financial market imperfections as measured by c.

Proposition A2. i) $s^{**}(c), R^{**}(c, s^{**}(c)), \underline{F}(c)$, and $\bar{F}(c)$ are decreasing in c. ii) $\tilde{s}(c), R^{**}(c, \tilde{s}(c)), \underline{v}(c)$, and $v^0(c)$ are increasing in c.

Proof: i). First, from equation A10 we obtain $\partial s^{**}/\partial c = -(g-1) < 0$. Next, recall from equation 7 that $R^{**}(s) = \alpha/(r + c - s)$. Inserting $s^{**}(c)$ from equation A10 into the right-hand side of this formula gives

$$R^{**}((c, s^{**}(c)) = \frac{\alpha}{(r + c)g - v},$$

from which we obtain $\partial R^{**}(c, s^{**}(c))/\partial c = -g\alpha/((r+c)g - v)^2 < 0$ – recall from Lemma A4 that in equilibrium $R^{**}(\cdot) > 1$ implying the positivity of the denominator.

Then, differentiating equations (A6) and (A7) with respect to c gives

$$\frac{\partial \underline{F}}{\partial c} = -\frac{\alpha}{(r + c)^2} \ln(\frac{\alpha}{r + c}) < 0$$

and

$$\frac{\partial \bar{F}}{\partial c} = -\frac{\alpha}{r + c} \left[\frac{\alpha}{r + c} (\ln(\frac{\alpha}{r + c - \bar{s}}) - 1) + \frac{1}{r + c - \bar{s}} \right] < 0,$$

in which the inequalities follow from Assumption A1 and $\bar{s} < 1 \leq r + c$.

ii) First, differentiating equation A8 gives

$$\frac{\partial \tilde{s}}{\partial c} = 1 + Fe^{-(\frac{\alpha + (r+c)F}{\alpha})} > 0.$$

Next, inserting $\tilde{s}(c)$ from equation A8 into the right-hand side of $R^{**}((c, \tilde{s}(c) = \alpha/(r + c - \tilde{s}(c))$ yields

$$R^{**}((c, \tilde{s}(c)) = e^{\frac{\alpha + (r+c)F}{\alpha}}$$

from which we immediately obtain $\partial R^{**}(c, \tilde{s}(c))/\partial c = FR^{**}(c, \tilde{s}(c))/\alpha > 0$.

Then, from equations (A11) and (A12) we get, respectively, $\partial \underline{v}(c)/\partial c = g - 1 > 0$ and

$$\frac{\partial \tilde{v}(c)}{\partial c} = g + Fe^{-\left(\frac{\alpha+(r+c)F}{\alpha}\right)} > 0. \blacksquare$$

Appendix D. Derivation of the firm's optimal R&D investment rule with an R&D tax credit.

We modify our theoretical model of section 3 by setting $s = 0$ and introducing instead a R&D tax credit rate $\tilde{\tau}_R \in [0, 1]$, which the firm receives whether or not it has corporate tax liability. In this case, we may rewrite the investor's payoff (2) as

$$\Pi^I(R, \pi^I) = (1 - \tau) \left[P\pi^I - (r + c)(R + F)\right]. \tag{A17}$$

and the firm's payoff (3) as

$$\Pi^E(\tilde{\tau}_R, R, \pi^I) = (1 - \tau) \left[P \left(\pi(R) - \pi^I\right)\right] + \tilde{\tau}_R R. \tag{A18}$$

As in section 3, we can seek a financing contract $\left(\pi^I, R\right) \in [0, \infty)^2$ that maximizes the firm's expected payoff. Thus, letting the investor's expected payoff from equation (A17) to be equal to 0 and solving the resulting equation for π^I gives

$$\pi^{I*}(R) = \frac{(r + c)(R + F)}{P}. \tag{A19}$$

After substitution of equations (1) and (A19) for equation (A18), the problem of seeking an optimal financing contract boils down to

$$\max_{R\in[0,\infty)} \Pi^E(\tau_R, R)_+ = (1 - \tau) \left[\alpha \ln R - (r + c - \tau_R) R - (r + c)F\right]_+. \tag{A20}$$

In equation (A20), $\tau_R = \tilde{\tau}_R/(1 - \tau)$ denotes the "adjusted" tax credit rate. Equation (A20) corresponds to the firm's objective function (6) save for s being replaced by τ_R. Clearly the optimal R&D investment decision rule with an R&D tax credit must be identical to the one given by equations (7)-(8) with τ_R replacing s.

Note from equation (A19) that the repayment promise is now independent of the R&D tax credit rate whereas in section 3 the repayment promise is contingent on the subsidy rate (see equation (5)). As equations ((3), (5), (A18) and (A19) show, now the firm claims the tax credit but has to promise a higher repayment to the investor than in section 3.

Appendix E: Counterfactual

Execution

For the counterfactual, we utilize the estimated parameter values and the assumed functional forms. We then draw shocks (ε_{it}, ζ_{it} η_{it}, μ_{it}) from their estimated (joint) distribution. We replace those draws in the top 1% with the value at the 99^{th}%. We also remove from the calculations the top 0.02% of observations with the highest simulated mean R&D investments. We use 100 simulation rounds.

Robustness

In Tables E1 and E2 we present results from our counterfactual when 1) we estimate the model using as cost of finance the estimated cost of finance based on balance sheet information, 2) ignoring (soft) loans Tekes gives and only use subsidies as our measure of s_{it} and 2) excluding the largest 3 firms in the estimation sample. The loans Tekes gives are soft in two senses: First, the interest rate a firm has to pay is subsidized; second, in case the project fails, the firm may not need to pay the (whole) loan back. We report the means of the same objects reported in the main text.

Table E1. Counterfactual results from the robustness tests

Balance sheet based cost of finance	R&D participation	R&D | R&D > 0	R&D	ratio (R&D)	profit	spillovers	welfare	ratio (welfare)
Laissez-faire	0.61	119 579	53 825	1	768 507	34 023	802 530	1
1^{st} best	0.63	278 492	128 046	2.38	725 403	82 875	821 264	1.02
2^{nd} best	0.61	266 438	124 606	2.32	728 572	79 397	820 956	1.02
No financial market imperfections	0.61	127 293	58 721	1.09	774 875	37 119	811 994	1.01
Tax credits	0.62	186 563	85 054	1.58	795 997	53 771	807 921	1.01
Subsidies	0.61	196 043	85 276	1.58	788 751	55 970	802 905	1.00
$s\|s > 0$		484 753		1.50				
Only Tekes subsidies	**R&D participation**	**R&D | R&D > 0**	**R&D**	**ratio (R&D)**	**profit**	**spillovers**	**welfare**	**ratio (welfare)**
Laissez-faire	0.62	177 494	93 613	1	1 071 675	56 106	1 127 781	1
1^{st} best	0.63	477 645	241 506	2.58	1 006 207	152 806	1 159 013	1.03
2^{nd} best	0.62	463 791	237 024	2.53	1 009 298	149 040	1 158 338	1.03
No financial market imperfections	0.62	177 509	93 625	1.00	1 071 698	56 113	1 127 811	1.00
Tax credits	0.63	281 237	150 497	1.61	1 117 539	90 199	1 137 305	1.01
Subsidies	0.62	234 441	121 442	1.30	1 079 765	74 050	1 119 216	0.99
$s\|s > 0$		483 026						
Excluding largest 3 firms	**R&D participation**	**R&D | R&D > 0**	**R&D**	**ratio (R&D)**	**profit**	**spillovers**	**welfare**	**ratio (welfare)**
Laissez-faire	0.62	200 633	103 615	1	1 198 377	56 997	1 255 374	1
1^{st} best	0.63	490 711	239 880	2.32	1 142 516	140 979	1 283 495	1.02
2^{nd} best	0.62	475 261	235 682	2.27	1 145 333	137 527	1 282 860	1.02
No financial market imperfections	0.62	200 646	103 626	1.00	1 198 399	57 003	1 255 403	1.00
Tax credits	0.63	295 412	154 354	1.49	1 241 097	84 928	1 263 048	1.01
Subsidies	0.62	255 250	127 601	1.23	1 205 536	71 470	1 247 116	0.99
$s\|s > 0$		459 925						

NOTES: The reported numbers are the means over all firms and simulation rounds for R&D participation, R&D investment, R&D conditional on a positive subsidy rate, profit, spillovers and welfare. Ratio (R&D) is the mean R&D in the regime in question divided by the laissez-faire mean R&D; ratio (welfare) is the mean welfare in the regime in question divided by the laissez-faire mean welfare.

Table E2. Counterfactual estimates

variable	balance sheet based cost of finance	only Tekes subsidies	excluding 3 largest firms
$\Pr[apply]$	0.18	0.15	0.15
$subsidy\, rate \mid s > 0$	0.42	0.42	0.39
τ_R	0.41	0.39	0.34
Government cost, $s \mid s > 0$ & $R\&D > 0$	84 796	59 146	56 937
Government cost, $\tau_R \mid R\&D > 0$	76 491	109 682	100 440
Government cost, s	34 846	28 833	24 908
Government cost, τ_R	34 872	58 694	52 480

NOTES: The figures are calculated over all simulation rounds and firms.