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Complexities in the simple optimization of wood production and carbon sinks

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Abstract

To resolve several open issues in the forestry and carbon literature, we apply an analytically solvable stand-level model for optimizing the values of wood production and carbon sinks. Nonmonotonic stand volume development is shown to lead to locally optimal finite and infinite rotations. With infinite rotation, forests are most valuable as a carbon storage, and this outcome is shown to depend on discount rate and, in specific situations, on initial stand age. A rotation maximizing the pure value of carbon sinks is proved to lengthen with discount rate. In contrast to existing understanding, zero discounting does not render carbon sink enhancement as superfluous; instead, maximizing the average carbon stock in standing forests and decaying wood products is optimal. This combined with wood production implies that optimal rotation is, excluding accidental parameter values, discontinuous at a zero discount rate. These results have far-reaching implications in the comparisons of carbon sink studies. Including carbon into the optimal rotation model changes the rationale of stand value and its development over the rotation.

Keywords: forest carbon sink; optimal rotation; discounting; carbon neutrality; climate and forest policy
JEL codes: Q23, Q54

1 Introduction

The United Nations and European Union's target of reaching zero greenhouse gas emissions by 2050 ("Net Zero" policy) places heavy demand on atmospheric carbon dioxide (CO2) removals. According to the IPCC (Lee et al. 2023), increasing forest land area and improved forest management are among the main options for carbon (C) removal together with future methods such as bioenergy, carbon capture, and storage (BECCS). In order to obtain a rough economic perspective, the global annual terrestrial *C* sink is ca. 10.3 Pg of CO_2 (Canadell and Raupach 2008), of which woodlands cover the dominant share. In 2022, the average price of an EU emission trading system emission allowance was ca. 84 USD (International Carbon Action Partnership 2022). At this price, the positive externality value of the annual *C* sink is ca. 865 billion USD. For comparison, forestry contribution to global GDP is at the magnitude of 660 billion USD (Li et al. 2022).

In economics, the seminal papers by Plantinga and Birdsey (1994) and van Kooten et al. (1995) have offered the basis for a literature on optimizing forests as a *C* sink together with their traditional value as sources of wood. Later economic research applies models with extended details and numerical methods to study particular cases and tree species (e.g. Boscolo and Vincent 2003, Parkatti and Tahvonen 2021). Besides economics, an extensive natural sciences-based literature analyzes payback periods, *C* debts, and trade-offs between the *C* sink and wood output (Fargione et al. 2008, Luyssaert et al. 2018, Peichl et al. 2022). As a whole, the literature (including the first economic papers) appears heterogeneous and often contradictory in the underlying basic principles. Our study aims to clarify the generic economic features for optimizing the values of forest *C* sinks. For this end, we apply an analytically solvable model, i.e. an approach that has been nearly absent in the literature since the publication of the seminal papers.

In addition to heterogeneous principles, the discussion on *C* sinks is characterized by close interest group involvement. For the forest industry, public policies for enhancing *C* sinks appear as competition over their crucially vital inputs. According to their common view, the largest contribution from forestry in attempts to reduce atmospheric CO_2 is obtained by taking the highest possible production of biomass as the leading goal.[1] In contrast, environmentalists emphasize the high *C* stock in old-growth forests.[2] Similar views can be found in research and without reconciliation (e.g. Lewis et al. 2019, Petersson et al. 2022). According to our view, this debate and the interpretation of highly variable numerical studies cannot be clarified without an analytically coherent theoretical basis.

Based on a multitude of numerical economic models with rather similar structures, the underlying theoretical setup could be expected to be fully analyzed (cf. the cake eating model for exhaustible resources). However, this seems not to be the case. Guided by the seminal papers (op. cit.), our model adds the *C* sink of growing trees and *C* release from wood products into the classic Samuelson (1976) and Faustmann (1849) model.[3] The first complication arises from the inherent nonconvexities that occur with long rotations where stand volume is decreasing. We show that, under well-grounded assumptions on volume development, both finite and infinitely long locally optimal rotations exist simultaneously. This implies that, on specific occasions, the optimal rotation and whether active forestry persists depend on the initial stand age and on discounting. These solution features are unclear in the existing literature but will be shown to have strong consequences e.g. on the study setup and the results interpretation of Roebroek et al. (2023), who find that completely removing all human intervention on the world's forests could increase forest biomass by only 15–16%.

Economic studies emphasize discounting the value of a *C* sink while ecological studies typically neglect discounting or only discount wood revenues in scenario-type analyses (Lundmark et al. 2018). However, the role of discounting is unclear also in economic studies. Plantinga and Birdsey (1994) write

[1] E.g. Skogforsk (2019).
[2] E.g. New York Times, Sep. 7, 2022 "Europe is scarifying its ancient forests for energy".
[3] The release of carbon from harvested trees and assumptions from underlying functions imply that this model does not coincide with the Hartmann (1976) or Strang (1983) forest model with *in situ* values.

that under a high discount rate, the rotation that maximizes the value of a pure *C* sink is infinitely long. In van Kooten et al. (1995), a higher discount rate shortens the optimal rotation. van Kooten et al. (2004) write: *"Since a zero discount rate on physical carbon implies that there is no difference between removing a unit of carbon from the atmosphere today, tomorrow or at some future time, this could be extrapolated to conclude that it does not matter if the carbon is ever removed from the atmosphere."* A similar view is shared in Boyland (2006), Johnston and van Kooten (2015), McDermott et al. (2015), and van Kooten et al. (2015): without discounting, it is optimal to delay the *C* sink enhancement indefinitely, i.e. the question of forest *C* sink is relevant only under discounting. This sheds quite a shadow onto the extensive noneconomic literature on forest *C* sinks that typically neglect discounting. However, none of the economic studies (op. cit.) justify their views analytically.

We prove that when optimizing pure *C* sinks, the rotation is longer the higher the rate of discount. Next, we show that without discounting, the solution maximizing the pure *C* sink value is clearly defined and maximizes the average (annual) *C* stock in forest stands and in wood products and the rotation may well be infinitely long. A similar rationale is present when maximizing both the sum of the net value of wood production and the value of a *C* sink under zero discounting. Thus, a zero discount rate by no means leads to infinite postponement of *C* sink enhancement. However, when the discount rate approaches zero, the effect of a *C* sink on the optimal rotation vanishes, implying that optimal rotation is discontinuous at the zero rate of discount. This is surprising and has not been recognized earlier. Besides the fact that understanding an economic model only if discounting is strictly positive is unsatisfactory[4], our results have strong consequences in the interdisciplinary *C* sink literature, where a large number of studies neglect discounting (e.g. Harmon and Marks 2001, Fargione et al. 2008, Creutzburg et al. 2017, Luyssaert

[4] Zero discounting outcomes have been extensively analyzed e.g. in the contexts of the exhaustible resources model (Gale 1967), supergames (Aumann and Shapley 1994), economic growth (Ramsey 1928), utilitarianism (Asheim et al. 2022), and forest economics (Mitra and Wan 1986).

et al. 2018, Roebroek et al. 2023) or only discount timber revenues (Lundmark et al. 2018, Kellomäki et al. 2019). Understanding the rationale of increasing forest *C* sinks to follow only because of discounting suggests that the extensive literature on forest *C* sinks without discounting is theoretically flawed. Our result shows that such a view is unwarranted. Additionally, our results explain why the results in studies without discounting may strongly differ *vis á vis* optimization studies with coherent discounting.

We show that a *C* sink as such does not support the maximum sustainable yield (MSY) and, when maximizing the value of a pure *C* sink, the MSY is optimal only without discounting and when *C* is permanently stored in harvested trees. Previous research emphasize that a *C* sink lengthens the rotation, but we prove that *a priori* the effect may equally be the reverse.

Including a *C* sink changes the stand value rationale, and we show that the value may develop nonmonotonically over the rotation and may well be lower just prior to a clearcut compared with the value of bare land. This is in sharp contrast to the classic model, where stand value increases exponentially over the rotation. Finally, we provide a new interpretation for why the "carbon neutrality" of forestry is fallacious and why the substitution of CO_2 incentive resources with wood, highly emphasized in noneconomic studies, is explicitly absent in economic *C* sink optimization.

We continue by specifying the model, and optimality conditions. This is followed by the existence and uniqueness analysis. Next, we show the properties of pure *C* sink maximization, including the outcome of zero discounting, and then the discontinuity of an optimal rotation. Finally, we present discussion and conclusions with some forest and climate policy implications.

2 Stand-level optimization of wood production and a C sink

Stand volume F (m^3 ha^{-1}) as a function of stand age t satisfies the assumptions *(A1)*:

$F \in C^3$, $F(0) = F'(0) = 0, F(t) > 0$ *for* $t \in (0,\infty)$, $F'(t) \to 0$ *and* $F(t) \to \hat{F}$ *when* $t \to \infty, F''(t) > 0$ *for* $t \in (0,\hat{t})$.
If $\arg\sup F(t) = \{\infty\}$, $F''(t) < 0$ *for* $t \in (\hat{t},\infty)$ *and* F''/F' *is decreasing in* $t \in (0,\infty)$.
If $\exists \tilde{t} < \infty$, *such that* $\hat{t} < \tilde{t}$ *and* $F'(\tilde{t}) = 0$, *then* $F'(t) < 0$ *for* $t \in (\tilde{t},\infty)$, $F''(t) < 0$ *for* $t \in (\hat{t},\widehat{t})$, $F''(t) > 0$ *for* $t \in (\widehat{t},\infty)$ *and* F''/F' *is decreasing in* $t \in (0,\infty)$, $t \neq \tilde{t}$.

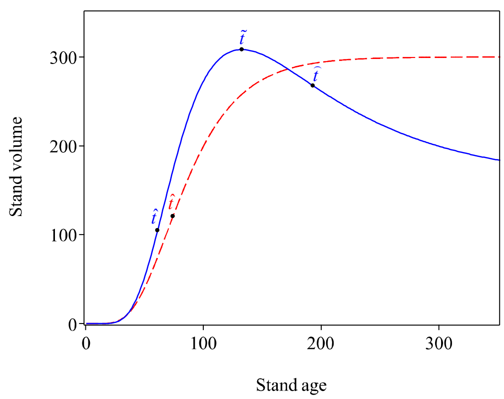

Figure 1. Monotonic and nonmonotonic stand development

Thus, stand volume either increases monotonically along a convex–concave path toward a maximum or reaches a maximum at some finite age $\tilde{t}$ and decreases toward a long-run level from above (Figure 1). The assumption that F''/F' is decreasing in t is satisfied by a variety of sigmoidal type of functions[5]. In the following we apply assumptions on these two alternatives throughout the analysis.

Let $p \geq 0$, $r \geq 0$, and $w \geq 0$ denote the stumpage price (per m^3), discount rate, and cost of regeneration (per ha) respectively. To include a C sink and release, let θ (≈ 0.7) denote C in tons of CO_2 per m^3 of wood biomass and $\omega(\approx 1.9)$ a biomass expansion factor that converts trunk volume into total biomass i.e. foliage, bark, stumps, branches, and roots. The social price of C is $\tau \geq 0$ (per tCO_2), implying that

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[5] Example for a monotonically increasing volume: $F(t) = a_1(1-e^{-a_2 t})^{a_3}, a_1, a_2 > 0,\ a_3 > 1$ and for volume development with a peak: $F(t) = 300(1-3e^{-0.01t})(1-e^{-0.03t})^{10}$.

the present value of C released from a harvested unit of wood is $\tau\theta\omega\beta$, where $\beta = \alpha/(\alpha + r)$ and α is the average rate of decay of harvested stand biomass including manufactured wood products. The problem of maximizing the values of wood production and a C net sink is

$$J(t^*) = J_f\left(t^*\right) + J_c(t^*) = \max_{\{t\in(0,\infty)\}} \left\{ -w + pF(t)e^{-rt} + \tau\theta\omega\left[\int_0^t F'(s)e^{-rs}ds - \beta F(t)e^{-rt}\right]\right\} / \left(1 - e^{-rt}\right), \quad (1)$$

where t^* is the optimal rotation, $J_f(t)$ is the value from wood production, and $J_c(t)$ is the value of a (net) C sink. Note that when $w > 0$, this setup assumes mandatory regeneration activity independent of the sign of bare land value (BLV), as is typical e.g. in Nordic countries.

Our specification (1) is in line with previous analytically studied models, albeit details differ. Plantinga and Birdsey (1994) assume that C release is instantaneous ($\alpha \to \infty$). Their volume function is either concave and monotonically increasing or concave and increasing for $t \in [0, \tilde{t}]$, after which the volume decreases. In van Kooten et al. (1995), the volume function is convex–concave–convex and stand volume approaches zero as $t \to \infty$. Regeneration cost is zero in both papers. Akao (2011) assumes that F implies a unique optimal rotation, leaves details unspecified as well as the relationship between F and a C (gross) sink. Tahvonen and Rautiainen (2017) apply the monotonically increasing volume function alternative in *(A1)*. None of these studies includes the dependence of the C release value on the discount rate. Our choice of linear relationship between timber volume and total C is in line with expansion factors that convert the stem dry mass into whole-tree dry mass (Lehtonen et al. 2004).

The first-order necessary optimality condition for a finite rotation period is

$$J'(t) = J'_f(t) + J'_c(t) = 0, \quad (2)$$

where

$$J'_f(t) = \left\{ pF'(t) - \frac{r\left[-w + pF(t)\right]}{1 - e^{-rt}} \right\} \left(e^{rt} - 1\right)^{-1} \equiv y_f(t)\left(e^{rt} - 1\right)^{-1}, \quad (3)$$

$$J_c'(t) = \tau\theta\omega\left\{F'(t)(1-\beta) - \frac{r\left[\int_0^t F'(s)e^{-rs}ds - \beta F(t)\right]}{1-e^{-rt}}\right\}\left(e^{rt}-1\right)^{-1} \equiv y_c(t)\left(e^{rt}-1\right)^{-1}. \tag{4}$$

In the following, we apply the definitions:

$$y(t) \equiv y_f(t) + y_c(t),\ \varphi \equiv p\hat{F} - w,\ \gamma \equiv \tau\theta\omega\left[\int_0^\infty F'(s)e^{-rs}ds - \beta\hat{F}\right],$$
$$J'(t^*) = 0,\ J_f'(t_f) = 0,\ J_c'(t_c) = 0.$$

Proposition 1: Assume $r > 0, p \geq 0, \tau \geq 0, w \geq 0$. *Then A) under monotonic volume development, the optimal rotation is unique and finite if* $\varphi + \gamma > 0$ *and infinite if* $\varphi + \gamma \leq 0$ *and B) if stand volume has a maximum at a finite age, these two solutions may exist simultaneously as two locally optimal solutions.*

Proof: Appendix 1.

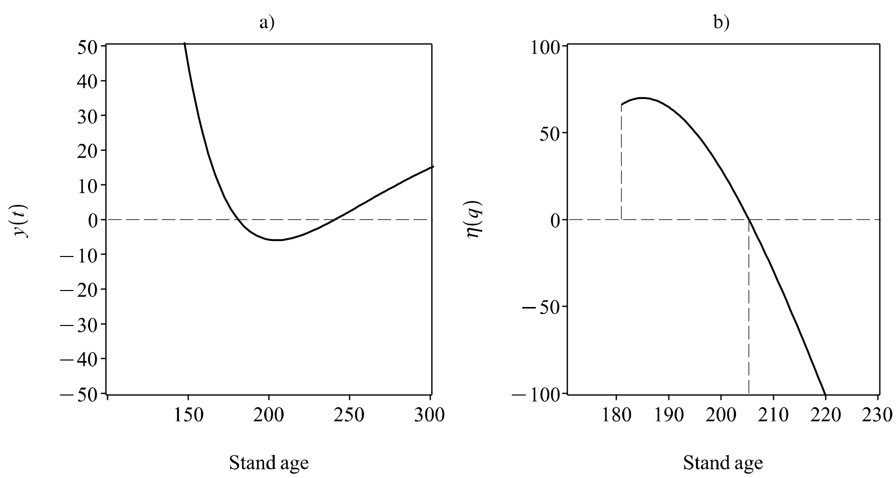

Figure 2. Finite and infinite optimal rotations.
a) Existence of simultaneous locally optimal finite and infinite rotations
b) Dividing initial age between finite and infinite rotation solutions (t* optimal finite rotation)
Parameter values: $F(t) = 300(1+3e^{-0.01t})(1-e^{-0.03t})^{10}$, $p = 20, \tau = 60, \theta = 0.7, \omega = 1.8909$, $r = 0.03$, $\alpha = 0.01464, w = 4200$, $t* = 181$, dividing initial stand age 205 .

When the two locally optimal solutions exist simultaneously, the rotation choice may depend on the initial stand age. If the finite rotation is globally optimal given bare land as the initial state, this rotation is obviously optimal if the initial stand age is lower than the optimal rotation age. However, if the initial stand age is higher, an immediate harvest continuing with a finite rotation is optimal only if the sum of the immediate net gain and the BLV exceeds the infinite horizon gain from the C sink, i.e. if

$$\eta(q) = pF(q) - \tau\theta\omega\beta F(q) + J(t^*) - \tau\theta\omega\int_q^\infty F'(s)e^{-r(s-q)}ds > 0, \qquad (5)$$

where q is the initial stand age. Figures 2a,b show a numerical example with locally optimal finite and infinite rotations, where continuing harvesting is optimal when $q \le 205$. However, it is optimal to abandon harvesting forever if stand initial age exceeds 205 years.

The effect of a C sink on the optimality of wood production (i.e. on the extensive margin of forestry) is not straightforward. Without a C sink, the optimality is determined by the sign of $\max_{\{t\in(0,\infty)\}} pF(t) - w$ when the initial state is bare land (and regeneration is mandatory). After adding a C sink (and given bare land), the optimality of wood production is determined by the sign of

$$\mu = \max_{\{t\in(0,\infty)\}} pF(t) - w + \tau\theta\omega\left[\int_0^t F'(s)e^{-rs}ds - \alpha F(t)/(\alpha + r)\right].$$

Thus, when the immediate revenues net of regeneration cost plus the value of the C (net) sink are negative with all rotation periods, it is never optimal to harvest and *vice versa*. The maximization of μ is attained either with $t = \tilde{t}$ or $t \to \infty$. The effect of the C sink on this harvesting choice is potentially positive (negative) if the expression in the square bracket $(\equiv \hat{\gamma})$ obtains a positive (negative) sign and is high enough to change the sign of μ. As $\partial\hat{\gamma}/\partial\alpha < 0$, and $\hat{\gamma}$ is positive when $\alpha = 0$ and negative when $\alpha \to \infty$, a faster release of C from wood products decreases the value of $\hat{\gamma}$. If $\hat{\gamma} < 0$, a high enough C price τ always implies $\mu \le 0$, i.e. the unoptimality of harvesting, whereas when $\hat{\gamma} > 0$, the C sink with a high enough C price implies the optimality of harvesting, albeit no harvesting exists without the C sink. Additionally, after including the C sink, the optimality of active forestry becomes dependent on discounting and $\hat{\gamma} \to 0$ as $r \downarrow 0$ or $r \to \infty$ and $\hat{\gamma}$ may increase, decrease, and chance its sign as a function of r.

The solutions in van Kooten et al. (1995) are computed numerically and optimal rotation is written to be either finite or infinite (uniqueness not studied). However, given their assumption $F(t) \to 0$ as $t \to \infty$, the latter solution is never optimal because the necessary optimality conditions (2)–(4) are not satisfied when $t \to \infty$.[6] Plantinga and Birdsey (1994) present sufficient optimality conditions for finite and infinite rotations but do not study uniqueness. Additionally, they write that an immediate clearcut is always optimal if the initial stand age exceeds the finite optimal rotation. This contradicts with our condition (5) and the numerical example in Fig. 2b. Their assumption concerning the concave stand volume function at ages where the volume is increasing is a problem, as no optimal solution with a positive rotation length for the timber production problem exists under this assumption and with no regeneration cost. Tahvonen and Rautiainen (2017) prove the uniqueness of an optimal finite rotation given monotonic F and $\varphi + \gamma > 0$. None of these authors further elaborate the question of C sinks or the optimality of active forest harvesting, nor does Akao (2011).

However, the question of active forest management and C sinks has been discussed elsewhere. Roebroek et al. (2023) obtain a result where completely removing all human intervention from the world's forests could increase forests biomass by only 15–16%. This outcome is presented as an upper-bound contribution of global forests in climate mitigation. Our results on local optima challenge this study setup and the derived conclusions: the total choke off harvesting may imply a low forest biomass, but this does not rule out the possibility of (optimal) solutions with long but finite rotations and much higher C stocks and sinks. Additionally, our result on the effect of the initial stand state emphasizes that it may be optimal to preserve old-growth forests, albeit increasing their amount would not be optimal.

[6] Their volume function is $at^b e^{-ct}$, $a,b,c > 0$.

As has been observed e.g. by Skytt (2021) and Stokland (2021), forest ecological research should greatly strengthen the understanding of stand volume and C stock development in old-growth unharvested forests.

2.1 Properties of maximizing a pure C sink

To obtain a more detailed understanding of the inclusion of a C sink into forest resource optimization, we next analyze it separately from the wood production problem. Thus, the problem is to

$$\max_{\{t\in[0,\infty)\}} J_c\left(t\right)=\tau\theta\omega\left[\int_0^t F'(s)e^{-rs}ds-\beta F(t)e^{-rt}\right]/\left(1-e^{-rt}\right), \tag{6}$$

where the regeneration cost is assumed to be included in the wood production part of the model.

Proposition 2. Given $J_c''(t_c)\neq 0$ *and* $r>0$*, the optimal finite C rotation* t_c *increases with* α*. Proof, Appendix 2.*

Given optimal solutions with finite rotation periods and increasing stand volume, a faster decay rate decreases both the net value growth and the interest cost of postponing the future C storage (Equation 4). The latter effect dominates, and it is optimal to postpone the future harvesting and emissions. When stand volume is decreasing at the age of optimal rotation, both effects postpone the optimal harvest timing. When the C release is instantaneous $(\alpha\to\infty)$ and stand volume increases monotonically, $J'\left(t_c\right)>0$ for $t\in(0,\infty)$ (Equation 4), the rotation maximizing the net value of a C sink approaches infinity.

Earlier analytical studies by Plantinga and Birdsey (1994), van Kooten et al. (1995), and Akao (2011) do not present analytical results for the effects of the decay rate on rotation period length, while Tahvonen and Rautiainen (2017) show that a higher β increases the rotation length.

Proposition 3. Given $J_c''\left(t_c\right)\neq 0$*, the optimal finite C rotation* t_c *increases with the discount rate r. Proof, Appendix 3.*

Compared with the classic rotation model, solving the effect of the discount rate is somewhat tedious and the effect is the opposite. Intuitively, a higher rate of discount implies postponement of a clearcut since the *C* release is costly. This result is not obtained in earlier papers. Plantinga and Bridsey (1994) assume an instantaneous *C* release ($\beta = 1$) and write that high enough discounting eliminates optimal harvesting. van Kooten et al. (1995) only note that a higher discount rate in their model with both wood and *C* objectives shortens the rotation in their numerical example.

Proposition 4. When $r \downarrow 0$ *and* $\alpha > 0$, $t_c \downarrow \tilde{t}_c > t_{msy}$ *and* $\tilde{t}_c$, *when finite is defined by*

$$F\left(\tilde{t}_c\right) + F'\left(\tilde{t}_c\right)/\alpha - \frac{\int_0^{\tilde{t}_c} F\left(s\right)ds + F\left(\tilde{t}_c\right)/\alpha}{\tilde{t}_c} = 0. \tag{7}$$

Proof: By L'Hopital's rule and partial integration we obtain from (6)

$$\lim_{r\downarrow 0} J_c = \left[-\int_0^t F'\left(s\right)sds + F\left(t\right)/\alpha + tF(t)\right]/t = \left[\int_0^t F(s)ds + F\left(t\right)/\alpha\right]/t, \tag{8}$$

where $\tau\theta\omega$=1. Differentiating w.r.t. *t* yields the first-order optimality condition for finite t_c:

$$\left[F\left(t\right) + F'\left(t\right)/\alpha\right]/t - \left[\int_0^t F\left(s\right)ds + F\left(t\right)/\alpha\right]/t^2 = 0\,. \tag{9}$$

The uniqueness follows by *(A1)* similarly as in *Proposition 1*. Multiplying (9) by *t* yields (7). Multiplying (7) by α yields

$$F\left(t\right)/t - F'\left(t\right) = \alpha\left[F\left(t\right) - \int_0^t F\left(s\right)ds/t\right] > 0, \tag{10}$$

implying that $\tilde{t}_c > t_{msy}$. □

We note that the necessary optimality condition (7) can as well be obtained by letting the interest rate approach zero in (4). To interpret the last expression in (8), note that the quantity of *C* in harvested biomass (wood products etc.) at the moment of clearcut is written as

$F+Fe^{-\alpha t}+Fe^{-\alpha 2t}+Fe^{-\alpha 3t}+...=\sum_{i=0}^{\infty}Fe^{-\alpha it}$ and equals $\sum_{i=0}^{\infty}Fe^{-\alpha(it+s)}$ at any moment $s\in[0,t]$ during the rotation. Thus, the average C stock in harvested biomass is

$$\int_0^t \sum_{i=0}^{\infty}Fe^{-\alpha(it+s)}ds/t=\frac{F}{\alpha t}.$$

This implies that maximizing the last expression of (8) coincides to maximizing the average C stock in uncut trees and in harvested biomass, and by condition (7), the maximum is attained when the marginal increase in C stock equals the C stock annual average. Thus, when $r\downarrow 0$, the C sink objective approaches the maximization of the long-run average C stock. This leads to the question concerning the suitable optimality criteria when the discount rate equals zero. In economic models, the commonly used choice is to maximize the long-run average (Dutta 1991), which is applied in the classic Samuelson (1976) paper for pure wood production. No alternatives to this objective under zero discount have been presented. This objective leads to the same rotation that is optimal as $r\downarrow 0$. In the model for pure C sinks, maximizing the long-run average leads to an analogous setup. Thus, we take

$$\tilde{J}_c=\left[\int_0^t F(s)ds+F(t)/\alpha\right]/t \tag{11}$$

as the objective to be maximized when the discount rate equals zero.

As a corollary, we can observe from (10) that when $r=\alpha=0$ the optimal rotation is the MSY rotation and that with $\alpha>0$ the rotation period increases. Additionally, the derivative of (8) shows that when

$$\alpha\geq\lim_{t\to\infty}\frac{F(t)-F'(t)t}{F(t)t-\int_0^t F(s)ds}$$

the infinite rotation is locally optimal (globally optimal and unique when F is monotonic).

Figure 3a plots equation (11), i.e. the C stock in forest and in wood products as function of the rotation period. Depending on the properties of stand growth and wood product decay this stock is maximized by finite (122 years) or infinite rotations. In Figure 3b, the optimal rotation increases with discount rate. As the 122 years rotation maximizes the average annual C stock, the average stock decreases with interest rate and rotation length. When $r \downarrow 0$, both the average C stock and the present value of the C sink converge toward the same value as implied by *Proposition 4*. Earlier analytical papers have not presented results for the pure C sink model when discount rate equals or approaches zero.

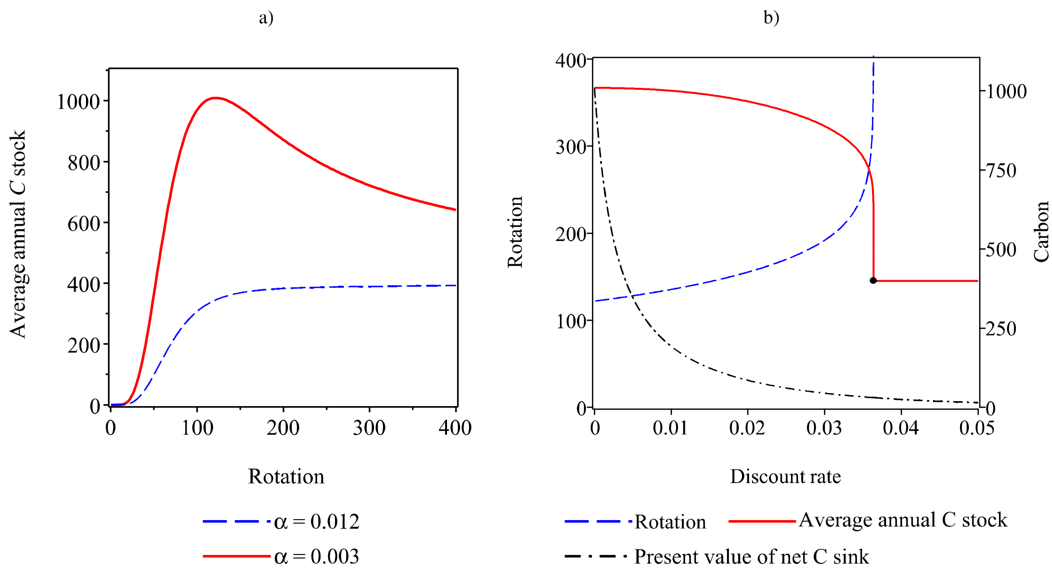

Figure 3. Maximizing the pure C sink. 3a) discount rate zero, 3b) varying discount rate. Note: $F(t) = 300(1-e^{-0.03t})^8$, $\alpha = 0.003$ (in 3b), $\omega = 1.9$, $\theta = 0.7$, $t_c\big|_{r=0} \approx 122$.

2.3 Optimal simultaneous wood production and C sink under zero discounting

Given the pure wood production objective and $\max_{\{t \in (0,\infty)\}} pF(t) > w$, it holds that $J_f \to \infty$ as $r \downarrow 0$, i.e. an unbounded BLV. Following Samuelson (1976) the objective with zero discounting is to maximize average annual net revenues. This objective leads to the same rotation that is obtained by the necessary optimality conditions when $r \downarrow 0$. Define t_{fr} as the optimal finite rotation for $\max_{\{t \in (0,\infty\}} \left[pF(t) - w\right]/t$.

Lemma 1. Given $\alpha > 0$, $r \downarrow 0$ implies that the optimal finite rotation to problem (1) approaches t_{fr}.

Proof: By (2)–(4) and (9) we obtain

$$\lim_{r\downarrow 0} J'(t) = \lim_{r\downarrow 0}\left(e^{rt}-1\right)^{-1}\left[pF'(t)-\left[-w+pF(t)\right]/t\right]+\tau\theta\omega\left\{\frac{F(t)+F'(t)/\alpha}{t}-\frac{\int_0^t F(s)\,ds+F(t)/\alpha}{t^2}\right\}.$$

As $\left(e^{rt}-1\right)^{-1}\to\infty$ when $r\downarrow 0$, and $\tau\theta\omega\left\{\left[F(t)+F'(t)/\alpha\right]/t-\left[\int_0^t F(s)\,ds+F(t)/\alpha\right]/t^2\right\}$ is finite when $\alpha>0$, it follows that $F'(t)-\left[-w+pF(t)\right]/t\to 0$ and $t^*\to t_{fr}$ when $r\downarrow 0$. □

Thus, as the discount rate approaches zero, the optimal rotation approaches the pure wood production solution, where the effect of the C sink disappears completely. Given the objective of maximizing the average net timber revenues and objective (8) for the C sink, the problem is to maximize

$$J(t)\big|_{r=0}=\left\{-w+pF(t)+\tau\theta\omega\left[\int_0^t F(s)\,ds+F(t)/\alpha\right]\right\}/t, \tag{12}$$

which leads to the necessary optimality condition for a finite $t^*\big|_{r=0}$:

$$J'(t)\big|_{r=0}t^2=p\left[F'(t)t-F(t)\right]+w+\frac{\tau\theta\omega}{\alpha}\left[F'(t)t-F(t)\right]+\tau\theta\omega\left[F(t)t-\int_0^t F(s)ds\right]=0.$$

When $w=0$, $\lim_{r\downarrow 0}t^*=t_{msy}$ by L*emma 1*. As $F(t)t-\int_0^t F(s)ds>0$, it holds that $t^*\big|_{r=0}>t_{msy}$, i.e. the optimal rotation is discontinuous at $r=0$. When $w>0$, $\lim_{r\downarrow 0}t^*>t_{msy}$ and $p\left[F'(t)t-F(t)\right]+w=0$ by *Lemma 1*, but as the solution to $\frac{\tau\theta\omega}{\alpha}\left[F'(t)t-F(t)\right]+\tau\theta\omega\left[F(t)t-\int_0^t F(s)ds\right]=0$ is unique (by *Proposition 4*), $t^*\big|_{r=0}=\lim_{r\downarrow 0}t^*$ can only be met by a single level of w. Thus, except for this accidental case, optimal rotation with $w>0$ is discontinuous at $r=0$. We summarize these findings as

Proposition 5: Given $\alpha>0$ and t_{fr} finite and ruling out an exceptional parameter constellation, the optimal rotation is discontinuous at $r=0$.

We further note that, since $\lim_{t\to\infty}\left[F'(t)t - F(t)\right] < 0$ and bounded and $\lim_{t\to\infty}\left[F(t)t - \int_0^t F(s)ds\right] > 0$ and bounded, a low enough $p + \tau\theta\omega/\alpha$ implies that $J'(t)\big|_{r=0} t^2 > 0$ as $t \to \infty$, implying that $t^*\big|_{r=0}$ is infinite although under $w = 0$ it holds that $\lim_{r\to 0^+} t^* = t_{msy}$. Thus, in this case the optimal rotation is infinitely long when discount rate is zero albeit it approaches the MSY rotation when discount rate approaches zero.

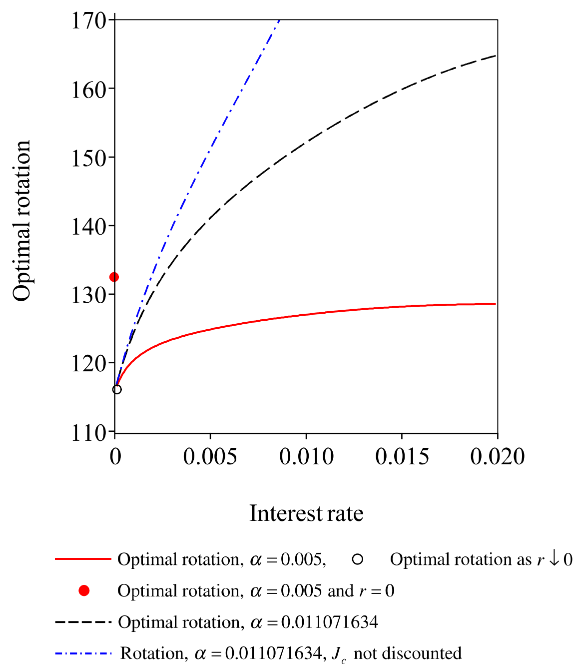

Figure 4. The discontinuity of optimal rotation at a zero discount rate. Parameter values: $F(t) = 300\left(1 - e^{-0.03t}\right)^8$, $w = 770,\ p = 30,\ \tau = 100,\ \theta = 0.7,\ \omega = 1.8909$.

In Fig. 4 $\lim_{r\downarrow 0} t^* = t_{fr} \approx 115.75$ and as proved in *Lemma 1*, the optimal rotation approaches this level independently of the level of α. Given $\alpha = 0.005$, the optimal rotation with a zero discount rate is $t^*\big|_{r=0} \approx 134.4$ and with $\alpha = 0.011$ the rotation with a zero discount rate is infinitely long. For comparison, Fig. 4 shows the outcome when only timber revenues are discounted.

Earlier analytical studies (Plantinga and Bridsey 1994, van Kooten et al. 1995, Akao 2011, Tahvonen and Rautiainen 2017) do not present results when the discount rate approaches or equals zero. However, according to e.g. van Kooten et al. (2004) and Boyland (2006) it is optimal to postpone *C* sink enhancement infinitely without discounting. Our analytical results do not support this frequently repeated understanding in showing that it is natural to maximize the average annual values of wood production and *C* stock in trees and wood products under zero discounting. The solution for this problem is well defined and does not imply that optimal *C* sink enhancement activities could be infinitely postponed.

A large literature investigates trade-offs and forest management alternatives without reference to discounting or interest rate (e.g. Fargione et al. 2008 Luyssaert et al. 2018, Roebroek et al. 2023). If the forest *C* sink issue was relevant only under discounting, the rationale of these studies would be difficult to justify. However, our findings (Propositions 4 and 5) show that the absence of discounting does not, as such, problematize these studies. However, our findings reveal that the results of these studies may strongly differ from studies with low discount rates.

2.4 The effects of C sink inclusion on optimal rotation

Similarly as in the classic rotation model, a higher timber price and lower regeneration cost shorten the optimal rotation. However, the effect of *C* price on rotation length is less straightforward:

Proposition 6. Given monotonic F *,* $r > 0$ *and a finite optimal rotation* t^* *with* $J''(t^*) \neq 0$*, the optimal rotation is an increasing function of C price* τ *, if* $w = 0$ *or* $\beta = 1$ *and a decreasing function when* $w \uparrow p\hat{F}$ *and* $\beta \downarrow 0$. *Proof, Appendix 4.*

The proof shows that the *C* rotation t_c can be longer than the wood production rotation t_f and *vice versa,* and by continuity it is equally possible that $t_f = t_c$. Thus, we obtain

Corollary 2. Given assumptions of Proposition 6, C valuation lengthens (shortens) the optimal rotation t, if* $t_c > t_f$ $(t_c < t_f)$ *and* $t^* = t_f$*, when* $t_f = t_c$.

Proof: When $t_c > t_f$, it holds that $y_c(t_f) > 0$ by the uniqueness of t_c, implying that $J'(t_f) > 0$ by the uniqueness of t^*. Thus, $t_f < t^*$. When $t_c < t_f$, it holds that $y_c(t_f) < 0$ by the uniqueness of t_c, implying that $J'(t_f) < 0$ by the uniqueness of t^*. Thus, $t^* < t_f$. Finally, $t_f = t_c$ implies $J'(t_f) = 0$ directly.□

In Plantinga and Birdsey (1994) rotation increases with C price, but they apply restrictive assumptions $w = 0$ and $t_c = \infty$. van Kooten et al. (1995) obtain the same result numerically. The result in Akao (2011), derived with a somewhat different model, is similar to ours.

2.5 Value of C sinks

The inclusion of a C sink impacts the value of a forest stand. Without C, the BLV and timber value q years after a clearcut are $J_f(q,t) = e^{-r(t-q)}\left[pF(t) + J_f(t)\right]$, where $0 < q \le t$ and t is any rotation period including the optimal rotation t^* or t_f. Thus, during the rotation (and after regeneration), the value is increasing exponentially. Accordingly, the present value of a net C sink develops as

$$J_c(q,t) = \tau\theta\omega e^{rq}\left\{\int_q^t F'(s)e^{-rs}ds + e^{-rt}\left[-\beta F(t) + \frac{\int_0^t F'(s)e^{-rs}ds - \beta F(t)e^{-rt}}{1-e^{-rt}}\right]\right\}. \qquad (13)$$

At the beginning of a rotation, the present value $J_c(0,t)$ is positive (by partial integration) and remain positive over the rotation if the C release from harvested trees is slow and β is close enough to zero. However, if α is high and β close to one, the present value of the C net sink is negative when $q \uparrow t$. In this case, a high enough C price changes the qualitative properties of stand value determination. This is shown in Fig. 5, where the total stand value decreases because of the decreasing value of the C sink. In this example, the BLV is higher compared with stand value just before clearcut $(5116 - 770$ *vs* $4053)$. Thus, valuing C sink implies profound changes in the most classic forest economic results.

Equation (13) and Fig. 5 raise the question of the C net sink value at the normal forest state. Recall that under pure wood production the value of a one-hectare normal forest is $[-w + pF(t)]/(rt)$. Applying (13) and partial integration, we obtain the value of a C net sink of a normal forest as

$$\frac{1}{t}\int_0^t J_c(q,t)dq = \tau\theta\omega\frac{1}{t}\frac{(1-\beta)F(t)}{r} = \tau\theta\omega\frac{1}{t}\int_0^\infty F(t)e^{-s(r+\alpha)}ds \geq 0. \tag{14}$$

The first equation in (14) states that the present value of a net C sink over an infinite horizon equals the value of the sink net of C released from harvested trees. The second equation shows that this is equivalent to the present value of C in the decaying stock of harvested wood material. By (14), the value of a net sink for the normal forest is strictly positive for any finite rotation length. In contrast, the value of a net sink is zero if the C release from harvested wood is instantaneous or the rotation approaches infinity.

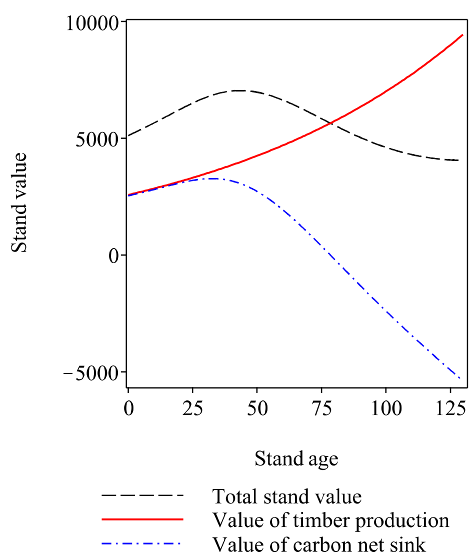

Figure 5. The stand value components over a rotation. Parameter values:
$F(t) = 300(1-e^{-0.03t})^8$, $w = 770$, $p = 30$, $r = 0.01$, $\tau = 25$, $\theta = 0.7$, $\omega = 1.8909$, $\alpha = 0.1608$ and $t^* \approx 129.8$.

3 Discussion and conclusions

Our study has developed the theoretically coherent and analytically solvable model initiated by Plantinga and Birdsey (1994) and van Kooten et al. (1995) for optimizing the values of wood production and C

sinks. Although many numerical economic studies on this subject have been published, the underlying analytical features of the generic model version are incompletely understood. We have shown that both the question of active forestry with wood production *versus* utilizing forests as pure *C* sinks and the role of discounting include complexities. Additionally, the effect of *C* sinks on the value of forest stand yield clear deviations *vis-à-vis* the classic rotation model.

In an influential study, Roebroek et al. (2023) observe that eliminating harvesting and other human influence on the world's forests will increase the aboveground biomass only by 15–16%, representing the global CO_2 emissions accrued over a four-year period. This increase in *C* storage is written to reveal the upper-bound contribution of forests in climate change mitigation. Our results show that complete elimination of harvesting does not guarantee the maximum *C* stock out from the atmosphere nor the *C* stock in forest biomass. A fraction of *C* stock is in harvested biomass, and the *C* stock in forests may obtain the maximum with a finite rotation. Further caveats and complexities arise when the question is viewed in an economic optimization setup based on value terms. Additionally, a comparison of the present and no-human intervention outcomes reflects a zero discount rate and, as we have shown, this may yield very different results compared with results under discounting.

We prove that when optimizing the pure *C* sink value, the optimal rotation is longer the higher the discount rate and the *C* release rate from wood products. Shorter rotations with a lower discount rate are somewhat surprising, but somewhat resemble the common view that climate policy demands for forest harvesting are less pressing if the planning horizon is long (Fargione et al. 2008, Skytt et al. 2021). Additionally, we prove that when discount rate approaches zero, the problem is to maximize the average *C* stock in forests and wood products. This objective is also well grounded when the discount rate is zero, and it may imply an infinitely long rotation when the *C* release back to the atmosphere is fast. These results are in sharp contrast with the existing view that without discounting, the economic rationale of forest carbon sinks disappears (e.g. van Kooten et al. 2004, McDermott et al. 2015). MSY tends to have an appealing role in forest sciences and policy, but we show that, excluding accidental cases, under the pure C sink objective it is optimal only with a zero discount rate and when *C* is permanently stored out from the atmosphere.

Maximizing both the values of wood production and *C* sinks reveals that the optimal solution is discontinuous at a zero rate of discount. This solution feature is surprising, albeit not entirely exceptional (cf. Dutta 1991). It follows from the facts that when discount rate approaches zero, the present value of wood production approaches infinity while the value of *C* sinks remains bounded, but when the rate of discount equals zero, the only conceivable objective is maximizing the average annual values of both wood production and *C* stock in forests and wood products. It is well possible that when the discount rate approaches zero, the optimal rotation is very short but infinitely long when the discount rate is zero. While stand value increases exponentially over the rotation in the classic rotation model, including the *C* sink completely changes the stand value determination, and stand bare land value may exceed the stand value just before harvesting.

One main line of argumentation in research and policy documents emphasizes the "*C* neutrality" of forestry (e.g. European Union 2009, Favero and Mendelsohn 2014). According to another closely related view, the positive climate policy contribution of forestry calls for the MSY to guarantee that forest industry output efficiently substitutes *C*-intensive alternatives such as fossil fuels (Lundmark et al. 2014, Gustavsson et al. 2017, Peichl et al. 2022). At the other end of the spectrum is the view that maximizing the *C* stock of natural forest ecosystems is preferable for mitigating climate change (e.g. Lewis 2019). Our generic model allows commenting on these controversies from economic and theoretical viewpoints.

If the optimal rotation for a pure *C* sink equals the optimal rotation for pure timber production, it could be possible to label forest harvesting and the use of harvested biomass as *C* neutral. However, we show that this can only happen accidentally. In addition, by emphasizing that *C* absorbed by trees is eventually released back to the atmosphere, the *C* neutrality argument assumes zero discounting. But given zero discounting, the *C* neutrality argument is problematic in light of our finding, which shows that zero discounting leads to a specific optimal rotation and therefore cannot express *C* neutrality. The *C* neutrality argument tries to justify any rotation similarly as the present economic literature in emphasizing that under zero discounting there is no difference whether a unit of carbon is removed today, tomorrow, or at any future date from the atmosphere.

A frequently repeated argument (e.g. Gustavsson et al. 2017) supports MSY by substitution of forest biomass for more *C*-intensive commodities and inputs. Together with a zero discount rate, this plays a central role e.g. in the setup by Fargione et al. (2008). This substitution effect is not discussed in economic forestry *C* sink studies nor does it enter explicitly in our generic setup. This is because any possible substitution effects are included as pecuniary externalities in the timber price. The price is higher especially if *C* emissions from competing commodities and inputs are included in emission permit markets or taxation. Thus, substitution effects *via* wood prices support timber production *vis á vis* increasing the *C* stock in forests. If emissions from fossil fuels remained uncontrolled, we are in the realm of the second-best outcome, which can be studied with a market-level forest model with endogenous prices and *C* sink by extending Akao (2011) and Tahvonen and Rautiainen (2017). Arguments referring to substitution benefits as a reason for not increasing forest *C* sinks are not valid in a first–best context. Nor is it possible to argue that the substitution effect as such would be enough to reach the full potential of forestry in economically efficient climate policy. Such arguments simply neglect the possibility to increase the *C* stock in standing forests.

Appendix 1, Proof of proposition 1

Proof: Given (A1) and (2)–(4) we obtain by the L'Hopital rule

$$\lim_{t\downarrow 0} J'(t) = \lim_{t\downarrow 0}\left\{ \frac{rwe^{rt}}{(e^{rt}-1)^2} + \frac{pF''}{re^{rt}} + \frac{\tau\theta\omega F''(1-\beta)}{re^{rt}} - \frac{pF''+\tau\theta\omega F''\left(e^{-rt}-\beta\right)}{-re^{-rt}(e^{rt}-1)+2r+(1-e^{-rt})re^{rt}} \right\} =$$

$$\lim_{t\downarrow 0} J'(t) = \lim_{t\downarrow 0}\left\{ \frac{rwe^{rt}}{(e^{rt}-1)^2} + \frac{pF''}{2r} + \tau\theta\omega\frac{F''\left(1-\beta\right)}{2r} \right\} \geq 0\,,$$

i.e. the value of $J'\left(t\right)$ is nonnegative when $t\downarrow 0$. If $w>0$, $\lim_{t\downarrow 0} J'\left(t\right)\to\infty$. The value may be zero when *w=0,* but by (1) it approaches zero from above. Let t_1 denote the lowest level of t implying $J'\left(t\right)=0$. In the following we apply the definitions

$$\sigma\left(t\right)\equiv J''\left(t\right)\Big|_{J'(t)=0} = \left\{\left[p+(1-\beta)\tau\theta\omega\right]F''\left(t\right) - r\left(p-\tau\theta\omega\beta\right)F'\left(t\right)\right\},$$
$$\phi\left(t\right)\equiv F''\left(t\right)/F'\left(t\right)+r\left(\tau\theta\omega\beta-p\right)/\left[p+\left(1-\beta\right)\tau\theta\omega\right].$$

A) Assume $F'\left(t_1\right)>0$. The $sign\left[\sigma\left(t_1\right)\right]=sign\left[\phi\left(t_1\right)\right]$ and is nonpositive. By *(A1)*, $\phi'\left(t\right)<0$. This rules out any finite $t_2>t_1$ with $J'(t_2)=0$ when F is monotonic. When $t\to\infty$, it follows that $y\left(t\right)\to -r(\gamma+\varphi)$. Given monotonic F and $-r(\gamma+\varphi)\geq 0$, finite optimal rotations are ruled out by an equivalent uniqueness argument as above, implying that the optimal rotation is infinite and unique. Thus, given monotonic *F,* optimal rotation is finite if $\varphi+\gamma>0$ and infinite if $\varphi+\gamma\leq 0$ and always unique.

B) Continue assuming that $F'(t_1)>0$. Suppose finite t_2 such that $t_1<t_2$ with $J'\left(t_2\right)=0$ and $F'\left(t_2\right)<0$. When $F'\left(t\right)<0$, the $sign\left[\sigma\left(t\right)\right]=sign\left[-\phi\left(t\right)\right]$. By $-\phi\left(t_2\right)\geq 0$ and $-\phi'(t)>0$, no t_3 with $J'\left(t_3\right)=0$ and $\sigma\left(t_3\right)\leq 0$ exists. Suppose $F'\left(t_2\right)=0$. This implies $\sigma\left(t_2\right)<0$ and a contradiction with $\sigma\left(t_1\right)\leq 0$. Thus, given $F'\left(t_1\right)>0$, at most one finite locally optimal solution exists with the locally optimal infinite solution that, however, exists iff $\gamma+\varphi\leq 0$.

Assume $F'(t_1)<0$, implying that $sign\left[\sigma(t_1)\right]=sign\left[-\phi(t_1)\right]\le 0$. As $-\phi'(t)>0$, a t_2 exists with $J'(t_2)=0$ and $\sigma(t_2)\ge 0$ iff $\gamma+\varphi\le 0$ but no finite $t_3>t_2$ with $J'(t_3)=0$.

Assume $F'(t_1)=0$, implying $F''<0$ and $\sigma(t_1)<0$ (by A1). As $-\phi'(t)>0$ for $t>t_1=\tilde{t}$, a t_2 with $J'(t_2)=0$ and $\sigma(t_2)\ge 0$ exists iff $\gamma+\varphi\le 0$ but no finite $t_3>t_2$ with $J'(t_3)=0$.

Thus, given nonmonotonic volume development, no more than one locally optimal finite solution exists together with the possible locally optimal solution with t being infinitely high. □

Appendix 2, Proof of Proposition 2

Set $\tau\theta\omega=1$ and differentiate $y_c(t_c)$ in (4) w.r.t. α:

$$\frac{\partial y_c(t_c)}{\partial\alpha}=-\frac{r}{(\alpha+r)^2}\left[F'(t)-r\frac{F(t)}{1-e^{-rt}}\right].$$

When $F'(t_c)\le 0$, the positive sign of this derivative follows directly. By the optimality condition (4) $F'(t)=r\left[\int_0^t F'(s)e^{-rs}ds-\beta F(t)\right]\left[\left(1-e^{-rt}\right)(1-\beta)\right]^{-1}$. Substituting this with $\partial y_c/\partial\alpha$ and rearranging gives

$$\frac{\partial y_c}{\partial\alpha}\Big|_{y_c=0}=-\frac{r}{\alpha+r}\frac{\int_0^t F'(s)e^{-rs}ds-F(t)}{1-e^{-rt}}>0\,.$$

As $\partial y_c(t_c)/\partial\alpha>0$ and $J_c''(t_c)<0$ by $J''(t_c)\ne 0$ and Proposition 1, the implicit function theorem implies that t_c is an increasing function of α. □

Appendix 3, proof of Proposition 3

Set $\tau\theta\omega=1$ and write the optimality condition (4) for any finite t_c as

$$\lambda \equiv y_c\left(1-e^{-rt}\right) = F'(t)\left(1-\beta\right)\left(1-e^{-rt}\right) - r\int_0^t F'(s)e^{-rs}ds + r\beta F(t) = 0\,.$$

Differentiation of λ with respect to (w.r.t) r yields

$$\frac{\partial\lambda}{\partial r} = F'(t)\left[te^{-rt} - \beta te^{-rt} + \left(1-e^{-rt}\right)\beta^2/\alpha\right] - \int_0^t F'(s)e^{-rs}ds + r\int_0^t F'(s)se^{-rs}ds + F(t)\beta^2.$$

By the optimality condition $F'(t)=r\left[\int_0^t F'(s)e^{-rs}ds-\beta F(t)\right]\left[\left(1-e^{-rt}\right)\left(1-\beta\right)\right]^{-1}$. Substitute this into $\partial\lambda/\partial r$:

$$\frac{\partial\lambda}{\partial r}\Big|_{y_c=0} = rte^{-rt}\,\frac{\int_0^t F'(s)e^{-rs}ds - \beta F(t)}{1-e^{-rt}} + r\int_0^t F'(s)se^{-rs}ds - \int_0^t F'(s)e^{-rs}ds\left(1-\beta\right). \qquad (3.1)$$

Note that the value of (3.1) approaches zero as $t\downarrow 0$. To clarify the sign of (3.1) compute

$$\frac{\partial\lambda^2}{\partial r\partial t}\Big|_{y_c=0} = \frac{\left(rt+e^{-rt}-1\right)e^{-rt}}{1-e^{-rt}}\left\{F'(t)\left(1-\beta\right) - r\left[\int_0^t F'(s)e^{-rs}ds - \beta F(t)\right]\left(1-e^{-rt}\right)^{-1}\right\} = 0. \qquad (3.2)$$

The term $rt+e^{-rt}-1$ is positive with $t>0$ and the term in (curly) brackets is zero by the first-order optimality condition, i.e. $y_c=0$. As $y_c>0$ for $0<t<t_c$, (3.2) shows that (3.1) is positive when $t=t_c$, which by $J_c''\left(t_c\right)\neq 0$ and the implicit function theorem implies that t_c is an increasing function of r. □

Appendix 4, proof of Proposition 6.

By (2) $J_f'(t)=0$ implies $F'(t)=[rF(t)-rw/p]/(1-e^{-rt})$. Thus, $t=t_f$ implies $y_f+y_c=\kappa\tau\theta\alpha r/(1-e^{-rt_f})$, where $\kappa=F(t_f)-\dfrac{w}{p}(1-\beta)-\int_0^{t_f}F'(s)e^{-rs}ds$. When $w=0$ or $\beta=1$, we obtain $\kappa>0$, implying by finite and uniqueness t^* that $t^*>t_f$. Thus, in Equations (2)–(4) $y_f(t^*)<0$ and $y_c(t^*)>0$, implying, by the implicit function theorem $J''(t^*)<0$ and $\partial J'(t^*)/\partial\tau>0$, that $\partial t^*/\partial\tau>0$.

When $w \uparrow p\hat{F}$, it follows by Proposition 1 that $t_f \to \infty$ and $\kappa \to \hat{F}\beta - \int_0^\infty F'(s)e^{-rs}ds < 0$ when β is low enough. Thus, by finite and uniqueness t^* it holds that $t^* < t_f$ and in Equations (2)–(4) $y_f(t^*) > 0$ and $y_c(t^*) < 0$. By the implicit function theorem, $J''(t^*) < 0$ and $\partial J'(t^*)/\partial\tau < 0$, we obtain $\partial t^* / \partial\tau < 0$. □

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