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Commitment and Conflict in Multilateral Bargaining *

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Abstract

We theoretically investigate the effects of strategic pre-commitment in multilateral dynamic bargaining. Each round features a commitment stage in which players can declare that they will reject any proposal giving them less than a self-imposed threshold. Such declarations bind in the ensuing voting stage with an exogenously given probability. We characterize the set of Markov perfect equilibria under all q-majority rules. Under unanimity rule, an inefficient equilibrium always exists, and efficient equilibria exist only if the probability that commitments bind is sufficiently large and the number of players is sufficiently small. Under any (super)majority rule, every equilibrium is efficient. The results suggest that the unanimity rule is particularly damaging if the number of legislators is large and the time lags between consecutive sessions are long.

Keywords: bargaining, commitment, conflict, delay, environmental agreements, international negotiations, legislative, majority, multilateral, unanimity

JEL Codes: C7, D7

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1 Introduction

"The unanimity rule has meant that some key proposals for growth, competitiveness and tax fairness in the Single Market have been blocked for years." (European Commission press release, Jan 15th 2019).

A fundamental constitutional choice facing organizations such as the European Union (EU) concerns the decision rules to be used in bodies such as the European Council of Ministers. Many organizations, including the EU, require the *unanimous* consent of all members at least for certain types of decisions. This rule has the important advantage that it ensures that all decisions reached constitute Pareto improvements over the status quo. However, politicians and expert observers alike have often complained that unanimity rule is responsible for inefficient delays and even gridlock. Prominent EU officials, including former and current Commission presidents Jean-Claude Juncker and Ursula von der Leyen, have therefore proposed the expanded use of qualified majority rule. The call for such a reform seems to have gained momentum as the number of member states has grown. Other international organizations, including the World Trade Organization (WTO), The United Nations Framework Convention on Climate Change (UNFCCC), and the North Atlantic Treaty Organization (NATO), face similar challenges.[1]

Despite this public debate, there is little game-theoretic literature formally investigating the potential for inefficient delay under unanimity vs. qualified majority rule.[2] The goal of the present paper is to take some steps towards understanding the potential for unanimity rule to cause inefficient delay. We therefore consider a setting

[1]The WTO was not able to meet the December 2002 deadline imposed in paragraph 6 of the Doha declaration on the TRIPS agreement since a single member prevented consensus (Ehlermann and Ehring, 2005). UNFCCC has not been able to reach a comprehensive and binding agreement on how to limit carbondioxide emissions (Pizer, 2006; Nordhaus, 2006; Stern, 2008; Gollier et al., 2015), rather less binding forms of agreements have been adopted. The most recent example of what appears to be consistent with aggressive commitment tactics is the public announcement of the Turkish president Tayyip Erdogan prior to the potential NATO membership bid by Finland and Sweden. Expert obervers interpret this as an attempt to extract concessions from the U.S. and other NATO members (https://www.bloomberg.com/news/articles/2022-05-17/what-turkey-wants-from-sweden-and-finland-in-nato-expansion-spat).

[2]Buchanan and Tullock (1965) investigate the constitutional choice among q-majority rule in a model that *assumes* that "decision costs" increase in q - as well as n. Discussing the challenges of climate negotiations, Weitzman (2019) suggested that "one could could try to argue that, other things being equal, transactions costs increase at least proportionally with the number of parties n," noting that an explicit formalization of this intuition is lacking.

where immediate agreement would be efficient. In addition to unanimity rule, we investigate the potential efficiency properties of qualified majority rules. Our model builds on the seminal Baron and Ferejohn (1989) model of legislative bargaining, to which it adds a capacity to precommit (Schelling, 1956; Crawford, 1982). In particular, we add a commitment stage at the beginning of each round of negotiations. At this stage, each player can attempt to commit to rejecting, in the subsequent bargaining stage, any proposal where she receives less than a self-imposed threshold. Between the commitment and bargaining stages, any such attempted commitment fails with an exogenously given probability.[3] The commitment status is assumed common knowledge at the bargaining stage. Thus, we pursue a complete information explanation for conflict. Once commitment attempts have been made and their success determined, one of the players is randomly drawn to make a proposal to which all others respond by either accepting or rejecting the offer. Agreement arises if, according to an exogenously given consent rule, sufficiently many players give their consent to the proposal.

We find that, under the unanimity rule, an inefficient (stationary) Markov perfect equilibrium (MPE) with delay always exists. Under a wide range of relevant parameter values, every MPE is inefficient. Moreover, as the number of players grows larger, delay and inefficiency become more severe. In contrast, there can be no delay or inefficiency in any stationary equilibrium under any less-than-unanimity rule. Giving up unanimity and instead requiring all but one party to agree is enough to restore full efficiency in our model. The mechanism underlying this result is that players compete for being included into the winning coalition.

Our model highlights the role of aggressive precommitment as a mechanism contributing to inefficient delay in unanimity decision making. In seminal work, Buchanan and Tullock (1965) posited a fundamental tradeoff between what they called the "external costs" and the "decision costs" associated with a given q-majority rule. The larger is the required majority, the smaller is the chance that a given individual will be harmed by a collective decision. Conversely, Buchanan and Tullock conjectured that more demanding majority requirements will be associated with greater expected "decision costs" in the form of delay and possibly gridlock. Our modeling approach

[3] We adopt the simplest such model where each player's probability of failing is independent of the failures of the commitment attempts of other players and, moreover, each individual commitment attempt has an equal chance of failing. Also, for the sake of simplicity, we assume that commitments need to be re-established at the commitment stage of each round.

provides a relatively simple game-theoretic micro-foundation for the latter conjecture. By design, we do not investigate external costs, and abstract from complexities generated by cross-externalities, asymmetric and incomplete information, and the shadow of alternative treaties (See Harstad, 2012, for instance). We would argue that most multilateral negotiations involve an important distributive element, and abstracting from the other dimensions allows for a tractable and transparent framework to understand the roles played by (i) commitment, (ii) decision rule, and (iii) number of parties involved on what Buchanan and Tullock coined the decision costs. It thereby contributes to a deeper understanding of such inefficiencies and how they might be reduced by moving to qualified majority rule. Surprisingly, our analysis suggests that decision costs may change discontinuously when abandoning unanimity rule in favor of any supermajority rule. We also extend the model to allow for asymmetries in commitment success probabilities and discount factors.

A key lesson in the existing multilateral bargaining literature is that when players' valuations are heterogeneous, those who need to be compensated the least will be included in the winning coalition, while more "expensive" players are excluded. This theoretical idea receives empirical support in the experiment by Miller et al. (2018). From the responder perspective, this also suggests that mispresenting private valuations (Tsai and Yang, 2010; Eraslan and Chen, 2014), or sending delegates with induced valuations higher or lower than those of the principal (Harstad, 2010) may pay off individually in these settings if there is not enough competition. The puzzle is to understand how the incentive in doing so depends on the decision rule and other players' decisions. Relatedly, Manzini and Mariotti (2005) show that when an alliance decides on how aggressive a delegate to send to bilateral negotiations, the delegate will be more aggressive when the alliance decides by unanimity than by majority. Negotiations in both stages are efficient, however. Our paper studies multilateral negotiations rather than bilateral ones and shows that an aggressive commitment is optimal under unanimity when success of commitments are stochastic, and that optimal commitments unavoidably lead to ineffiency.

Among bilateral bargaining models where players may build a reputation for obstinacy (e.g. Myerson, 1991; Abreu and Gul, 2000; Compte and Jehiel, 2002; Kambe, 1999; Fanning and Wolitzky, 2022), some (but not all) predict inefficient delay.[4] No-

[4]Myerson (1991) introduces obstinate types in a reputational model of bargaining and illustrates how delay occurs to screen the true type. In Kambe (1999), agreement may be immediate even if

tice that such models effectively assume unanimity rule. While these models have generated interesting insights and have been widely applied, they quickly become intractable when extended to n-player multilateral settings.[5] An exception is Ma (2021), who analyzes the three-player case with majority rule. He shows that there exists an equilibrium with efficient outcomes and that player's equilibrium payoff in that equilibrium is non-monotone in the probability of obstinacy.

In our model, the success of the commitment is not based on an attempt to mimic a behavioral type. Rather, the commitment technology is exogenously given. Our modeling approach is thus admittedly simpler. In a multilateral bargaining model with obstinate types, each player would have to track the beliefs about the obstinacy of each of the other players and these would have to be updated both based on the proposals and the rejections made. The optimal actions would then depend on these beliefs. Thus, the dimensionality of the model grows exponentially with the number of players. Although the simplicity of the present complete information commitment model abstracts from asymmetric information aspects of commitment, it is the simplicity that makes the model scalable from a bilateral to a multilateral bargaining setting and allows to compare general decision rules and analyze how frictions are affected by the number of parties.

The origins of both reputational bargaining and the present approach can be traced back to Schelling (1956), who argued informally that being committed to an aggressive bargaining position can be advantageous within negotiations, and discussed several means by which such pre-commitment could be achieved. One example is that a negotiating party may make its initial bargaining position public, such that it would incur a prohibitive cost if it were to back down from this position. Another example involves sending delegates who have limited and observable mandates to agree only on certain terms. In both cases, the announced bargaining position will be based on justifications of some form. Exogenous events may undermine these justifications in the interim, giving the bargainers an "excuse" to reconsider the position. For example, in climate negotiations, the basis for the bargaining position or mandate could be that

negotiators are obstinate with a positive probability. In Abreu & Gul (2000), delay occurs in the continuous time limit when players are obstinate with a strictly positive probability. See Fanning & Wolitzky (2022) for a recent review.

[5] One would have to keep track of the beliefs of players who differ in terms of what they have proposed/rejected and how often. The dimension of the space of beliefs on which strategies would potentially be conditioned grows exponetially with the number of players.

measures to combat the climate change in each country should be proportional to per capita net emissions. Suppose that between the date when the commitment position goes public and the following COP meeting starts, new scientific evidence is published that net emissions per capita in the respective country turn out much higher than expected. Then the room of manouvre suddenly expands for the negotiating party and the commitment fails.

Crawford (1982) formalizes some of Schelling's arguments in a bilateral bargaining framework with both strategic ex-ante pre-commitment and ex-post revoking of commitments.[6] He shows that with sufficiently low success probability of individual pre-commitments, both players make aggressive pre-commitments in the unique equilibrium, which is inefficient since the commitments are mutually incompatible.[7] Ellingsen and Miettinen (2008) show that impasse may be considerably more likely and inefficiency more severe if there is a small cost of commitment. Ellingsen and Miettinen (2014) generalize the results to a dynamic infinite horizon setting. We extend Ellingsen and Miettinen (2014) to the multilateral case and compare the performance of various decision rules.[8]

Complete information models of rational multilateral bargaining, which build upon the seminal model by Baron and Ferejohn (1989), typically predict immediate agreement without delay in all stationary equilibria.[9] This is true whether unanimity or any type of majority is required, and independently of the number of players (Banks and Duggan, 2000; Eraslan and Evdokimov, 2019). Compte and Jehiel (2010) study

[6] For various modeling approaches, see Crawford (1982); Muthoo (1996); Myerson (1991); Fearon (1994, 1997); Levenotoğlu and Tarar (2005); Jackson and Morelli (2007); Güth et al. (2004) for instance. Subtle differences in assumptions about the way commitment is built and how it may be lost, inclusive the cost of establishing and revoking commitments, lead to differences in predicted outcomes. See Muthoo (1999) and Miettinen (2022) for reviews.

[7] Putnam (1988) suggests informally that a two-level game framework should be used to study interdependencies between national and international politics. He suggests that "one effective way to demonstrate commitment to a position in [international] bargaining is to rally support from one's constituents" [at the national level] and that "such tactics may have irreversible effects on the constituents' attitudes, hampering subsequent ratification of a compromise agreement." (1988, p.450) A recent bilateral model which captures some of Putnam's intuition is provided by Basak and Deb (2020) where concession costs depend on the political support, which in turn depends on the stochastic realization of an underlying state of nature.

[8] Extending the model to $n > 2$ players also necessitates a slight modification of the commitment technology, as discussed below.

[9] An exception is Wolitzky (2023) where imperfectly observed offers or claims result in inefficient conflict with positive probability. It is a moral hazard explanation of conflict in that non-observable or -verifiable actions result in inefficiency.

the effect of the voting rule on the division of real voting power between the parties and find, in the single-dimensional case, that unanimity shifts effective power away from the proposer to the parties with most demanding positions. Their model is simlar to ours in that there is complete information about the positions. However, these are exogenously given, and their model does not feature any inefficiencies. Majority decision making can be more efficient for the sake of its information aggregation properties (See Bouton et al., 2018); our paper highlights the fact that (super)majority decision making promotes efficiency even when information is complete if players are capable of tying their hands to force concessions from others. [10]

In addition to the present paper, there are other complete information explanations for delay in multilateral settings. Efficient delay and inefficient immediate agreements may arise due to fluctuations over time in the total surplus which is being shared (Merlo and Wilson, 1995, 1998). Eraslan and Merlo (2002) show in bargaining with a stochastic surplus that unanimity rule is always efficient but majority rule may lead to inefficiencies: a proposer may be better off buying a majority into an inefficient agreement than passing and waiting for an efficient realization of the pie. Agranov et al. (2020) find experimental support for the theory. In non-stochastic environments and focusing on stationary equilibria, inefficient delay may arise if the principal or proposer negotiates with others sequentially one at a time or in smaller groups (Cai, 2000; Iaryzcower and Oliveros, 2019) or if there are several simultaneous offers at each round and thus free-riding among proposers (Kosterina, 2019). Yildirim (2007, 2018) show that there can be inefficiencies due to endogenous recognition probabilities. Ali (2006) generalizes the analysis Yildiz (2003) to multilateral settings and shows that, unlike in bilateral negotiations, persistent optimism may lead to delay in multilateral settings if unanimity decision making is applied.

The paper is structured as follows. Section 2 presents the model. Section 3 analyzes the simplest multilateral case of three players. Section 4 presents the general analysis and the main results. Section 5 concludes.

[10] As in our analysis, these models assume that immediate agreement is efficient. Some authors have analyzed settings in which delay can be efficient, in which case majority rule may lead to inefficient early agreement such that unanimty becomes the more efficient rule. An example is Model 6 in Banks and Duggan (2006) as well as Eraslan and Merlo (2017), discussed below.

2 Model

The negotiation game involves $n > 2$ players and takes place in discrete time with infinite horizon. In each period $t \in \{1, 2, ...\}$, actions are taken in two stages – the *commitment stage* and the *bargaining stage*. At the commitment stage, players can attempt to make short-lived commitments which last at most the current period.[11] That is, each player i chooses a *commitment attempt* $x_i \in [0, 1]$, where $x_i = 0$ (or indeed any value below a player's continuation payoff) can be interpreted as a choice not to commit. In between the two stages, each player's commitment attempt may fail independently with probability $1-\rho$. With a nod to Schelling's (e.g. 1960, p. 40, 1966, p. 44) original contributions, we will say that a player whose commitment attempt fails "has a *loophole*". The probability that a commitment attempt is "successful" is ρ. The realization of the attempt, the *commitment status*, is denoted by s_i and equals x_i with probabiliy ρ and 0 with probability $1 - \rho$.

At the bargaining stage, each player becomes the proposer with probability $1/n$, in which case that player's commitment, if successful, loses its strength.[12] With probability $(n-1)/n$, a player becomes a responder. The proposer proposes a deal $d = (d_1, ..., d_N)$, with $\sum_{i=1}^{N} d_i \leq 1$, where we refer to d_i as the "offer" made to player i. Each player then votes to *accept* or *reject*. Yet, any responder i with commitment status $s_i > d_i$ will automatically reject the proposal. The proposed deal is implemented if at least q players (including the proposer) vote to accept. If not, a new period begins with the commitment stage. If a deal d is implemented in period t, player i's payoff equals $\delta^{t-1} d_i$. Players are impatient, with (common) discount factor $\delta \in (0, 1)$.

Our equilibrium concept is (stationary) Markov Perfect Equilibrium. A stationary *Markov strategy* for player i, denoted ψ_i, specifies a commitment attempt x_i chosen at the commitment stage in any period, a (possibly mixed) proposal strategy for any realized commitment status profile $s = (s_1, ..., s_n)$, and an "accept" / "reject" action given any proposed deal d (the latter being relevant only at player i nodes

[11] In this respect, our model differs from Ellingsen and Miettinen (2014), which involves persistent commitments in a bilateral context. Limiting persistence to at most one period is necessary to make the model tractable for larger n, as otherwise the dimensionality of the state space on which even stationary strategies would be defined grows exponentially.

[12] This assumption significantly simplifies the analysis, and increases the chances for the existence of an efficient equlibrium. Note that commitments can be strategically valuable only in the responder role, hence no player would wish to extend the commitment technology to be binding on proposers. See Appendix A.5 for a robustness exercise in which we relax this assumption and show that our main results are unaffected.

where $s_i \leq d_i$). A (stationary) Markov Perfect Equilibrium (MPE) is a collection of stationary Markov strategies $\psi^* = (\psi_1^*, ..., \psi_n^*)$ which induce a Nash Equilibrium after every history. As is common in the literature on Baron-Ferejohn bargaining, we assume that players do not use weakly dominated strategies at the voting stage.

Lemma 1. *Let $(x_1^*, ..., x_n^*)$ be the vector of commitment attempts that is part of a MPE strategy profile ψ^*. And let $(v_1^*, ..., v_n^*)$ be the vector of expected utilities associated with that equilibrium. Then the equilibrium strategies in the* bargaining stage*, given any commitment status profile $s = (s_1, ..., s_n)$ satisfy the following conditions:*

1. *Responder i votes to accept iff she is offered at least $d_i \geq \hat{x}_i(s_i) \equiv max\{\delta v_i^*, s_i\}$.*
2. *Let C_i be the "cheapest coalition" consisting of $q-1$ responders other than i. If $\delta v_i \leq 1 - \sum_{j \in C_i} \hat{x}_j(s_j)$, then when player i proposes, he offers $\hat{x}_j(s_j)$ to each member j of C_i, the residual to himself, and votes to accept. (If there are multiple "cheapest" coalitions, he may randomize between them.) Otherwise, he makes a proposal that fails.*

Define $\pi_i(x|\psi^*)$ as the expected utility that would *be achieved by player i if, at the initial commitment stage, (1) players attempted committing to $x = (x_1, ..., x_n)$ and (2) all players followed the equilibrium strategy ψ^* starting at the immediately ensuing bargaining stage.*[13] *Then the commitment attempts x_i^* satisfy the following:*

$$x_i^* \in \arg\max \pi_i(x_i, x_{-i}^*|\psi^*)$$

Finally, the equilibrium expected utilites v_i^* satisfy

$$v_i^* = \pi_i(x^*|\psi^*).$$

Proof. See Appendix **A.1**. □

[13] This payoff is pinned down by the conditions already outlined for the bargaining stage and by the additional condition that, in case no deal is made, player i's expected utility is given by δv_i^*. (The formal details are cumbersome to express explicitly but will be clearly developed in the subsequent analysis.)

3 The case of three players

In this section, we illustrate our main results within a simple three-player model, focusing on *symmetric* equilibria in which all players make the same commitment attempt x^*. A more general analysis for n players and asymmetric equilibria is presented in Section 4. We characterize equilibria under majority and unanimity rule. In the three player game, commitment attempts can conceivably be of two types: A "moderate" commitment is such that a deal can be reached even if both responders' commitment attempts are successful, implying immediate agreement and efficiency. An "aggressive" commitment is such that a deal is possible only if (at least) one responder has a loophole, implying a positive probability of inefficient delay.

Subsection 3.1 considers majority rule and shows that all symmetric equilibria are efficient. In subsection 3.2, we show that under unanimity rule, an inefficient symmetric equilibrium with "aggressive" commitments always exists. Finally, subsection 3.3 shows that an efficient equilibrium with "moderate" commitments exists under unanimity rule only if the probability that commitments stick (ρ) is large enough.

3.1 Efficiency of majority rule

Consider the case of three players and majority rule, i.e. $q = 2$. It is easy to see that a commitment profile in which no player attempts to commit constitutes an equilibrium. In such an equilibrium, the common expected equilibrium payoff is $v^* = 1/3$. Due to stationarity, the common continuation value is δv^*. Any player who deviates to a commitment above δv^* would simply be left out of any coalition. And as long as the other players remain uncommitted, agreement is immediate. Thus such a deviation cannot pay off.

Indeed, this equilibrium is unique. To see this, suppose first that there exists a symmetric equilibrium in which all players make an "aggressive" commitment attempt $x^* > 1 - \delta v^*$, where again v^* is the common equilibrium payoff. Then a deal occurs only if at least one responder has a loophole, and it cannot include a committed responder. Conditional on being responder and committed, a player's payoff is either zero (when the other responder has a loophole and a deal is made) or δv^* (when the other responder is also committed and no deal is made). Then a deviation to $y = 1 - \delta v^*$ pays off, as the deviator will be paid $y > \delta v^*$ whenever he is responder and both commitments stick, and his payoff is unaffected in all other events - a contradiction. Therefore $x^* \leq 1 - \delta v^*$, implying immediate agreement and efficiency.

Pushing on, suppose there exists an efficient symmetric equilibrium with commitment $x^* > \delta v^*$. Then a committed responder is included with positive probability only in the event where both commitments stick. At least one player is included with probability less than one in such events. Then an arbitrarily small downward deviation to $x^* - \epsilon$ results in her being included with probability one, and has no effect in other events - a contradiction. It follows that any symmetric equilibrium involves $x^* \leq \delta v^*$, equivalent to not committing, and thus outcome-equivalent to the standard Baron-Ferejohn framework.

Proposition 1. *In the three-player game under majority rule, there exists a unique symmetric equilibrium involving no commitments.*

3.2 Aggressive equilibrium under unanimity rule

Next, consider unanimity rule, i.e. $q = n = 3$. Suppose that a symmetric stationary equilibrium with aggressive commitments exists, and denote the (common) expected equilibrium payoff by v^*. Due to stationarity, each player's continuation value is δv^*. An aggressive commitment is such that $1 - 2x^* < \delta v^*$, meaning that the proposer would not be willing to pay both responders if both commitments succeed. Thus, an aggressive commitment is "targeted" to the event that *one* responder ("the other") has a loophole. The largest commitment to which a proposer will concede in that event is given by

$$x^* = 1 - 2\delta v^*. \tag{1}$$

This commitment leaves both the proposer and the uncommitted responder indifferent between a deal and continuation: they each receive the continuation value δv^*, and the player who is the only one to succeed with her commitment receives the residual.

Given that all players commit to x^*, three things can happen with positive probability in a given round. Suppose without loss of generality that player 1 is proposer. If one responder has a loophole (say, player 3) then player 1 proposes $d = (\delta v^*, x^*, \delta v^*)$. If instead *both* responders' commitments fail, player 1 proposes $d = (1-2\delta v^*, \delta v^*, \delta v^*)$. Note that we can then write $d = (\delta v^* + (x^* - \delta v^*), \delta v^*, \delta v^*)$, reflecting the fact that, in the event of an "extra" loophole (beyond the minimum number required for agreement), the proposer secures an additional "chunk" of size $(x^* - \delta v^*)$. We will encounter this type of logic again in the more general analysis. Finally, if both responders are committed, player 1 makes any proposal where at least one responder is offered less than the committed share and thus the game proceeds to round $t+1$. Thus, the equilibrium involves a positive probability of inefficient delay.

Concretely, the probability of agreement in a given round is $1-\rho^2$. Conditional on agreement, the sum of payments is one. Therefore, the expected discounted sum of utilities is $\sum_i v^* = \frac{1-\rho^2}{1-\delta\rho^2}$, and so the expected equilibrium payoff for each player is given by

$$v^* = \frac{1-\rho^2}{3(1-\delta\rho^2)}$$

Note that v^* is strictly less than $1/3$, reflecting the inefficiency. Equilibrium payoffs increase, and commitments become less aggressive, if players are more patient (δ increases) or if they are less likely to succeed with their commitments (ρ decreases).

The following proposition establishes that this equilibrium always exists in the three-player game under unanimity rule.

Proposition 2. *In the three-player game under unanimity rule, there always exists a symmetric MPE with aggressive commitments and delay.*

To prove the proposition, let us verify that the commitment profile (1) constitutes part of an equilibrium. By construction, if all players attempt commitment $x^* = 1 - 2\delta v^*$, there can be no beneficial deviations at the bargaining stage. Consider then deviations at the commitment stage.

A commitment matters only if the player is drawn to respond (recall that the commitment is automatically relaxed if one is drawn to propose). A deviation to not commiting ($x_i = 0$) would imply that player i will definitely receive δv^* as responder. Sticking to x^* yields a strictly larger payoff with positive probability, namely when the other responder's commitment attempt fails. Thus, the deviation does not pay off. Deviating to a *more* aggressive commitment does not pay off either, since no proposer would ever concede when such a commitment sticks. A deviation down to a *less* aggressive commitment can only pay off if it increases the probability of a deal in the ensuing bargaining stage. In equilibrium, the only event in which no deal is reached is when both responders remain committed. In order to induce a deal in this event, a deviating responder would have to choose a less aggressive commitment position y such that $\delta v^* \leq 1 - x^* - y$. Therefore, any deviation that increases the probability of a deal satisfies $y \leq 1 - \delta v^* - x^* = \delta v^*$. Thus, such a deviation does not pay off.

3.3 Efficient equilibrium under unanimity rule

Suppose that there exists a symmetric equilibrium with "moderate" commitments, such that agreement occurs even if both responders' commitments "stick". As above, denote the (common) expected equilibrium payoff by v^*. (Clearly, $v^* = 1/3$.) Then the largest commitment to which a proposer will concede in the event that both responders' commitments "stick" is given by

$$x^* = \frac{1 - \delta v^*}{2}. \tag{2}$$

This commitment is targeted to leave the uncommitted proposer indifferent between a deal and continuation while allowing two reponders who succeed to each have their commiment shares. Since $x^* - \delta v^* = (1 - 3\delta v^*)/2 \geq 0$, each responder's commitment share is greater than her continuation value δv^*. The proposal where the committed responders receive x^* and the proposer receives δv^* will thus be passed, and the agreement is immediate even in the event that both responders succeed in committing. This equilibrium is thus efficient. The following proposition establishes that this equilibrium exists if and only if $\rho \geq 1/2$.[14]

Proposition 3. *In the three-player game under unanimity rule, an efficient equilibrium exists if and only if* $\rho \geq 1/2$.

To understand why an efficient equilibrium may fail to exist, note that if either (or both) commitment attempts fail in such an equilibrium, the proposer offers each uncommitted responder the continuation value, thus allowing her to keep

[14]Assuming that the proposer always has a loophole significantly simplifies the analysis. One might wonder whether it biases results for or against the existence of an efficient equlibrium. For $n = 3$ and unanimity rule, it can be shown that an alternative model in which proposer loopholes occur randomly admits an efficient equilibrium if and only if $\rho \geq \sqrt{2/5} \approx .63 > \frac{1}{2}$. That is, the condition for the existence of an efficient equilibrium is more demanding in such a model than in the version we are analyzing.

$\delta v^* + k(x^* - \delta v^*)$, where k is the number of responders with a loophole. The fact that there are positive probability events where the proposer receives more than her continuation value creates an opportunity to deviate upwards at the commitment stage. Specifically, players may increase their commitment in an attempt to appropriate the extra "chunk" of size $(x^* - \delta v^*)$ that otherwise accrues to the proposer in the event that their commitment succeeds and the other responder has a loophole. Such a deviation to $y = x^* + (x^* - \delta v^*)$ will leave the proposer indifferent between continuation and passage in the event that one responder has a loophole, i.e. it satisfies $\delta v^* = 1 - y - \delta v^*$. As before, the deviation matters only if the deviating player is drawn to respond and the commitment succeeds. Conditional on this event, the deviating player will *gain* $(x^* - \delta v^*)$ in the event that the other responder has a loophole (probability ρ), and *lose* $(x^* - \delta v^*)$ in case the other responder does not have a loophole (probability $1 - \rho$). Therefore, this deviation does not pay off (and the efficient equilibrium exists) if and only if $\rho \geq 1/2$.

4 General analysis

We now turn to a more general analysis of the n-player game. Subsection 4.1 shows that for any $q-$majority rule with $q < n$, no inefficienct equilibrium exists. Instead, the (essentially) unique equilibrium involves no commitments above the common continuation value and immediate agreement (Theorem 1).

We will then turn to unanimity rule. In Subsection 4.2, we begin by constructing symmetric commitment profiles, each of which is tailored to achieve agreement only when a certain number of commitment attempts fail. In subsection 4.3, we show that the most aggressive symmetric profile, requiring all but one of the commitments to fail, always constitutes part of a Markov Perfect Equilibrium (Theorem 2).

In subsection 4.4, we show that the *efficient* symmetric profile, which permits agreement even when all commitments succeed, constitutes an equilibrium if and only if ρ is sufficiently large (Lemma 2). Building on this, we prove the more general result, that *all* Markov Perfect Equilibria (symmetric or not) are inefficient if and only if ρ is sufficiently small (Theorem 3).

Given that these results are developed in a discrete time framework, it is of interest to ask whether the results would extend to a continuous time framework with frequent negotiations. In subsection 4.6, we analyze such a version of our model and show that both an efficient equilibrium and inefficient equilibria exist when the length of the time period separating consecutive negotiation rounds tends to zero.

4.1 Effiency of q-majority rules

When majority rule is used, players who commit aggressively may be left out of winning coalitions when successful. It is therefore intuitive to expect Bertrand-like competition in commitments, possibly leading to full efficiency. Although the formal details turn out to be more complicated (see Appendix for a complete proof), this intuition is helpful to understand the following result.

Theorem 1. *Under any q-majority rule with $q < n$, there exists an (essentially) unique MPE in which no player attempts a commitment exceeding his continuation value and agreement is immediate. Expected payoffs are $v_i^* = \frac{1}{n}$ for all i, all players are included with the same probability as responders, and each included responder is paid $\frac{\delta}{n}$.*

The equilibrium is unique in the sense that in any equilibrium, $x_i^* \leq \delta v_i^*$ for each i and all players are included with the same probability as responders. Multiplicity remains only in the sense that each player can choose *any* commitment attempt satisfying $x_i^* \leq \delta v_i^*$, and players could randomize differently over coalition partners when proposing - so long as all players are included with the same probability overall.

The intuition for this result is as follows. Those who attempt to commit too aggressively will be left out of any winning coalition when successful. This implies that their continuation payoff must be lower than that of players whose commitment attempts permit inclusion to some winning coalition with positive probability. Therefore, players with aggressive commitments have room to deviate down to a less aggressive commitment, such that in fact everyone must be included with positive probability. If all responders commit in a way such that they are included with positive probability, at least one will have an incentive to "undercut" another's commitment in order to be included more often. This triggers a Bertrand-like competition in commitments. In equilibrium, no player's commitment can exceed their continuation value. Thus, no aggressive commitments are made. The outcome is efficient and coincides with that of the Baron and Ferejohn (1989) model.

The analysis starting from the next subsection shows that, contrary to supermajority decision making, commitment strategies are used in any equilibrium, and inefficiency due to delay is often unavoidable under unanimity.

4.2 Symmetric commitment profiles under unanimity rule

We now turn to the analysis of unanimity rule. As can be anticipated from the three player example, any symmetric MPE will have the property that there is some number $o \in \{0, ..., n-2\}$ such that at least o responders must have a loophole for an agreement to be reached. The commitment attempt x^* must be such that the proposer will be indifferent between failure and agreement when exactly o responders have a loophole. (Otherwise, any player can benefit by slightly increasing his commitment attempt without affecting the probability of agreement.) That is, $\delta v^* = 1 - (n-1-o)x^* - o\delta v^*$. Furthermore, the expected payoff in a symmetric equilibrium v^* is fully determined by assuming that all players attempt committing to x^* in every period.

These observations together constitute a set of necessary conditions for a symmetric commitment profile to be part of an MPE. We therefore begin by deriving, for any given $o \in \{0, ..., n-2\}$, *candidate* commitments, denoted $x^S(o)$ and associated expected payoffs $v^S(o)$ that satisfy these conditions. Rearranging the condition for proposer indifference, we obtain the candidate

$$x^S(o) = \frac{1 - (o+1)\delta v^S(o)}{n-1-o}, \tag{3}$$

If all players commit to $x^S(o)$, agreement will be reached whenever *at least* o responders have a loophole. It will be useful to establish the following definition.

Definition 1. The probability that *at least* k out of m responders have a loophole is denoted

$$\eta(k, m) = \sum_{l=k}^{m} f(l, m),$$

where $f(l, m) = \begin{pmatrix} m \\ l \end{pmatrix} (1-\rho)^l \rho^{m-l}$ is the pdf of a binomial probability distribution with "success" (i.e. loophole) probability $(1-\rho)$.

Assuming that all players consistently commit to $x^S(o)$ in every period, the probability that agreement is reached at period t is $\eta(o, n-1)(1-\eta(o, n-1))^{(t-1)}$. Thus, the expected total payoff would be $\eta(o,n-1)\sum_{t=1}^{\infty}[(1-\eta(o,n-1))\delta]^{(t-1)}$, and so the expected payoff associated with this candidate profile is given by

$$v^S(o) = \frac{1}{n}\frac{\eta(o,n-1)}{1-\delta\left(1-\eta(o,n-1)\right)}. \tag{4}$$

Note that $v^S(o)$ is decreasing in o, with $v^S(0) = \frac{1}{n}$ and $v^S(o) < 1/n$ for all $o > 0$. Thus, if $x^S(o)$ with $o > 0$ constitutes a symmetric MPE commitment profile, the corresponding equilibrium is inefficient, and the inefficiency is larger the more loopholes are required for a deal to be made. At the same time, $x^S(o)$ increases with o and thus, conditional on succeeding with one's own commitment, the earned share when the deal arises is larger the higher is o. Thus, commitment profiles with higher o generate longer conflict duration, greater inefficiency and greater asymmetries in the shares that the parties receive conditional on reaching an agreement. The symmetric commitment profile candidates can therefore be naturally ordered from the least aggressive and efficient ($o = 0$) to the most aggressive and inefficient ($o = n-2$). Our subsequent analysis will focus on these polar cases. In addition, in Subsection 4.5 we analyse the symmetric equilibria in the intermediate cases, where commitments target h loopholes with $0 < h < n-2$.

As stated at the beginning of this subsection, the candidate profiles were constructed to satisfy the *necessary* condition that the proposer be made indifferent in the event that exactly o responders have a loophole. This implies that no player wishes to deviate at the commitment stage in a way that does not affect the probability of agreement. To verify that a given candidate profile indeed constitutes an MPE, we must additionally verify that no player wishes to engage in *larger* deviations. In the following subsections, we investigate the incentives for such deviations from the least ($o = 0$) and most ($o = n-2$) aggressive candidate profile, respectively.

4.3 Existence of inefficient equilibrium under unanimity rule

Consider the most aggressive symmetric commitment profile, in which all players attempt commitment to

$$x^S(n-2) = 1-(n-1)\delta v^S(n-2).$$

This profile is such that each player aims to extract the entire surplus from agreement (beyond the sum of continuation values) in the case where all other responders have a loophole. If this profile is part of a symmetric MPE, the associated expected payoff equals

$$v^S(n-2) = \frac{1}{n}\frac{\eta(n-2,n-1)}{1-\delta\left(1-\eta(n-2,n-1)\right)},$$

where the probability that an agreement arises in each period, $\eta(n-2,n-1) = (n-1)\rho(1-\rho)^{n-2} + (1-\rho)^{n-1}$ is very small if n and ρ are large. In that case, the expected delay at this profile is long, thereby severely undermining efficiency.

In order to check whether this is an equilibrium, we must verify that no player wishes to deviate at the commitment stage. Note that the agreement requires that there are at least $n-2$ loopholes among the $n-1$ responders. Hence, agreement occurs only in two cases: (i) only one of the commitment attempts succeeds or (ii) none of the commitment attempts succeed. In both cases, $n-1$ agents will get exactly the continuation value, and the residual, $1-(n-1)\delta v^S(n-2)$, is secured either by the committed responder or by the proposer.

To understand the effects of deviations, it will be useful to note that deviating to any $y \neq x^S(n-2)$ affects the deviator's payoff only if (a) she is drawn to respond and (b) her commitment sticks. Therefore, the analysis that follows focuses exclusively on the payoffs achieved in that event.

Following any upward deviation to $y > x^S(n-2)$, the proposer will not want to make a deal even in the most favorable instance where all other responders have a loophole; so a more aggressive commitment cannot be profitable. Consider then a deviation to less aggressive commitments, $y < x^S(n-2)$. Since the payoff achieved conditional on commitment success and agreement would then be lower, such a deviation can only be beneficial if the probability of an agreement is increased. Therefore, a profitable deviation must have the property that the proposer will concede to it in cases where the deviator's own commitment attempt as well as $k \geq 1$ others succeed. The largest commitment that will be met when (at most) $k = 1$ additional responder's commitment sticks is such that the proposer is left with her commitment value after giving $x^S(n-2)$ to one succesful responder, y to the deviator, and $\delta v^S(n-2)$

to the $(n-3)$ other responders, i.e.

$$\delta v^S(n-2) = 1 - x^S(n-2) - y - (n-3)\delta v^S(n-2)$$

But substituting $x^S(n-2) = 1-(n-1)\delta v^S(n-2)$ we see that this boils down to

$$y = \delta v^S(n-2).$$

Therefore in the only event where the commitment matters, the deviator's payoff drops to the continuation value $\delta v^S(n-2)$, whereas if she stays with the equilibrium demand, she will get $x^S(n-2)$ in case all of the other responders have a loophole.

It's clear that the argument can be extended to say that for all $k > 1$, there is also no profitable commitment that would be met if the deviating player's and k additional commitments stick, i.e. deviations of the type

$$y = 1 - kx^S(n-2) - (n-1-k)\delta v^S(n-2),$$

since *a fortiori* these commitments would be strictly *smaller* than the continuation value. We can conclude that there always exists an MPE in which all players attempt the most agressive symmetric commitment $x^* = x^S(n-2)$. This is enough to establish the following theorem.

Theorem 2. *Under unanimity rule, an inefficient equilibrium always exists.*

Before moving on, we comment briefly on the comparative statics of this most aggressive equilibrium. The comparative statics with respect to ρ is obvious: delay increases in ρ. The number of loopholes at a given round among the $n-1$ responders follows the binomial distribution.[15] The number of required loopholes $n-2$ increases with the number of players n. It is thus intuitive that the duration of the conflict increases with the number of players. Moreover, since the duration of conflict increases with n, the expected arrival date of the deal also increases with n and thus the equilibrium payoff, $v^S(n-2)$, and the continuation value, $\delta v^S(n-2)$, which is allocated to each flexible responder when the deal is done, decreases with n. Therefore, the fraction of the pie that the deal allocates to the unique successful player, $x^S(n-2)$,

[15] This approaches the normal distribution with mean $(n-1)(1-\rho)$ and standard deviation $\sqrt{(n-1)(1-\rho)\rho}$ as n tends to infinity.

and thus the difference of the final payoffs, $x^S(n-2)-\delta v^S(n-2)$, increases as the number of players increases.

4.4 Existence of efficient equilibria under unanimity rule

Now consider the opposite extreme, a symmetric commitment profile requiring no loopholes, i.e. $o=0$. If all players commit to this profile, agreement is immediate and thus $v^S(0)=1/n$. Given this, the optimal commitment characterized in equation (3) yields,

$$x^S(0)=\frac{1}{n-1}\left[1-\frac{\delta}{n}\right]. \tag{5}$$

That is, in the event that all responders succeed with their commitment, the $n-1$ responders are sharing what's left after the proposer is permitted to keep $\delta v^S(0)=\frac{\delta}{n}$. But when there are loopholes, the proposer will secure a surplus above $\delta v^S(0)$. In particular, he will earn an extra "chunk" of size $\left(x^S(0)-\delta v^S(0)\right)$ for every responder that has a loophole. The potential availability of these "chunks" implies a potential incentive to engage in more aggressive commitments.

Concretely, consider a deviation to a more aggressive commitment $y>x^S(0)$. Any such deviation will have the property that it will be met only if at least $k\geq 1$ of the *other* responders have a loophole. The most aggressive commitment that will be met in the event that there are exactly k loopholes is

$$y_k=x^S(0)+k\left(x^S(0)-\delta v^S(0)\right). \tag{6}$$

Thus, a player deviating to y_k is increasing his demand by k "chunks" of size $\left(x^S(0)-\delta v^S(0)\right)$, which he is hoping to capture from the proposer in all events where at least k responders have a loophole.

Let us consider the payoff consequences of such a deviation conditional on the deviator's commitment succeeding. In all cases where *fewer* than k of the other $n-2$ responders have a loophole, agreement will fail and the deviating player will *lose* $\left(x^S(0)-\delta v^S(0)\right)$. This occurs with probability $1-\eta(k,n-2)$. In all cases where *at least* k of the other $n-2$ responders have a loophole, the deviating player will *gain* $k\left(x^S(0)-\delta v^S(0)\right)$. This occurs with probability $\eta(k,n-2)$. Therefore a deviation aiming at $k\geq 1$ loopholes pays off if $\eta(k,n-2)\,k\left(x^S(0)-\delta v^S(0)\right)>$

$(1 - \eta(k, n-2)) \left(x^S(0) - \delta v^S(0)\right)$, which boils down to

$$\eta(k, n-2) > \frac{1}{k+1}.$$

It follows that an efficient symmetric equilibrium exists if and only if this condition is violated for every conceivable deviation. That is, if and only if $\eta(k, n-2) \leq \frac{1}{k+1}$ for all $k \in \{1, ..., n-2\}$. Lemma 5 in the Appendix implies that if the condition is satisfied for $k = 1$, then it is satisfied for all $k = \{1, ..., n-2\}$. Thus, the efficient symmetric equilibrium exists *if and only if* the probability of having at least one out of $n-2$ loopholes is at most 1/2, i.e. $1 - \rho^{n-2} \leq 1/2$.[16] Let us denote the minimal ρ which satisfies this existence condition by $\hat{\rho}$. Then we have established the following result regarding the existence of efficient *symmetric* equilibria.

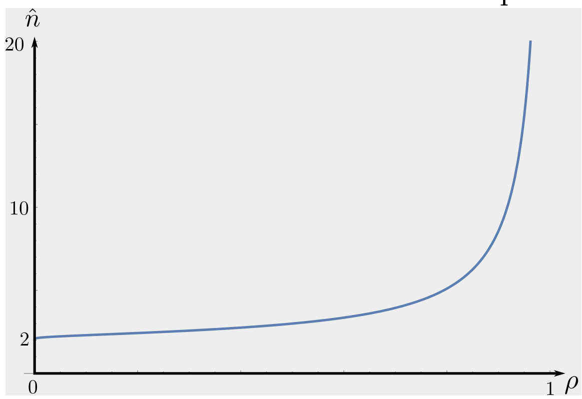
Lemma 2. *An efficient symmetric MPE requiring no loopholes for agreement to be reached exists iff* $\rho \geq \hat{\rho} = (\frac{1}{2})^{1/(n-2)}$.

Note that Lemma 2 refers only to *symmetric* MPE. A natural question to ask is whether efficient *asymmetric* equilibria can exist even when the condition of the Lemma is violated. The answer is no, as can be established by extending part of the logic underlying the Lemma as follows. (For details, see the proof of Theorem 3). Suppose that an efficient equilibrium exists in which players are not identically committed. Order players according to the size of the "chunk" $(x_i^* - \delta v_i^*)$ that becomes available when their own commitment fails. Then consider a deviation by player 1, with the smallest such "chunk", in which he increases his commitment attempt by exactly $(x_1^* - \delta v_1^*)$. As in the case of a symmetric equilibrium, player 1 will then *gain* this extra "chunk" (at least) whenever at least one other responder has a loophole. Conversely, he *may (but need not) lose* that same "chunk" in all other events. Therefore a *sufficient* condition on ρ for this deviation to pay off is exactly the same as for a deviation to y_1 from the symmetric equilibrium. This establishes that the conditon on ρ identified in Lemma 2 is *necessary* for the existence of an efficient equilibrium (symmetric or not). And since Lemma 2 provides a sufficient condition, the following more general result follows.

Theorem 3. *Under unanimity rule, all MPE are inefficient iff* $\rho < \hat{\rho} = \left(\frac{1}{2}\right)^{\frac{1}{n-2}}$.

[16] Notice that the condition for the existence of the efficient equilibrium does not depend on δ, as would be typical for non-stationary equilibria supported by trigger strategies.

Figure 1: Maximum n such that an efficient equilibrium exists



Proof. See Appendix A.4. □

Note that, as n increases, the probability that at least one loophole arises also increases, so that any efficient equilibrium will eventually be destabilized. The condition in the theorem can therefore be equivalently stated in terms of the maximal n for which an efficient equilibrium exists, denoted by $\hat{n} = 2 - \frac{\ln 2}{\ln \rho}$. Note that $\hat{n}$ is increasing in ρ and approaches infinity as ρ tends to 1. This is displayed in Figure 1. For $\rho < \frac{1}{2}$, $\hat{n} < 3$, so an efficient equilibrium does not exist for any $n > 2$. However, as ρ approaches zero, commitments become less relevant. In the limit, the model reduces to the standard Baron-Ferejohn game, and all equilibria are efficient.[17] Remark also that $\hat{n}$ rapidly falls as we move down from ρ close to one. For example, $\hat{n}$ is small (less than 10) even for $\rho = 0.9$.[18]

4.5 Intermediate equilibria under unanimity rule

So far we have found that the most aggressive equilibrium exists independently of the parameter values. The efficient equibrium exists only if the probablity of a loophole or the number of players is sufficiently small. In between these two extremes, additional equilibria requiring an intermediate number of loopholes, $1 \leq o \leq n-3$, may exist. In

[17] The expected payoff in any equilibrium (including the most inefficient) approaches $1/n$ as ρ approaches zero.

[18] One might ask whether the non-existence of efficient equilibria is driven by the assumption that commitment attempts are chosen simultaneously. If instead players moved sequentially in the commitment stage, would efficient equilibria always exist? The answer is no. For $n = 3$, it can be shown that an efficient equilibrium of the sequential move version exists under unanimity rule only if $\rho \geq \frac{1}{2}(9 - \sqrt{65}) \approx 0.47$.

this subsection we will characterize the full set of such equilibria and how it depends on the number of players and the probability of a loophole.

Consider a profile with symmetric commitments characterized in (3) targeting to at least $o \in \{1, n-3\}$ responders with loopholes. As above, we will now study whether and under which conditions this consititutes an equilibrium. In this intermedate case, we need to consider both upward and downward deviations. Both types of deviations affect the deviating player's payoff only if she becomes a responder and her commitment attempt succeeds. So like above we can conduct the analysis conditional on that event.

Let us begin by considering downward deviations, to less aggressive commitments. Such deviations have the property that they may be met even if strictly fewer than o responders have a loophole. The largest commitment y that will be met when there are at least $o-1$ responder loopholes is given by

$$\delta v^S(o) = 1 - y - (n-1-h)x^S(o) - (h-1)\delta v^S(o).$$

If we substitute the expression for $x^S(o)$, we obtain $y = \delta v^S(o)$. Therefore, just as in the most aggressive equilibrium, the deviator's payoff is reduced to the continuation value, and thus a deviation designed to make agreement possible with one fewer loopholes is not profitable. It is clear that the argument can be extended to say that for all $k > n-1-h$, there is also no profitable commitment that would be met when there are even fewer than $h-1$ loopholes, since *a fortiori* these commitments would be strictly *smaller* than the continuation value. This is enough to establish the following lemma.

Lemma 3. *Consider a symmetric commitment profile where all players commit to $x^S(o)$ for some $h \in \{1, ..., n-1\}$. Then a unilateral downward deviation to any $y < x^S(o)$ is not profitable.*

Next consider a deviation to a more aggressive commitment $y > x^S(o)$. Such a deviation must have the property that, for some $k \geq 1$, it will be met if at least $o+k \geq o$ of the *other* responders have a loophole. I.e. the deviation is targeted to succeed when at least k loopholes occur *in addition* to the o required in equilibrium. As before, such a deviation makes a difference only in case the deviating player ends up being a responder and her own commitment attempt succeeds. In addition, we can restrict attention to events where at least o of the other $(n-2)$ responder commitments have a loophole since, if there were fewer, the payoff would be $\delta v^S(o)$ in every case. In this contingency, the outcome will depend on how many of the remaining $(n-2-o)$ responders have loopholes. Then by the same reasoning leading up to equation (6), the most aggressive commitment y that will be met with k *additional* loopholes (i.e. $o+k$ in total) is of the form

$$y_k = x^S(h) + k\left(x^S(o) - \delta v^S(o)\right).$$

To see whether such a deviation pays off, note again that we can condition on the event that (a) the deviating responder's commitment sticks and (b) at least o of the other $(n-2)$ commitments attempts fail. Then the tradeoff is as follows. In all cases where *fewer* than k additional loopholes appear (out of $n-2-h$ chances), the deviating player will *lose* $(x^S(o) - \delta v^S(o))$. This occurs with probability $1 - \eta(n-2-o,k)$. In all cases where at least k additional loopholes occur, the deviating player will *gain* $k\left(x^S(o) - \delta v^S(o)\right)$. This occurs with probability $\eta(n-2-o,k)$. So a deviation aiming at $k \geq 1$ additional loopholes strictly pays off if and only if $\eta(n-2-o,k)k\left(x^S(o) - \delta v^S(o)\right) > (1-\eta(n-2-o,k))\left(x^S(o) - \delta v^S(o)\right)$ or simply

$$\eta(n-2-o,k) > \frac{1}{k+1}. \tag{7}$$

This establishes that an equilibrium requiring $o \in \{1, n-3\}$ loopholes exists iff the reverse of (7) holds for $k = 1, ..., n-o-2$. To gain intuition, define $g = n-o$. Then, since the equilibrium requires at least $n-g$ loopholes, there are up to $(n-2)-(n-g) = g-2$ chunks of the pie of size $x^S(o) - \delta v^S(o)$ that accrue to the proposer in case more than $n-g$ loopholes appear. The player who deviates upwards is aiming to extract some number k of these chunks from the proposer, at the cost of higher probability of provisional impasse. There are up to $g-2$ variants of upward deviations which are ordered in terms of aggresiveness, k. Each of them generates a benefit of k times the size of the chunk times the probability of at least k additional loopholes $\eta(g-2,k)$. Each deviation labelled by k is also associated with a provisional loss of the probability of less than k additional loopholes $(1-\eta(g-2,k))$ times one chunk of size $x^S(o) - \delta v^S(o)$. The condition checks for each of these deviations, that the deviation does not pay off. It is a necessary and a sufficient condition since we verified above in Lemma 3 that a downward deviation is never profitable. The bargaining stage strategies are optimal in an obvious manner that we are familiar with from the existing literature (see subsection 1).

As in our analysis of the efficient equilibrium, the characterization can be simplified further. Lemma5 in the Appendix implies that whenever a deviation targeting $k = 1$ additional loopholes does not pay off, no larger deviation will pay off either. That is, if the condition $\eta(n-2-o,k) \leq \frac{1}{k+1}$ is satisfied for $k = 1$, then it is satisfied for all $k = \{1, ..., n-o-2\}$. Defining $o = n - g$ we have established that for any $g \in \{3, ..., n-1\}$, a symmetric equilibrium requiring at least $(n-g)$ loopholes exists iff $\eta(g-2,1) \leq \frac{1}{2}$, which can be written $g \leq 2 - \frac{\ln(2)}{\ln(\rho)}$ or equivalently $\rho > 2^{-\frac{1}{g-2}}$. Combining this insight with Theorems 3 and 2 yields our main result.

Theorem 4. *For any $o \in \{0, ..., n-2\}$, a symmetric equilibrium requiring o loopholes exists iff any of the following equivalent conditions hold:*

- $\eta(n-2-o,1) \leq \frac{1}{2}$
- $n \leq \bar{n}(o,\rho) \equiv o + 2 - \frac{\ln(2)}{\ln(\rho)}$
- $\rho > \underline{\rho}(o,n) \equiv 2^{-\frac{1}{n-2-o}}$.

The corresponding commitment profile involves $x^S(o) = \frac{1-(o+1)\delta v^S(o)}{n-o-1}$, and the expected payoff is $v^S(h) = \frac{1}{n} \cdot \frac{\eta(n-1,o)}{1-\delta(1-\eta(n-1,o))}$.

In order to appreciate the substance of this result, we should emphasize that it is an if-and-only-if statement. For example, setting $o = 0$ yields that the most efficient equilibrium exists *only* if $n < \bar{n}(0,\rho)$ which of course coincides with the $\hat{n}$ already derived earlier in subsection 4.4. For $n > \bar{n}(0,\rho)$, a profile in which no player attempts to commit in a way that would cause delay does not constitute an equilibrium. Thus, the inefficiency is *not* just a consequence of coordination failure. Even if all players expected others not to commit, each individual player would have an incentive to deviate from such a profile.

Notice also that the expected period of reaching an agreement in an equilibrium with n players and $o > 0$ required loopholes equals $\sum_{t=0}^{\infty} t[(1-\eta(n-1,o;\rho))]^{t-1}\eta(n-1,o;\rho) = 1/\eta(n-1,o;\rho)$. This is increasing in the number of required loopholes, o, and the strength of commitment ρ. Yet, it is decreasing in the number of players n, reflecting the fact that any given number of loopholes, o, is more likely to arise when there are more players. However, it is precisely this effect that makes deviations to more aggressive commitments more profitable, such that an increase in n will eventually destabilize an equilibrium with o required loopholes. For arbitrary n and ρ, the most efficient equilibrium that exists requires at least

$$\underline{o}(n,\rho) \equiv \begin{cases} n-2 & \rho \leq \frac{1}{2} \\ n-2+I\left(\frac{\ln(2)}{\ln(\rho)}\right) & \rho \in \left(\frac{1}{2}, 2^{-\frac{1}{n-2}}\right) \\ 0 & \rho \geq 2^{-\frac{1}{n-2}} \end{cases} \tag{8}$$

loopholes, where $I\left(\ln(2)/\ln(\rho)\right)$ is the smallest integer larger than $\ln(2)/\ln(\rho)$. Thus, for $\rho < \frac{1}{2}$, only the most inefficient equilibrium exists. As ρ gets larger, additional more efficient symmetric equilibria exist. And for $\rho \geq 2^{-\frac{1}{n-2}}$, all symmetric equilibria, including the fully efficient equilibrium, exist. Notice, yet, that $\underline{o}(n,\rho)$ is increasing in n and, as we discovered at the end of the previous subsection, the efficient equilibrium will eventually destabilize and cease to exist.[19] Therefore, the expected delay before reaching an agreement increases in n.

Theorem 5. *The shortest delay in any equilibrium is increasing in n.*

Proof. We know that for each n, the most efficient equilibrium is either the efficient equilibrium or the profile with $h > 0$ required loopholes such that $\eta\left(n-2-o,1\right) \leq \frac{1}{2}$ and $\eta\left(n-2-(o-1),1\right) = \eta\left(n-1-o,1\right) > \frac{1}{2}$. Find the largest n for which $1-\rho^{n-2} \leq 1/2$. Then $\eta\left(\bar{n}(0,\rho)-2,1\right) = 1-\rho^{n-2} \leq 1/2$ and the efficient equilibrium exists and yet $\eta\left(\bar{n}(0,\rho)+1-2,1\right) > 1-\rho^{n-1} > 1/2$. Thus the efficient equilibrium does not exist when $n = \bar{n}(0,\rho)+1$. Yet, by (8), the equilibrium requiring one loophole does exist in that case. The expected delay clearly increases when moving from $\bar{n}(0,\rho)$ to $\bar{n}(0,\rho)+1$ players in that case. These facts also imply that $\rho^{\bar{n}(0,\rho)-1} < \frac{1}{2} \leq \rho^{\bar{n}(0,\rho)-2}$. Or, $1-2^{-\frac{1}{\bar{n}(0,\rho)-2}} \geq 1-\rho > 1-2^{-\frac{1}{\bar{n}(0,\rho)-1}}$. Thus, for $l,k = 0,1,...$ and for $\bar{n}(0,\rho) > 5$,we have that $\begin{pmatrix} \bar{n}(0,\rho)+k \\ 1+k+l \end{pmatrix}(1-\rho) - \begin{pmatrix} \bar{n}(0,\rho)-1+k \\ k+l \end{pmatrix} < \begin{pmatrix} \bar{n}(0,\rho)+k \\ 1+k \end{pmatrix}(1-\rho) - \begin{pmatrix} \bar{n}(0,\rho)-1+k \\ k \end{pmatrix} < \begin{pmatrix} \bar{n}(0,\rho) \\ 1 \end{pmatrix}(1-\rho) - \begin{pmatrix} \bar{n}(0,\rho)-1 \\ 0 \end{pmatrix} \leq \begin{pmatrix} \bar{n}(0,\rho) \\ 1 \end{pmatrix}(1-2^{-\frac{1}{\bar{n}(0,\rho)-2}}) - \begin{pmatrix} \bar{n}(0,\rho)-1 \\ 0 \end{pmatrix} = \bar{n}(0,\rho)(1-2^{-\frac{1}{\bar{n}(0,\rho)-2}})-1 < 0$ and thus $= \eta\left(n, \underline{o}(n+1,\rho)\right) -$

[19] $2^{-\frac{1}{n-2}}$ is increasing in n.

$\eta\,(n-1, \underline{o}(n,\rho)) = \sum_{l=\underline{o}(n,\rho)+1}^{n} \rho^{n-l}(1-\rho)^{l-1}\left[\begin{pmatrix} n \\ l \end{pmatrix}(1-\rho) - \begin{pmatrix} n-1 \\ l-1 \end{pmatrix}\right] < 0$ showing that expected delay increases whenever ρ is such that the efficient equilibrium exists with six or more players. In order to show that the claim holds also for $\bar{n}(0,\rho) \in \{3,4,5\}$, remark first that $\begin{pmatrix} n-1 \\ l-1 \end{pmatrix} / \begin{pmatrix} n \\ l \end{pmatrix} \geq 1/2$ holds when $l-1 \geq (n-1)/2$. Thus the only remaining non-obvious case is when there are 5 players and ρ is such that the equilibrium requiring one loophole is the most efficient one. In that case, there are four responders and $\begin{pmatrix} 5 \\ 2 \end{pmatrix} > \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. The probability of an agreement changes from $1-\rho^4$ to $1-\rho^5-5(1-\rho)\rho^4$. The difference equals $-\rho^5+5\rho^5-5\rho^4+\rho^4 < 0$. $\square$

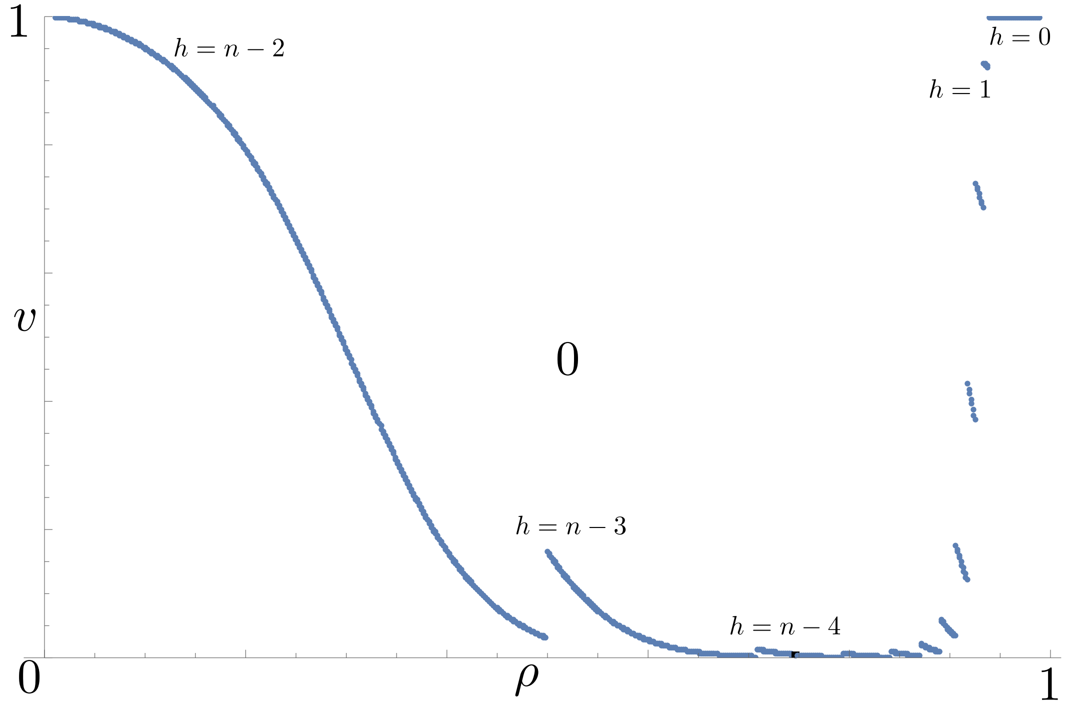
The theorem shows that, although increasing the number of players increases the chances of reaching an agreement at an equilibrium profile with a given number of required loopholes, the strategic incentives also change and more aggressive commitments become more attractive. This destabilizes the most efficient equilibrium. When there is one more player in the game, the most efficient equilibrium also requires one more loophole. This is associated with, not shorter, but longer delay.

The effect of changing ρ has an analogous, but reverse logic. Efficiency decreases and delay increases in a given equilibrium profile as commitment strength ρ increases. Yet, at the same time, increasing commitment strength, also eventually has the effect that more efficient equilibria emerge to the set of equilibria.

Theorem 3 states that under unanimity rule, efficient equilibria exist only for $\rho \geq \hat{\rho} = \left(\frac{1}{2}\right)^{\frac{1}{n-2}}$. On the other hand, expected payoffs in any equilibrium approach $1/n$ (i.e. full efficiency) as ρ tends to zero. This suggests a non-monotone relationship between ρ and expected payoffs: For small ρ, only the most aggressive equilibrium exists, and the associated payoffs are decreasing in ρ. But for $\rho \geq \hat{\rho}$, a fully efficient equilibrium (also) exists, and so expected payoffs *could* be higher if that equilibrium were played. (Note, however that the inefficient equilibrium continues to exist as well.) A reasonable question to ask is: what is the relationship between ρ and the expected payoff in the *most efficient* equilibrium?

Focusing on symmetric equilibria and fixing n, only the most inefficient equilibrium, requiring $n-2$ loopholes for agreement, exists for ρ close to zero. The expected payoff in that equilibrium is decreasing in ρ. For ρ large enough, a more efficient

Figure 2: Expected payoff in most efficient symmetric equilibrium ($n = 13$, $\delta = 0.9$)



equilibrium, requiring $n - 3$ loopholes, exists (in addition to the most inefficient), with a larger expected payoff than the first (but still short of $1/n$). Again, the payoff in this equilibrium is decreasing in ρ. We can find ranges of ρ for which the symmetric equilibrium requiring at least o loopholes exists, for each o from $n-2$ to 0. Equilibriua with fewer and fewer loopholes keep on appearing sequentially as ρ increases towards 1. Thus, there are $n - 1$ ranges of ρ, beginning at ρ close to zero (where *only* the most inefficient equilibrium exists) and ending at ρ close to one (where *all equilibria* from most inefficient to fully efficient exist).

The expected payoff in the most efficient symmetric equilbrium, as a function of ρ, thus indeed follows a non-monotone pattern as depicted in the following figure, which is is plotted for values $n = 13$, $\delta = 0.9$

4.6 Frequent negotiations under unanimity rule

So far we have considered several institutional features of multilateral negotiations, such as which kind of decision rule is being applied, how many parties there are in the negotiations, whether commitment positions can be formulated, and how likely they are to succeed. Let us now analyse another institutional feature in multilateral negotiations, namely how frequently negotiation rounds take place. Unlike in many face-to-face bilateral negotiations, it is by no means obvious that in international multilateral negotiations, offers could be generated very frequently.[20]

In such institutionally rich settings, it is not entirely obvious how to think about the frequency of negotations. A commonly used approach in the bargaining literature is to consider a continuous time formulation where discounting over two consecutive negotiation rounds is parametrized by the time gap between the rounds, t, such that delay between the rounds is discounted by factor $\delta = exp(-rt)$. Here r is the discount rate reflecting the cost associated with the passage of a naturally given time interval such as a year, and t is the delay between negotiation rounds expressed as a fraction of the natural time interval. For example, if negotiation rounds take place once a year and the yearly interest rate equals 3%, then $\delta = exp(-0.03) \approx 0.97$; if negotiation rounds occur once in every six months $\delta = exp(-0.03/2) \approx 0.985$. A typical question addressed in this setting is what happens when all institutional frictions constraining frequency of rounds are lifted and the rounds rather follow each other in an almost continuous sequence. Formally, what happens when t approaches zero?

In order to formulate that limit, we must first take a stand on what happens to the process of commitments and the exogenous, but stochastic, arrival of loopholes. A straigthforward generalization of the model presented in the previous sections would assume that each player's individual loophole arrivals follow an i.i.d. memoryless Poisson process with an arrival rate of λ. Then within a time period of length t, the probability of zero arrivals, i.e. that no loophole arises for this player, is $\rho = exp(-\lambda t)$. Notice that this implies that the arrival rate of loopholes among the $n-1$ responders is then just $(n-1)\lambda$. Not surprisingly, the probability of loophole arrival within a given round of negotiations tends to zero as the length of negotiation round, t, tends to zero. Thus, the periodic probability of commitment success ρ tends to one. Moreover, among the known properties of Poisson processes is that the probability of two arrivals at exactly the same time is zero.[21] Thus, when t tends to zero, two or

[20] In the European Union, a summit where all heads of state gather together takes place once every six months (June and December), the Doha round of the WTO has had nine comprehensive meetings since the start of the round in 2001 (and are by and large inconclusive by the time of writing this manuscript). In climate change negotiations, general meetings (Conference of the Parties, COP) take place once a year (the 25th COP was organized in Madrid in December 2019 and ended without any conclusive agreement on measures or timeline on how to reach the targets set in Paris 2015).

[21] This is due to the fact that a Poisson process is orderly and thus simple.

more loopholes never arrive at the same time.[22]

Since ρ tends to one as t tends to zero, the efficient equilibrium exists in the limit.[23] The equilibrium payoff equation in the discrete time formulation of the model (4) can be adjusted to the continuous time formulation, as a function of n, h, λ, r and t, as follows

$$v^S(o) = \frac{1}{n} \cdot \frac{\eta(o, n-1)}{1 - exp(-rt)(1 - \eta(o, n-1))}.$$

Given that at most one loophole arrives in a Poisson process at any point in time, two cases are of special interest in the limit where the time period length tends to zero. The first is the efficient equilibrium profile, with corresponding payoff

$$v^S(0) = \frac{1}{n},$$

and second the symmetric commitment profile requiring just one loophole, with associated payoff

$$v^S(1) = \frac{1}{n} \cdot \frac{\eta(1, n-1)}{1 - exp(-rt)(1 - \eta(1, n-1))} = \frac{1}{n} \cdot \frac{1 - (exp(-\lambda t))^{n-1}}{1 - exp(-rt)(exp(-\lambda t))^{n-1}}.$$

It can be shown that, in the limit,the symmetric commitment profile $x^S(1)$ constitutes an equilibrium commitment profile and thus there is an equilibrium requiring one loophole.[24] Applying l'Hôspital's rule yields the limit payoff

$$lim_{t \to 0} v^S(1) = \frac{1}{n} \cdot \frac{\lambda(n-1)}{r + \lambda(n-1)}. \tag{9}$$

Equation (9) reveals that, when offers are generated very frequently, the efficiency losses in the inefficient one-loophole equilibrium increase in the discount rate r and

[22]Despite its simplicity and tractability, this approach implicitly assumes some unrealistic features, above all that, as t tends to zero, (i) commitments can be reformulated at an increasing pace after a loophole arrival (at the beginning of the commitment stage following the arrival), and (ii) thus that the loopholes last for an decreasing length of time as t tends to zero. In more realistic formulations, one might decouple the process of re-establishing commitments from the frequency of negotiation rounds, but we leave that for future research.

[23]To confirm this, it is easy to see that staying flexible, deviating down, or deviating up in the most aggressive equilibrium cannot pay off since $v^* > exp(-rt)v^*$for any $t > 0$.

[24]See section 4.5. of our working paper version (Miettinen and Vanberg, 2020).

decrease in the loophole arrival rate λ and the number of players n.[25]

The existence of the aggressive equilibrium, requiring $n-2$ loopholes, is independent of the parameter values and thus it also exists in the limit.[26] Recall that two or more loopholes never arrive at the same time and thus, in the limit, that equilibrium has a zero equilibrium payoff, $v^S(n-2) = 0$. Thus, the equilibrium commitments are very aggressive $lim_{t\to 0} x_h^S(n-2) = 1$ much like in the inefficient equilibrium of the bilateral Nash demand game where each party demands the entire pie. Indeed, in this limit case, the efficient equililibrium always exists. Therefore, $lim_{t\to 0} v^S(0) = 1/n > lim_{t\to 0} v^S(1) > lim_{t\to 0} v^S(n-2) = 0$, and thus any ineffciency is due to a coordination failure like in the Nash demand game. This is in contrast to the case of less frequent negotiations, in which the efficient equilibrium does not exist, and thus inefficiency is unavoidable in equilibrium and not due to coordination failure.

Proposition 4. *When the length of the time period t tends to zero, both efficient and inefficient equilibria exist under unanimity rule.*

In a discrete time environment with strictly positive delay between bargaining rounds, Theorem 3 says that an efficient equilibrium does not exist when ρ is sufficiently small. In such a setting, a deviation to a more aggressive commitment can be profitable because the proposer faces a strictly positive cost of delay if she fails to concede. When negotiations are sufficiently frequent, such a deviation loses its leverage, as play will return to the efficient path with negligible delay. Thus, proposers would not concede to such deviations, and an efficient equilibrium always exists (Proposition 4). In contrast, the forces underlying Theorem 2 extend to more frequent negotiations: Still, the most inefficient equilibrium always exist, because the only deviation that could increase the probability of agreement (or shorten the expected delay) requires the deviator to accept his continuation value. Furthermore, proposers are willing to concede to the aggressive commitment in the event of sufficiently many loopholes, because failure to do so would yield a very low continuation value, given that the inefficient commitments will be immediately re-established. Proposition 4

[25] Notice also that, because the arrival probability of the first loophole is independent of the length of the time period, the expected length of conflict and thus the equilibrium payoff $v^S(1)$ are independent of the length of the time period, too.

[26] We can show that, in the limit, all symmetric commitment profiles for $o = 0, ..., n-2$ are equilibrium commitment profiles, but all equilibria with $o > 1$ never lead to an agreement since two or more loopholes never arrive in the same period (Miettinen and Vanberg, 2020).

therefore concludes with the existence of an inefficient equilibrium and the credibility of aggressive commitments also in frequent negotiations.

5 Conclusion

International and supranational organizations, including the World Trade Organization and the European Union, typically require unanimous consent to make decisions in important policy areas. This requirement is designed to guarantee that all decisions are mutually beneficial. However, many experts and practioners have crtiticized unanimity rule on the grounds that it often leads to excessive delay and gridlock. Such complaints have fueled calls for the expanded use of *qualified majority rule (QMR)* as a possible means to increase decision making ability.

In this paper, we have presented a tractable model of multilateral bargaining that predicts inefficient delay under unanimity decision making. The mechanism our model highlights is that unanimity rule creates incentives for players to commit to a tough bargaining stance prior to negotiating. Our analysis suggests that the associated inefficiencies grow more severe when the number of players is large. We also show that any less-than-unanimity rule circumvents this problem in the context of our model. Our analysis also suggests that inefficiencies can be reduced by increasing the frequency of negotiations.

Naturally, our model is highly stylized and abstracts from potentially important details of the applied contexts mentioned. Nevertheless, the stylized predictions fit the empirical patterns in the EU and WTO contexts that we have used to motivate our analysis. In both of these contexts, the number of parties is large and has increased prior to the observed impasses: the WTO Doha round failed after the enlargement of the organization in the late 1990's and early this Millennium, and EU decision making in sensitive areas has stalled ever since the enlargement of 2004.

As we mentioned, many EU practitioners have proposed expanding the use of *QMR* to sensitive areas such as foreign policy or taxation.[27] Our analysis would seem

[27]There are currently two qualified majority rules in use: "when the Council votes on a proposal by the Commission or the EU's High Representative for Foreign Affairs and Security Policy, a QM is reached if [...] 55% of EU countries vote in favour - i.e. 16 out of 28 [and these countries represent] at least 65 % of the total EU population. When the Council votes on a proposal not made by the Commission or the High Representative, a decision is adopted if [it is supported by] 72 % of EU [countries representing] at least 65 % of the EU population." (https://eur-lex.europa.eu/summary/glossary/qualified_majority.html, accessed 12.6.2020)

to lend support to this proposal in that we would predict a sharp rise in efficiency. An important caveat to this interpretation is that our model assumes that agreement is indeed efficient. In reality, the expanded use of *QMR* could increase the risk of decisions that impose external costs on non-consenting members, an aspect that is absent from our model. Thus, there exists a trade-off between the expected "external costs" and anticipated "decision costs," to borrow terminology introduced by Buchanan and Tullock (1965). By design, our analysis restricts attention to the latter aspect.

Perhaps due to the anticipation of such external costs, the proposal to expand the use of *QMR* has faced significant resistance from a number of member states. This is well documented among others by Koenig (2020), who conducts a confidential expert survey with diplomatic sources and concludes that only six member states support the expanded use of *QMR* (as currently defined). Given this resistance, it is perhaps interesting to emphasize that our model suggests that even a move to an *all-but-one rule* - preventing a *single member* from blocking agreement - may help to dramatically improve the EU's decision making capacity. It seems reasonable to assume that the "external costs" to be expected from such an all-but-one rule will be substantially lower than those to be expected under the currently established - and much less demanding - *QMR*. It stands to reason that a proposal to introduce a novel, highly demanding, *QMR* may be more acceptable to many member states than the extension of the currently established procedures.[28]

A Appendix

A.1 Proof of Lemma 1

Part 1. If $d_i \neq \delta v_i$, this follows from the elimination of weakly dominated strategies. Given this, it is without loss of generality to assume that responders vote "yes" when $d_i = \delta v_i = \hat{x}_i(s_i)$. To see this, suppose there exists an equilibrium in which some player i votes "no" on a proposal d (made by some proposer j given some commitment

[28] Another recommendation which warrants further investigation is to organize bargaining rounds more frequently. However, this policy conclusion hinges on the implicit assumption that commitment positions can be re-established at the beginning of the following round independently of how soon the next round arrives. This implies that the frequency reduces the short-term advantage of committed parties over the uncommitted ones, thereby undermining the incentive to deviate to a more aggressive commitment.

status profile s) in which i is being offered exactly $\hat{x}_i(s_i) = \delta v_i$. If i is not pivotal, there exists an outcome-equivalent equilibrium in which he votes "yes" in this event. Suppose that i is pivotal. That is, the proposal *fails* in equilibrium but it *would pass* if i were to vote "yes". Suppose that the equilibrium proposal at this history is $y \neq d$ and it passes. If j (the proposer) weakly prefers y over d, there exists an outcome-equivalent equilibrium in which i would vote "yes" on d (so it would pass) but j does not propose it. If j strictly prefers d over y, then she could make a proposal arbitrarily close to d (increasing d_i slightly) and improve over proposing y (recall that i is pivotal), a contradiction. Finally, suppose that the proposal made in this event fails (it could be d itself). Then the cheapest available coalition given s must cost at least $1 - \delta v_j$. (Otherwise j would strictly prefer to make a proposal that passes.) If the cheapest coalition costs *strictly more* than $1 - \delta v_j$, there is an outcome-equivalent equilibrium in which i votes "yes" on d but j does not propose it (strictly preferring delay). If the cheapest coalition costs *exactly* $1 - \delta v_j$, at least one of the responders in the cheapest coalition (say, k) must be committed to $x_k > \delta v_k$ (since the sum of continuation values is strictly less than one). Then if k reduces his commitment by an arbitrarily small amount, j will make a deal that includes k and k improves his payoff (relative to the proposal failing). The cost of this deviation is arbitrarily small, and so k's payoff increases, a contradiction.

Part 2. This is immediate if the price of the cheapest coalition is not equal to $1 - \delta v_i$. What remains to be shown is that i must make a proposal that passes in the case where the cheapest coalition costs exactly $1 - \delta v_i$. To see this, suppose that there exists some event (i.e. a profile of commitment statuses s and a proposer i) such that $\delta v_i + \sum_{j \in C_i} \hat{x}_j(s_j) = 1$ for the cheapest coalition C_i, but the proposer makes a proposal that fails. Then at least one $k \in C_i$ must then be committed to $x_k > \delta v_k$ (since the sum of continuation values is strictly less than one). Then if k reduces his commitment by an arbitrarily small amount, i will make a deal that includes k and k improves his payoff (relative to the proposal failing). The cost of this deviation is arbitrarily small, and so k's payoff increases, a contradiction.

Equilibrium commitments The condition $x_i^* \in \arg\max \pi_i(x_i, x_{-i}^* | \psi^*)$ follows from the one-stage deviation principle, which requires that for each i there must not exist a commitment attempt $\tilde{x}_i \neq x_i^*$ with the property that player i could strictly increase his payoff by deviating to $\tilde{x}_i$ in the initial commitment stage and reverting to equilibrium play thereafter. (See Fudenberg and Tirole (1991) Theorem 4.2 and note that our game satisfies their Definition 4.1 due to discounting.)

A.2 Proof of Theorem 1

A.2.1 Lemma 3

Lemma 4. It is without loss of generality to assume the following:

(a) $x_i \geq \delta v_i$ for all i.

(b) If player j is never included as responder when his commitment sticks, then $x_j = \delta v_j$.

Proof. Part (a) is obvious. Assume it for what follows. For part (b), suppose - seeking a contradiction - there exists j who is never included when his commitment sticks, but $x_j > \delta v_j$.

Suppose there is a commitment status profile s and a proposer $k \neq j$ such that (i) j is uncommitted in s, (ii) *every* set of the cheapest $(q-1)$ responders includes j, and (iii) the cost of these sets (denote it $C(I_k) = \sum_{i \in I_k} \hat{x}_i(s)$) satisfies $C(I_k) < 1 - \delta v_k$. Then player j could deviate to commitment attempt $\delta v_j + \epsilon$, and for ϵ sufficiently small, a deal that includes j would be made whenever k is proposer and faced with a commitment profile identical to s except that j is committed. Clearly j benefits from this deviation as without it he obtains at most δv_j when committed. A contradiction. Thus there does not exist a commitment status profile of the type considered.

Suppose that there a commitment status profile s and a proposer $k \neq j$ such that (i) j is uncommitted in s, (ii) *every* set of the cheapest $(q-1)$ responders includes j, and (iii) the cost of these sets satisfies $C(I_k) = 1 - \delta v_k$. (The difference to the previous case is that k is indifferent so that j cannot ensure a deal with a commitment attempt larger than δv_j.) Then it follows that in each of the cheapest coalitions I_k, at least one responder $i \neq j$ must be committed to $x_i > \delta v_i$. (Otherwise $C(I_k) = \sum_{i \in I_k} \delta v_i < 1 - \delta v_k$.) But then consider a modified event, identical to s except that some such i has a loophole. Then still the coalition that included him, I_k, is the cheapest set of $(q-1)$ responders, but now $C(I_k) < 1 - \delta v_k$ and hence the argument above applies. A contradiction. Thus again there does not exist a commitment status profile of the type considered.

Thus, there *does not exist* a status profile s and a proposer $k \neq j$ such that (i) j is uncommitted in s, (ii) *every* set of the cheapest $(q-1)$ responders includes j, and (iii) the cost of these sets (denote it $C(I_k) = \sum_{i \in I_k} \hat{x}_i(s)$) satisfies $C(I_k) \leq 1 - \delta v_k$. That is, there is no commitment status profile s such some player k *must* make a deal that includes an uncommited j. It follows that j could deviate to commitment attempt δv_j (or smaller), and all proposers could continue to make the same deals (or not make deals) as they do prior to that deviation. Therefore, there exists an equivalent equilibrium strategy profile that is identical except $x_j = \delta v_j$ (or smaller) and equilibrium play is the same in all events. □

A.2.2 Proof of the Theorem

Proof. Order players such that

$$x_1 \leq x_2 \leq .... \leq x_q \leq ... \leq x_n$$

and assume without loss of generality that $x_i \geq \delta v_i$ and that $x_j = \delta v_j$ for any player who is never included when his commitment sticks. (See Lemma 4.)

Define $H = \{h : x_h > x_q\}$, and assume that H is not empty. Then any $h \in H$ is never included when his commitment sticks. Then $\delta v_h = x_h > x_q$, and so in fact h is *never included as responder* (even with a loophole). Define $L = \{l : x_l \leq x_q\}$, and note that it is not empty. Then $v_h > v_l$ for all $h \in H$ and $l \in L$.

Let $z = \#L$. Define P_L, P_H as the average probability that a member of L and H makes a deal when proposing. It can be argued that $P_L \geq P_H$.[29] Let $P_F = 1 - \frac{z}{n}P_L - \frac{n-z}{n}P_H$ be the probability that no deal is reached in a given round. Note that all deals include *only* members of L as responders. Let C_H be the average total payments to responders in L when a member of H makes a deal. Then the average expected payoff among members of H is

$$\bar{v}_h = P_F \delta \bar{v}_h + \frac{1}{n} P_H (1 - C_H)$$

[29] Consider any commitment status profile s such that $h \in H$ would make a deal when proposing. Let I_h be the set of included responders. Then $\delta v_h \leq 1 - \sum_{j \in I_h} \hat{x}_j(s)$. Take any $l \in L$. If $l \notin I_h$, then $\delta v_l < 1 - \sum_{j \in I_h} \hat{x}_j(s)$, and so L can make a deal with I_h. If $l \in I_h$, then since $x_h = \delta v_h$, we have $\delta v_l \leq \hat{x}_l(s)$ and $\delta v_h = \hat{x}_h(s)$ and thus $\delta v_l \leq 1 - \sum_{j \in I_h \setminus \{l\}} \hat{x}_j(s) - \hat{x}_h(s)$. Therefore l can make a deal with $(I_h \setminus \{l\}) \cup \{h\}$. Thus for any commitment status profile at which a member of h makes a deal, any member of L will also make a deal.

and the average expected payoff among members of L is

$$\bar{v}_l = P_F \delta \bar{v}_l + \frac{z}{n} P_L \frac{1}{z} + \frac{n-z}{n} P_H \frac{C_H}{z},$$

where the second part of the sum reflects the fact that whenever a member of L makes a deal, the entire pie is shared in some way between the z members of L. But then

$$(1 - \delta P_F)(\bar{v}_l - \bar{v}_h) = \frac{P_L - P_H}{n} + P_H \frac{C_H}{z} > 0,$$

contradicting $v_h > v_l$ for all $h \in H$ and $l \in L$. It follows that H is empty, i.e. $x_j = x_q$ for all $j \geq q$.

$$x_1 \leq x_2 \leq .... \leq x_q = x_{q+1} = ... = x_n.$$

Now redefine $H = \{\delta v_h = x_q\}$ and $L = \{\delta v_l < x_q\}$. (Note that these sets are exhaustive because $\delta v_j \leq x_j \leq x_q$ for all j.) Suppose both sets are nonempty. Let us redefine $z = \#L$, as well as the other notation introduced above. Note that once again $v_h > v_l$ for all $h \in H$ and $l \in L$, and the argument in footnote 29 still applies, thus $P_L \geq P_H$.

Suppose $z \geq q$. Then any $h \in H$ is *never included as a responder*. (To see this, suppose a deal does include some $h \in H$. Then h is paid x_q. Moreover, at least one $l \in L$ is excluded. It follows that l is committed to x_q. But then l can deviate to $x_q - \epsilon$ and he will be included for ϵ arbitrarily small.) Then the argument above can be repeated to show that the average payoffs satisfy $\bar{v}_l > \bar{v}_h$, a contradiction.

Thus $z \leq q - 1$. Then any deal must include all members of L either as proposer or responder. (To see this, suppose a deal is made that does not include some $l \in L$. Then it includes at most $q - 2$ members of L, and so must include at least one responder $h \in H$, who is paid x_q. It follows that l is committed to x_q. But then l can deviate to $x_q - \epsilon$ and he will be included for ϵ arbitrarily small.) In addition to the notation already introduced, let $C_L = (q - z)x_q$ be the total payment made to responders in H when a member of L makes a deal. Then the average expected payoff among members of H is

$$\bar{v}_h = P_F \delta \bar{v}_h + \frac{n-z}{n} P_H \frac{1 - C_H}{n-z} + \frac{z}{n} P_L \frac{C_L}{n-z}$$

and the average payoff for members of L is

$$\bar{v}_l = P_F \delta \bar{v}_l + \frac{z}{n} P_L \frac{1 - C_L}{z} + \frac{n - z}{n} P_H \frac{C_H}{z},$$

where again each element of the sums reflects the way that the pie is shared among the members of both sets in the event of a deal. Then

$$\begin{aligned}
\bar{v}_L - \bar{v}_h &= P_F \delta (\bar{v}_L - \bar{v}_h) + \frac{P_L (1 - C_L)}{n} - \frac{P_H (1 - C_H)}{n} + \frac{n - z}{nz} P_H C_H - \frac{z}{n (n - z)} P_L C_L \\
&= P_F \delta (\bar{v}_L - \bar{v}_h) + \frac{P_L - P_H}{n} + \left(\frac{1}{n} + \frac{n - z}{nz} \right) P_H C_H - \left(\frac{1}{n} + \frac{z}{n (n - z)} \right) P_L C_L \\
&= P_F \delta (\bar{v}_L - \bar{v}_h) + \frac{P_L - P_H}{n} + \frac{P_H C_H}{z} - \frac{P_L C_L}{n - z}
\end{aligned}$$

Then using $P_F = \left(1 - \frac{z}{n} P_L - \frac{n-z}{n} P_H\right)$,

$$\bar{v}_L - \bar{v}_h = \left(1 - \frac{z}{n} P_L - \frac{n - z}{n} P_H \right) \delta (\bar{v}_L - \bar{v}_h) + \frac{P_L - P_H}{n} + \frac{P_H C_H}{z} - \frac{P_L C_L}{n - z},$$

and thus

$$\begin{aligned}
(1 - \delta) (\bar{v}_L - \bar{v}_h) &= \frac{P_L - P_H}{n} + \frac{P_H C_H}{z} - \frac{P_L C_L}{n - z} - \frac{1}{n} (z P_L + (n - z) P_H) \delta (\bar{v}_l - \bar{v}_h) \\
&= \frac{P_L - P_H}{n} (1 - z \delta (\bar{v}_l - \bar{v}_h)) + \frac{P_H C_H}{z} - \frac{P_L C_L}{n - z} - P_H \delta (\bar{v}_l - \bar{v}_h) \\
&= \frac{P_L - P_H}{n} [1 - z \delta \bar{v}_l - (n - z) \delta \bar{v}_h] + P_H \left(\frac{C_H}{z} - \delta \bar{v}_l \right) + P_L \left(\delta \bar{v}_h - \frac{C_L}{n - z} \right)
\end{aligned}$$

Note that all elements of the sum are positive: The first because the sum of continuation values is less than one, the second because $C_H \geq z \delta \bar{v}_l$, given that any deal includes all members of L and each is paid at least his continuation value. Finally, the last is positive because $C_L = (q - z) \delta \bar{v}_h$ since any deal made by a member of L includes $q - z$ members of H who are each paid exactly their (common) continuation value. Thus $\bar{v}_l \geq \bar{v}_h$, contradicting $v_h > v_l$ for all $h \in H$ and $l \in L$.

It follows that one of the sets, L or H, must be empty. Suppose H is empty, i.e. $\delta v_j < x_q$ for all j. Then by Lemma 4, all $i \geq q$ are included with positive probability when committed to $x_q > \delta v_i$. But any such deal would have to exclude some responder, and so that responder must also be committed to x_q. Then this responder could deviate to $x_q - \epsilon$ for ϵ arbitrarily small and be included. A contradiction.

Thus, the set L is empty and therefore $x_q = \delta v_j$ for all j. Combined with the ordering already established, it follows that

$$\delta v_i = x_i = x_j = \delta v_j$$

for all i, j. Thus, since no player is committed to more than his continuation value, the unique equilibrium is both efficient and symmetric. $\square$

A.3 Lemma 4

Lemma 5. *If there exists $\hat{k} \geq 1$ such that $\eta\left(\hat{k}, m\right) \leq \frac{1}{\hat{k}+1}$, then $\eta(k, m) \leq \frac{1}{k+1}$ for all $k > \hat{k}$.*

Proof. Note that $\eta(k, m) = \sum_{l=k}^{m} f(l, m)$, where $f(l, m) = \left(\begin{array}{c} m \\ l \end{array}\right)(1-\rho)^l \rho^{m-l}$ is the probability of l 'successes' (loopholes) in a binomial experiment with m trials and success (loophole) probability $(1-\rho)$. It is sufficient to show the following: "If there exists $k \geq 2$ such that $\eta(k, m) > \frac{1}{k+1}$ then $\eta(k-1, m) > \frac{1}{k}$." Suppose there exists $k \geq 2$ such that $\eta(k, m) > \frac{1}{k+1}$. Suppose $k < m(1-\rho)+1$, then $k-1 < m(1-\rho)$, implying that $(k-1)$ is below the median of the binomial, and so $\eta(k-1, m) > \frac{1}{2} \geq \frac{1}{k}$. Suppose $k \geq m(1-\rho)+1$. Since the binomial distribution is discrete log concave, it has the property that $\frac{f(h,m)}{\eta(h,m)}$ is non-decreasing in h (see An (1997) Proposition 10), which implies $\eta(k-1, m) \geq \frac{f(k-1,m)}{f(k,m)} \eta(k, m)$. Further, it can be shown that $\frac{f(k-1,m)}{f(k,m)} = \frac{k}{m+1-k} \frac{\rho}{1-\rho}$. Therefore $\eta(k-1, m) \geq \frac{k}{m+1-k} \frac{\rho}{1-\rho} \eta(k, m) > \frac{k}{m+1-k} \frac{\rho}{1-\rho} \frac{1}{k+1}$. The last expression is increasing in ρ, and we have $(1-\rho) \leq \frac{k-1}{m}$ (see above). Therefore, this expression is greater than $\frac{k}{(m+1-k)(k+1)} \frac{1-\frac{k-1}{m}}{\frac{k-1}{m}} = \frac{k}{k^2-1} > \frac{1}{k}$. $\square$

A.4 Proof of Theorem 3

Proof. Consider an efficient equilibrium and let the associated commitment profile be $\{x_i\}_{i=1}^n$. Note that $\sum_{i=1}^n v_i = 1$ and for any i, $\sum_{j \neq i} x_j \leq 1 - \delta v_i$ (Otherwise, i would not make a deal as proposer in the event that all responder commitments stick, contradicting efficiency). Also, note that $x_i > \delta v_i$ for all i. To see this, suppose there is i with $x_i \leq \delta v_i$. Conditional on being responder, player i's payoff is δv_i. (Agreement is certain, he is always included, and his "price" is δv_i irrespective of his commitment status.) If he deviates to $y = 1 - \sum_{j \neq i} \delta v_j$, he will receive $y > \delta v_i$ with positive probability (when all other responders have a loophole) Moreover, this (one shot) deviation cannot yield a payoff below δv_i in any other event, since i must vote yes for agreement to be reached. Therefore the deviation pays off.

Without loss of generality, order players such that $0 < x_1 - \delta v_1 \leq ... \leq x_n - \delta v_n$. Suppose player 1 deviates to $y_1 = x_1 + (x_1 - \delta v_1)$. Then, conditional on Player 1 being responder with successful commitment, agreement will occur whenever at least one (other) responder has a loophole. (To see this, let $i \neq 1$ be the proposer and suppose any $k > 1, k \neq i$ has a loophole. Then $\sum_{j \neq i} \hat{x}_j(s) \leq y_1 + \delta v_k + \sum_{j \notin \{1,i,k\}} x_j = \sum_{j \neq i} x_j + (x_1 - \delta v_1) - (x_k - \delta v_k) \leq \sum_{j \neq i} x_j \leq 1 - \delta v_i$, and so i will make a deal.) In that event (i.e. whenever there is at least one loophole among the $n-2$ other responders), Player 1's payoff increases by $(x_1 - \delta v_1)$ as a result of the deviation. In all other events (i.e. if no responder has a loophole), the deviation *may* (but need not) result in a deal not being reached, in which case player 1 would lose $(x_1 - \delta v_1)$ as a result of the deviation. Thus, a *lower bound* for the net benefit of the deviation equals $\eta \cdot (x_1 - \delta v_1) - (1 - \eta) \cdot (x_1 - \delta v_1)$, where $\eta = \eta(1, n-2)$ is the probability of at least 1 loophole among the $n-2$ responders other than 1. Therefore a *sufficient condition* for the deviation to pay off is that this be strictly positive, equivalently $\eta(1, n-2) > \frac{1}{2}$, or $\rho < \left(\frac{1}{2}\right)^{\frac{1}{n-2}}$. It follows that an efficient equilibrium (symmetric or not) does not exist if $\rho < \left(\frac{1}{2}\right)^{\frac{1}{n-2}}$. By Lemma 2, an efficient (symmetric) equilibrium exists iff $\rho \geq \left(\frac{1}{2}\right)^{\frac{1}{n-2}}$. □

A.5 A modified model with proposer commitment

Our analysis makes use of the simplifying assumption that commitments bind only responders. In this section, we analyse a version of the model in which proposers can be committed. For simplicity, we focus on symmetric equilibria in the three player game. We show that our results are qualitatively robust to this modification.

Efficiency of majority rule Let $n = 3$ and $q = 2$. Suppose (seeking a contradiction) there exists a symmetric equilibrium with inefficient delay. Then $x^* = 1 - \delta v^*$ (the commitment is tailored to form a coalition consisting of one committed and one uncommitted player). Agreement occurs if and only if at least one player (proposer *or* responder) has a loophole. One such event is when player 1 is proposer and has a loophole, and players 2 and 3 are both committed. In this event, either player 2 or player 3 is included with a probability strictly less than one. Let this player be i.

Suppose player i deviates to commitment attempt $y = x^* - \epsilon$. For ϵ small enough, this deviation does not affect i's payoff as proposer (if committed, he still obtains $1 - \delta v^* = x^*$ when at least one responder has a loophole, and δv^* otherwise). As responder, it affects his payoff only in the event that the proposer has a loophole and both responders are committed. By construction, there is *at least one* such event where i is included with probability strictly less than one. Following the deviation, he will be included for sure in *all* such events. Thus for ϵ small enough, player i's payoff increases following the deviation, a contradiction. This establishes that *no symmetric equilibrium with inefficient delay exists.*

Next, assume that an *efficient* symmetric equilibrium exists. Then $2x^* \leq 1$ (a winning coalition can be formed even when all players are committed). Suppose (seeking a contradiction) that $x^* > \delta v^*$. Agreement occurs in all events. One such event is when player 1 is proposer and all players are committed. Then either player 2 or player 3 is included with probability less than one in this event. Let it be player i. Then if player i deviates to $y = x^* - \epsilon$, he is included for sure in this event (and possibly other events). For ϵ small enough, the cost of this deviation is arbitrarily small and the benefit is discrete. A contradiction. Thus $x^* \leq \delta v^*$ in any efficient equilibrium. All such x^* are substantively equivalent to $x^* = 0$, i.e. no player commits. It is easy to see that this always constitutes an equilibrium, as any deviation to $y > \delta v^*$ would result in the deviator being excluded as a committed responder. This establishes that, *under majority rule, there is an (essentially) unique symmetric equilibrium involving no commitments.*

Inefficiency of unanimity rule Consider now $q = n = 3$. First, we prove that an equilibrium with "aggressive" commitments and delay always exists. This symmetric equilibrium involves commitment attempt $x^* = 1 - 2\delta v^*$. (It is targeted to achieve agreement when only one player's commitment sticks.) Suppose all players commit to x^*. An upward deviation to $y > x^*$ cannot pay off because no agreement can occur when such a commitment sticks, resulting in a payoff of δv^* instead of a positive probability of receiving x^*. A downward deviation can be beneficial only if it increases the chance of a deal. Thus, it would have to allow for a deal in the event that *two* players' commitments stick: $y \leq 1 - x^* - \delta v^* = \delta v^*$. But this deviation would guarantee a payoff of δv^* instead of a positive probability to earn x^*. Thus, no deviation (up or down) can be beneficial and thus *under unanimity rule, the most* *aggressive commitment attempt always constitutes an equilbrium.*

Next, we prove that an efficient equilibrium exists only for ρ large enough. To see this, suppose that an efficient symmetric equilibrium exists. Then $x^* = 1/3$ (targeted to form a coalition consisting of 3 committed players). Consider an upward deviation to $y > x^*$ such that (at least) one loophole is needed for a deal to be made. The best such deviation is $y = x^* + (x^* - \delta v^*)$, seeking to capture the single "chunk" that becomes available when one player has a loophole. This deviation can affect the deviator's payoff only when his own commitment sticks, thus we can condition our analysis on this event. Then, agreement will no longer occur if both others' commitments stick. Thus, with probability ρ^2 the deviator loses $(x^* - \delta v^*)$. The only other condition under which the deviation affects his payoff is when he is responder and at least one other player has a loophole. (This occurs with probability $\frac{2}{3}(1-\rho^2)$.) In these cases, he gains$(x - \delta v^*)$ (the extra chunk). Summing up, conditional on his commitment sticking, the change in the deviator's expected payoff is given by $\frac{2}{3}(1-\rho^2)(x^* - \delta v^*) - \rho^2(x^* - \delta v^*)$. This is positive iff $\rho > \sqrt{\frac{2}{5}} > 1/2$. It follows that *under unanimity rule, an efficient symmetric equilibrium exists iff* $\rho \leq \sqrt{\frac{2}{5}}$, a condition that is less demanding than what we obtain in the model where the proposer is automatically uncommitted.

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