Persistent Unemployment, Sovereign Debt Crises, and the Impact of Haircuts

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Abstract

After 2008, the Southern European economies suffered a strong and persistent increase in unemployment. Rising government bond spreads necessitated the implementation of austerity policies. Austerity, however, may increase unemployment. If workers lose human capital during unemployment spells, the economy’s future production potential and thus the fiscal capacities to serve public debt will decline, aggravating a sovereign debt crisis. Debt renegotiations can help to avoid the costs of austerity. I introduce skill loss during unemployment in a dynamic stochastic model of sovereign debt with long-term debt and endogenous haircuts to study optimal fiscal policy in sovereign debt crises. In a quantitative exercise, I find that with higher intensity of the skill loss, ex ante, debt issuance declines and fiscal policy becomes less pro-cyclical. The government strategically uses fiscal capacities both to soften rising unemployment, reducing long-run productivity losses, and to support hiring when external conditions are more favorable.

Keywords: sovereign debt, debt renegotiations, haircuts, unemployment, austerity

JEL-Codes: E44, E62, F34, F41

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1 Introduction

During the Great Recession, the Southern European countries experienced a strong and persistent increase in unemployment. At the same time, rising spreads on sovereign bonds necessitated the implementation of austerity measures. However, procyclical fiscal policy may have adverse effects on employment, production, and tax revenues and thus elevate debt problems.

Beside the short-run costs from unemployment in form of lower production and higher expenditures on unemployment transfers, long-run effects on the countries’ production potential may occur. Empirical evidence suggests that workers’ human capital may depreciate during spells of unemployment (e.g. Dinerstein et al., 2022), such that an economy with increasing long-term unemployment becomes less productive. Debt renegotiations are an option to avoid the costs of fiscal consolidation.

The experiences of the Southern European countries during the Great Recession give rise to three important questions. How does skill loss during unemployment spells affect optimal fiscal policy in the presence of default risk and what is the impact of such fiscal policy on macroeconomic outcomes? What are the consequences of debt renegotiations on unemployment in the short and long run? Do creditors suffer smaller losses if they agree on haircuts at the beginning of a crisis, enabling the sovereign to avoid the loss of production potential through austerity? To study these questions, this paper introduces skill loss during unemployment in a dynamic stochastic model of sovereign debt with endogenous haircuts, long-term debt, and matching frictions.

The model features a small open economy populated by infinitely-lived households and a continuum of identical profit-maximizing firms. Households consist of workers. Employed workers face an exogenous job separation risk. Following Sterk (2016), workers are either high-skilled or low-skilled. If a high-skilled worker does not find a new job in the period of job separation, her human capital depreciates. Low-skilled unemployed workers regain their skills after one period of employment. In each period, firms decide on posting vacancies taking into account the realization of a productivity shock, the job separation rate, the average skills of job seekers, and a tax rate on sales. Matching follows a Cobb-Douglas function and depends on the number of job searchers and the number of vacancies posted.

The government finances public consumption and unemployment transfers by raising sales taxes and issuing external debt. Following Chatterjee and Eyigungor (2012), external debt is long-term and matures probabilistically. Risk-neutral private foreign investors borrow at the risk-free interest rate and provide credit. They have complete information about the current state of the economy and demand a premium which reflects the endogenous risk of renegotiations. The government cannot commit to repay its debt and has the option to enter debt renegotiations, in which case it suffers a one-time utility cost as in Bianchi et al. (2018). In the period of renegotiations, the government is excluded from international financial markets and bargains with the foreign creditors on the total surplus of an agreement. In the following period, the government regains
access to foreign credits with reduced debt obligations.

In a quantitative exercise, I apply the model to Portugal. The policy functions imply that the government enters debt renegotiations when employment and exogenous productivity are low and the debt level is high. With lower employment, the share of low-skilled workers increases and the expected lower productivity of job seekers reduces the willingness of firms to hire. It follows that, if debt is low and unemployment increases, the government issues debt and reduces taxes to increase the firms’ benefit from hiring. Thus, there is a positive feedback between employment and expansionary fiscal policy. At high debt levels, the government is borrowing constrained and prefers to enter debt renegotiations to avoid the negative impact of austerity on employment and the resulting productivity costs from human capital depreciation during unemployment spells.

From an ex ante perspective, a higher intensity of the skill loss during unemployment reduces public debt issuance and the pro-cyclicality of fiscal policy. Instead of entering renegotiations more frequently to reduce taxes, the government finds it optimal to issue less debt such that debt-to-GDP declines. This result is due to a general equilibrium effect. For a given debt and employment level, the probability of renegotiations increases in the intensity of the skill loss. Higher interest spreads would require larger tax hikes to service debt amplifying the procyclicality of fiscal policy. In general equilibrium, however, since interest rates are higher, the government is more borrowing constrained and issues less debt. Lower debt service obligations reduce taxes and dampen tax hikes during economic downturns. Thus, taxes become less procyclical. A recalibration exercise confirms the finding of a reduced procyclicality and thus emphasizes the motive of the government to avoid strong tax hikes which would be followed by a larger increase in unemployment and the loss of workers’ productivity. With increasing skill loss, firms increase job creation and the employment level is generally higher. There are two opposing forces at work. On the one hand, with a higher intensity of the skill loss, the productivity of job seekers is lower and newly hired workers are less productive such that the benefits from hiring decline. On the other hand, firms benefit from preventing skill depreciation of newly displaced workers by posting costly vacancies. The second effect dominates. Related to the model implication of less procyclical taxation in the presence of a higher intensity of the skill loss, cross-country evidence for OECD countries suggests that fiscal policy in economies with higher GDP per capita is less procyclical (c.p. Vegh and Vuletin, 2015).

From an intermediate perspective, responses to a negative productivity realization show that the government avoids as strong tax hikes as in the absence of human capital depreciation. The government also allocates resources to later periods to support hiring with stronger tax cuts when the negative external factors become weaker. The tax cut response in later periods follows from taking into account that hiring in times of high separation rates implies an increased loss of newly formed employment relationships in the following period.

To study the renegotiation dynamics, I run a series of simulations. I find that renegotiations are preceded by periods of increasing unemployment rates and rising debt-to-GDP ratios. Unem-
employment decreases after a haircut despite high shares of low-skilled job seekers, bad exogenous productivity realizations and high separation rates because the government has more fiscal space to reduce taxes and to improve the firms’ benefit from hiring. Due to low tax rates, the government starts accumulating debt again, such that the strong expansionary fiscal policy is short-lived and unemployment temporarily increases again.

I use my model to evaluate what would have been the optimal debt rescheduling decision for Portugal. First, I employ an analysis in which I choose a series of productivity shocks such that the spread of Portuguese government bonds from 2008 to 2017 is matched. Then, I perform a counterfactual analysis with debt renegotiations in 2011, 2012, and 2013, where 2011 was the year in which Portugal was granted official financial assistance. While the model predicts repayment as optimal decision, debt renegotiations in 2011 would have reduced unemployment immediately by 0.86 percentage points and in the medium run by up to 2.18 percentage points. Later restructurings are followed by smaller unemployment reductions. However, foreign creditors benefit from later haircuts because of reduced outstanding debt obligations at the time of renegotiations. The recovery rate increases from 17.16% in 2011 to 21.74% if debt is rescheduled in 2013.

In a robustness analysis, I consider variations of the sovereign’s bargaining power to assess the impact of the outcome of debt renegotiations. A higher bargaining power and thus larger haircuts make the government less reluctant to enter debt renegotiations at low debt levels. Due to the higher probability of debt renegotiations, the government faces higher bonds spreads and therefore accumulates less debt. The long-term average of the unemployment rates shows very little variation in the borrower’s bargaining power. On the one hand, lower debt accumulation implies a declining debt service and lower tax rates. On the other hand, more fiscal space allows for stronger tax reductions with a persistent, positive impact on employment. Because of the strong effect on the borrowing ability, welfare is higher when the borrower’s bargaining power and thus the size of the haircut is low.

**Related Literature.** The paper builds on three strands of the literature. First, this paper is related to the literature on fiscal policy and sovereign debt crises. Cuadra et al. (2010) rationalize pro-cyclical fiscal policy in a model with endogenous default risk and endogenous fiscal policy. Arellano et al. (2019), de Ferra (2018), and Kaas et al. (2020) study the impact of fiscal policy on firms’ access to credits during sovereign debt crises. Only few papers so far have considered unemployment dynamics in the context of sovereign debt crises and I contribute to this literature by the introduction of skill loss during unemployment. Bianchi et al. (forthcoming) study the trade-off between unemployment reduction through expansionary fiscal policy and increasing borrowing costs in a two-sector economy with downward-rigid nominal wages and fixed exchange rates. Na et al. (2018) consider the impact of devaluations on unemployment around sovereign defaults. Anzoategui (2022) quantifies the difference in the macroeconomic outcome of Spain during the European sovereign debt crisis when fiscal policy follows estimated pre-crisis rules in-
stead of fiscal consolidation. Balke and Ravn (2016) integrate matching frictions in a sovereign debt model and conclude that procyclical fiscal policy is optimal in normal times, but austerity is optimal during crisis times.\textsuperscript{1} Scholl (2023) focuses on the political consequences of progressive taxation in a sovereign debt model with heterogeneous agents and unemployment risk. All these papers abstract from persistence of unemployment and the transmission of the impact of distortionary fiscal policy to future periods, which is my focus here.\textsuperscript{2} Balke (2022) studies the impact of sovereign risk on unemployment via financial intermediaries. She finds that a default is accompanied by peaking unemployment rates since banks cannot provide the financial assistance required by firms to pay wages and vacancies. While her emphasis is on persistent unemployment as a cost of default in the absence of distortionary fiscal policy, unemployment in my model is amplified by fiscal consolidation. I provide an endogenous mechanism to explain variations in fiscal policy cyclicality by incorporating skill loss during unemployment.\textsuperscript{3}

Second, the paper is related to the literature on skill loss during unemployment. Pissarides (1992) employs an overlapping generations model to show that unemployment can become persistent if unemployed workers lose a part of their human capital. The reason is a reduction in vacancy posting as firms’ incentives to hire decrease with the skill level. The lower number of jobs implies an extended unemployment duration.\textsuperscript{4} Sterk (2016) uses a quantitative version of the model of Pissarides (1992) to study the occurrence of multiple steady states in labor market dynamics. In a similar set-up, Laureys (2021) focuses on quantitative effects of hiring externalities. The literature has further considered the contribution of differences in unemployment history to wage dispersion (Ortego-Marti, 2016) as well as the influence of skill loss on labor market fluctuations (Lalé, 2018) and the output and welfare costs of business cycles (Walentin and Westermark, 2022). Laureys (2014), Esteban-Pretel and Faraglia (2010), and Acharya et al. (2022) study the impact of monetary policy in the presence of human capital depreciation. I use a search and matching model with skill loss during unemployment that closely follows Laureys (2021) and Sterk (2016) to study the optimal fiscal policy during sovereign debt crises. Like Laureys (2021), I restrict to parameterizations for which no multiplicity of steady states occurs.

Third, this paper builds on the literature on sovereign debt renegotiations. Yue (2010) endogenizes debt recovery rates by incorporating Nash bargaining in a sovereign debt model. Further contributions include dynamic bargaining with incomplete informations (Bai and Zhang, 2012), the dependence of recovery rates on the debtor’s (Sunder-Plassmann, 2018) and the external creditors’s business cycle (Asonuma and Joo, 2020), and stochastic bargaining to generate delays in

\textsuperscript{1}Shi (2018) proposes a similar model, but abstracts from distortions of fiscal policy.

\textsuperscript{2}The persistent impact of distortionary fiscal policy during sovereign debt crises has only been studied in the context of the government’s ability to raise taxes (Pappadà and Zylberberg, 2019) and regional migration (Gordon and Guerron-Quintana, 2019).

\textsuperscript{3}In an alternative approach, Joo (2014) uses news shocks to explain differences in the cyclicality of fiscal policy across countries.

\textsuperscript{4}Another strand of the literature builds on Ljungqvist and Sargent (1998, 2008), where longer unemployment durations and bigger skill losses arise from higher unemployment benefits which alter the workers’ labor supply decisions.
debt renegotiations (Benjamin and Wright, 2013; Bi, 2008). While these papers assume that re-
structurings follow a default, Asonuma and Trebesch (2016) allow for preemptive restructurings. 
Hatchondo et al. (2014) study the possibility of voluntary debt exchanges with benefits to debtor 
and creditor. All these papers use endowment economies. Asonuma and Joo (2019) consider rene-
gotiations in a production economy with productive public capital. I contribute to the literature 
by studying endogenous haircuts in a production economy with skill loss during unemployment.

The remainder of the paper is structured as follows. In Section 2, I discuss the empirical evidence 
on unemployment and sovereign default risk in the Southern European countries during the Great 
Recession. I further review the empirical evidence on labor market dynamics during sovereign 
debt crises. In Section 3, I describe the theoretical model. Section 4 presents the quantitative 
results. Section 5 concludes.

2 Empirical Evidence on Austerity, Unemployment, 
Skill Loss, and Sovereign Default Risk

During the European Sovereign Debt crisis, several economies faced a large increase in unemploy-
ment. Interest rates on bonds increased because of increasing doubts on the countries’ ability to 
serve their debt. Figure 1 takes Spain and Portugal as examples and presents in the upper panels 
the spread between 10-year Spanish (Portuguese) and German government bonds (blue, left axis) 
and the cycles of HP-filtered real GDP (red, right axis) for the years from 2002 to 2016. The lower 
panels show the percentage share of long-term unemployed in the total number of unemployed 
(blue, left axis) and the unemployment rate (red, right axis).

While long-term interest rates before the crisis did not differ across the Euro zone, spreads strongly 
increased from 2008 to 2012 until the announcement of the Outright Monetary Transactions 
(OMT) program. Real GDP dropped, accompanied by rising unemployment rates and a lagged 
increase in long-term unemployment. Unemployment in general and long-term unemployment in 
particular remained at persistently high levels and did not return to pre-crisis levels until 2016.

Empirical evidence suggests that increased long-term unemployment is costly because of human 
capital depreciation during longer unemployment spells. Direct evidence is provided by Edin and 
Gustavsson (2008) who study the effect of unemployment on measurable skills. Using Swedish 
individual-level data, they find a skill reduction equivalent to 0.7 years of schooling in response 
to one year out of work. Similarly, Dinerstein et al. (2022) estimate a skill depreciation rate of 
4.3% using Greek administrative data on teacher’s assignments. Indirect evidence comes from 
studies on the impact of longer unemployment spells on re-employment wages, where theory 
suggests human capital depreciation and stigma effects to cause negative effects (Lopes, 2021). 
For Portugal, Lopes (2021) documents a reduction of re-employment wages of around 0.5% per 
additional month of unemployment duration. Negative effects last for at least four years after re-
employment. Centeno and Novo (2009) find an even larger wager reduction of 1%. Similar effects
Notes: The upper panels show the spread between the interest rates on 10-year Spanish (Portuguese) and German bonds (blue line, left axis) and the cycles of HP-filtered real GDP (red line, right axis) of Spain (Portugal) from 2002 to 2016 in the left (right) column. The lower panels show the share of long-term unemployed in percent of total unemployment (blue line, left axis) and the unemployment rate (red line, right axis). All data is from Eurostat.

have been found for other countries. Schmieder et al. (2016) consider German data and find a daily wage reduction of almost 1% per additional month of unemployment duration. Addison and Portugal (1989) report a wage reduction of around 1% in response to a 10% unemployment duration increase for the US between 1979 and 1984.\(^5\) Ortego-Marti (2017b) finds that wages depend on a person’s entire unemployment history. He uses a search and matching model with skill loss during unemployment to explain the observed differences in total factor productivity across OECD countries. Overall, the empirical literature implies that higher unemployment may be costly no only due to lower production in the short run, but also because of lower productivity in the medium and long run.

Empirical evidence further suggests that fiscal policy contributed to the unemployment increase during the European Sovereign Debt crisis. In response to rising bond spreads and a reduction of revenues because of the recession in the aftermath of the financial crisis, the Southern European

\(^5\)A broader literature has studied the effects of job loss on re-employment wages. For overviews, I refer to the excellent surveys of Couch and Placzek (2010), Fallick (1996), and Kletzer (1998).
Figure 2: Cyclicality of Fiscal Policy and Change in Unemployment

Notes: The figure plots the change in the unemployment rate between 2008 and 2012 against the correlation between the percentage changes of real GDP and the tax index of Vegh and Vuletin (2015) between 1996 and 2013 for 26 OECD economies. Unemployment data is from OECD.

Economies implemented strong fiscal consolidation measures. Alesina et al. (2015) construct a narrative data set on fiscal consolidation plans. For Portugal between 2010 and 2013, they find government spending cuts of 0.1% to 0.8% of GDP and revenue increases via tax hikes of 0.4% to 0.6% of GDP per year. The austerity policies, however, had adverse economic effects: Blanchard and Leigh (2013) find that spending cuts and tax hikes caused a further increase of the unemployment rate. In a related study, Vegh and Vuletin (2014) provide empirical evidence suggesting that the increase in unemployment was stronger for countries with a more procyclical fiscal policy. They find that the change in unemployment varied positively with the correlation between GDP and government spending and conclude that countries with more fiscal space before 2008 could respond more actively and reduce the duration and the intensity of the crisis.6 Figure 2 shows the correlation between unemployment and cyclicality of fiscal policy, expressed by the correlation between the percentage changes of real GDP and the tax index of Vegh and Vuletin (2015) between 1996 and 2013 for 26 OECD economies, using unemployment data from the OECD. The red OLS regression lines suggest that countries with a more procyclical fiscal policy suffered from a larger increase in unemployment, in line with empirical evidence by Agnello et al. (2014) and Ball et al. (2013) for spending- and tax-based consolidations in OECD countries between 1978 and 2009.

Debt renegotiations with a following debt service reduction could have been an alternative to avoid austerity policies. However, evidence on the effect of debt renegotiations on unemployment is scarce. During the European Sovereign Debt crisis, only Greece renegotiated its debt in 2012, 6Vegh and Vuletin (2014) argue that these results confirm the finding of reduced duration and intensity of crises in Latin American countries with countercyclical fiscal policy, specifically Brazil and Chile after 1998.
receiving a haircut of 100 billion euro on its privately held debt and being granted official financial assistance in return to the implementation of austerity policies. Despite the debt reduction, the debt ratio remained above 160\% of GDP in 2013. Greek GDP further dropped and unemployment persistently increased.\(^7\) The observations for Greece are in contrast to the experience with emerging economies. Levy Yeyati and Panizza (2011) examine 20 default episodes between 1980 and 2006 and find that in the quarters before defaults unemployment rates remain constant or increase and in the quarters following a default unemployment rates tend to decrease.

3 The Model

3.1 The Environment

I consider a small open economy inhabited by identical infinitely lived households and a continuum of identical profit-maximizing firms. Households consist of employed and unemployed workers, own the firms, and derive utility from private consumption. Following Sterk (2016) and Laureys (2021), workers differ in their skills. Skill loss occurs during unemployment. Firms use labor as input and are subject to productivity shocks and matching frictions. In each period, they choose their optimal amount of hiring. The government imposes sales taxes and issues external debt on international financial markets. Debt contracts mature probabilistically as in Chatterjee and Eyigungor (2012), are not enforceable and subject to default risk. In each period, the government can decide to enter debt renegotiations. In renegotiation periods, the government loses access to international financial markets and suffers an exogenous one-time utility cost which depends on the realization of the productivity shock. The utility cost can be interpreted as a loss of reputation to the incumbent.\(^8\) Following Yue (2010), debt renegotiations take the form of one-round Nash bargaining between the policymaker and the international creditors. Renegotiations last one period and end with an agreement on the government’s debt obligations in the following period.

**Workers.** At the beginning of each period, a share \(n_t \in [0, 1]\) of workers is employed. Employed workers lose their job with exogenous probability \(\rho_x\). I follow Pries (2008) and assume the job separation rate \(\rho_{x,t} = \rho_x(z_t) \in [0, 1]\) to be a function of productivity shocks \(z_t\).\(^9\) Following the realization of the productivity shock, firms post vacancies and hire such that unemployed workers find a job with probability \(\rho_{f,t} \in [0, 1]\). Workers are heterogenous with respect to their skills and are either high-skilled (\(H\)) or low-skilled (\(L\)). As in Sterk (2016), high-skilled workers become low-skilled after one period of unemployment. In particular, high-skilled workers lose a fraction

\(^7\)C.p. Zettelmeyer et al. (2013) and Eurostat data.

\(^8\)The formulation of the utility cost is similar to Bianchi et al. (2018) where the cost depends on the realization of an endowment shock. Alternative formulations of exogenous utility costs have been considered by Chang (2007), Müller et al. (2019), and Roch and Uhlig (2018).

\(^9\)With constant separation rates, productivity shocks generate too small employment fluctuations in standard matching models with flexible wages.
\( \xi \) of their productivity. Low-skilled workers who find a job become high-skilled in the following period. The share \( p_t \) of low-skilled job seekers is

\[
p_t = \frac{u_t}{u_t + p_{x,t}(1 - u_t)},
\]

where \( u_t = 1 - n_t \) is the fraction of unemployed workers at the beginning of a period before the productivity shock is realized. The denominator denotes the total share of job seekers consisting of previously and newly unemployed workers. I assume employed workers receive skill-dependent wages \( w_t^{\{H,L\}} \). Job seekers who cannot find a job obtain transfers \( T_t^{\{H,L\}} \) which equal a fixed share \( \Omega \) of the respective wage \( w_t^{\{H,L\}} \), such that newly unemployed workers receive higher payments.

In my quantitative exercise in Section 4, I will calibrate the model based on annual data. Thus, I will interpret workers that lose their job at the beginning of a period and cannot find a new job in the same period as short-term unemployed. Short-term unemployed workers are still high-skilled and thus receive higher transfers. A worker who is left without a job for more than a period, equivalent to more than a year, will be considered as long-term unemployed.

**Firms.** Production uses labor as input, follows a constant return to scale production technology \( f(n_t), f : \mathbb{R}_+ \to \mathbb{R}_+ \), and is subject to productivity shocks \( z_t \). I assume that productivity \( z_t \in \mathcal{Z} \) has a compact support, \( \mathcal{Z} = [\underline{z}, \overline{z}] \subset \mathbb{R}_+ \), and follows a Markov process with transition function \( \mu(z_{t+1}, z_t) \). Firms pay sales taxes \( \tau_t \). After job separation, firms decide on hiring \( h_t = n_{t+1} - (1 - \rho_{x,t})n_t \), where \( n_{t+1} \) is the new optimal employment level. I follow Sterk (2016) and assume that firms cannot observe the skill status of new hires before hiring such that search for new hires is entirely random. However, I assume that firms know the skill level directly after hiring and pay wages \( w_t^{H} \) and \( w_t^{L} \) accordingly. A share \( p_t \) of hires is low-skilled with productivity reduced by a fraction \( \xi \). In each period, firms post a number of vacancies \( v_t \) at a fixed cost \( \kappa > 0 \) per unit. When production is linear in labor, firms’ per-period profits \( \Pi_t \) are given by

\[
\Pi_t = (1 - \tau_t)z_t(n_{t+1} - \xi p_t h_t) - (n_{t+1} - p_t h_t)w_t^H - (p_t h_t)w_t^L - \kappa v_t,
\]

where

\[
n_{t+1} = (1 - \rho_{x,t})n_t + h_t
\]

\[
h_t = a_t v_t
\]

\[
h_t \geq 0.
\]

Condition (3) is the transition equation of employment. Condition (4) defines hiring as the product of the vacancy yield \( a \) and vacancy posting. Condition (5) is the non-negativity constraint on hiring.

Matching follows a Cobb-Douglas function such that the number of hires is given by

\[
h_t = s_t^\alpha v_t^{1-\alpha},
\]
where \( s_t = u_t + \rho x_t (1 - u_t) \) denotes the number of job seekers. As in Pissarides (2000), wage setting follows from repeated Nash bargaining on the total surplus of a job and is described in detail in Appendix B.\(^{10}\) Households receive the firm profits.

**Households.** The representative household derives utility from private consumption \( c_t \). Let preferences be described by a per-period utility function \( u: \mathbb{R}_+ \rightarrow \mathbb{R} \), which is continuous, twice differentiable and strictly increasing in \( c_t \), concave in \( c_t \), and satisfies the Inada conditions. The household finances private consumption with profits from firms \( \Pi_t \), wage income from employed workers and transfers from unemployed workers. A share \( n_{t+1} \) of workers is employed and receives a wage \( w_t^{(H,L)} \), which depends on the skill level of the worker. A share \( p_t \) of newly employed workers is low-skilled \((L)\), while all other employed workers are high-skilled \((H)\). Unemployed workers receive transfers \( T_t^{(H,L)} \). Only newly unemployed workers are high-skilled and receive the higher transfer payment \( T_H \).

The household budget constraint is given by

\[
    c_t = \Pi_t + (n_{t+1} - p_t h_t) w_t^H + (1 - p_t)(1 - n_{t+1}) T_t^H + p_t h_t w_t^L + p_t (1 - n_{t+1}) T_t^L. \tag{6}
\]

**Government.** The government has access to incomplete international financial markets. It uses revenues from sales taxes and issuance of non-contingent bonds to finance public consumption \( g_t \) and unemployment transfers \( T_t^{(H,L)} \). I follow Chatterjee and Eyigungor (2012) and assume a bond to mature in the next period with probability \( \delta \) and otherwise to imply a coupon payment \( \psi \). I assume for simplicity that public consumption does not provide utility and set the size of public consumption to a constant share \( \gamma \) of private consumption such that \( g_t c_t = \gamma \). Let total transfer payments \( T_t \) be the sum of transfers to high-skilled and low-skilled unemployed:

\[
    T_t = (1 - p_t)(1 - n_{t+1}) T_t^H + p_t (1 - n_{t+1}) T_t^L.
\]

If the government repays its debt, the government budget constraint reads as

\[
    g_t + T_t = \gamma_t z_t (n_{t+1} - \xi p_t h_t) - q_t (b_{t+1} - (1 - \delta) b_t) + (\delta + (1 - \delta) \psi) b_t, \tag{7}
\]

where \( q_t \) denotes the unit price of a bond of size \( b_{t+1} \) when the government faces the productivity shock \( z_t \) and the firms choose the current period employment level \( n_{t+1} \). In the following, similar to Aguiar et al. (2022), I will assume without loss of generality that the coupon payment \( \psi \) is equal to the real interest rate \( r \). To prevent the government from choosing maximal debt dilution before renegotiations, I follow Hatchondo et al. (2016) and impose a lower bound on the bond price for new debt, \( q(z, b', n') \geq q \).\(^{11}\)

If the government decides to enter renegotiations, it suffers an exogenous one-time utility cost

\(^{10}\)In the set-up with productivity shocks and sales taxes, endogenous wages ensure that the net present value of a firm is non-negative.

\(^{11}\)In a similar approach, Chatterjee and Eyigungor (2015) use the observation of limits on the expected immediate default risk of newly issued bonds in sovereign debt markets and restrict new debt issuance to levels where the default probability does not exceed an exogenous threshold.
\( \chi(z_t) \) and is excluded from international financial markets for the rest of the period. The size of the utility cost \( \chi(z_t) \) is exogenously determined by the realization of the productivity shock \( z_t \), similar to the cost in Bianchi et al. (forthcoming) and Bianchi et al. (2018) who consider an income utility cost depending on the realization of an endowment shock. The budget constraint reads as

\[
g_t + T_t = \tau_t z_t (n_{t+1} - \xi p_t h_t). \tag{8}
\]

The government regains access to international financial markets in the period after debt renegotiations.

**International Creditors.** There is a continuum of identical infinitely-lived international creditors. International creditors are risk-neutral and borrow from international markets at the constant risk-free interest rate \( r \). They have perfect information about the state of the economy. International creditors demand a risk premium and internalize the risk of debt renegotiations and the expected return from an agreement.

**Timing.** The timing is as follows. At the beginning of a period, job separation occurs. The policymaker observes the realization of productivity \( z_t \) and chooses its optimal policies. The firms take the public sector policies as given and post vacancies. After hiring, production takes place with initially employed workers and new hires. At the end of the period, separated workers that have not found a job become low-skilled. If the government chooses to enter debt renegotiations at the beginning of the period, there will be an agreement on the new debt level before the firms decide on hiring.

### 3.2 Recursive Equilibrium

In equilibrium, firms take the government’s policy decision as given and maximize the expected discounted life-time profits subject to a non-negativity constraint on hiring. The government maximizes the expected life-time utility of households taking into account the optimal response in firms’ decisions.\(^{12}\) In each period, the government can choose to repay its debt or to enter debt renegotiations, where the government suffers a utility cost. Sovereign debt renegotiations take the form of one-time Nash bargaining between the foreign creditors and the policymaker. Foreign creditors are risk-neutral, borrow or lend from international financial markets at the risk-free interest rate \( r \), have perfect information about the state of the economy, and charge a risk premium. The following subsections describe the optimization problems of the firms and the government, the details of the debt renegotiations, and the zero-profit condition of foreign creditors. The formal definition of the recursive equilibrium is provided in Appendix A.

\(^{12}\)Ortigueira (2006) shows that the quantitative results of a Markov equilibrium may change if the public and the private sector choose their policies simultaneously.
3.2.1 The Private Sector

The firms take the public sector policies as given and maximize their expected discounted lifetime profits. I assume that firms discount future profits with the stochastic discount factor of the households. As sales taxes are uniform, the decisions on vacancy posting and hiring are identical for all firms. The optimality condition of the firms is given by:

\[(1 - \tau)z(1 - \xi p) - (1 - p)w^H - pw^L + \lambda
+ \beta \int_{z'} \frac{u'(c')}{d'(c)} (1 - \rho_x(z')) \left( (1 - \tau')z'\xi p' - p'(w^{H'} - w^{L'}) + \frac{\kappa'}{a'} - \lambda' \right) dz' = \frac{\kappa}{a},\]  

(9)

where \(\lambda\) is the Lagrange multiplier from the non-negativity constraint on hiring. The term on the right-hand side denotes the hiring costs per new hire. On the left-hand side, the term in the first line gives the expected profit from a newly hired worker to the firm in the current period. The term in the second line describes the present discounted expected benefit from having a high-skilled worker in the next period instead of hiring a new potentially low-skilled worker.

3.2.2 The Public Sector

In each period, the policymaker chooses between two options:

\[V(z, b, n) = \max \left\{ V^R(z, b, n), V^D(z, b, n) \right\}.\]  

(10)

\(V^R(z, b, n)\) denotes the value function of the government in case of debt repayment. \(V^D(z, n)\) is the value function in case of sovereign debt renegotiations. The discount factor \(\beta \in [0, 1]\) is common for all individuals in the economy.

When the government chooses to repay its debt, the value function solves:

\[V^R(z, b, n) = \max_{b', \tau, g} \left\{ u(c) + \beta \int_{z'} V(z', b', n') \mu(z', z) dz' \right\},\]  

subject to

\[g + T = \tau z(n' - \xi ph) - q(z, b', n')(b' - (1 - \delta)b) + (\delta + (1 - \delta)\psi)b\]
\[c = \Pi + (n' - ph)w^H + phw^L + T\]
\[\frac{q}{c} = \gamma\]
\[q(z, b', n') \geq q\]
\[p = \frac{1 - n}{1 - n + \rho_x n}\]
\[n' = N(z, \tau, n), h = H(z, \tau, n), \]

where the private sector policies \(n'\) and \(h\) follow from the optimality condition (9).

If the policymaker chooses to enter debt renegotiations, the economy is excluded from international financial markets for the rest of the period and the policymaker suffers a one-time utility cost \(\chi(z)\).

The government and external creditors bargain to find an agreement on the new debt level \(\tilde{b}\). The
renegotiation mechanism is described in detail in Section 3.2.3. In the following period, the government can reenter financial markets and decides on serving the reduced debt stock. The value function associated with debt renegotiations is given by:

\[ V^D(z, b, n) = u(c) - \chi(z) + \beta \int_{z'} V(z', \tilde{b}(z, b, n), n') \mu(z', z) dz' \]  

subject to

\[
\begin{align*}
g + T &= \tau z(n' - \xi ph) \\
c &= \Pi + (n' - ph) w^H + ph w^L + T \\
\frac{g}{c} &= \gamma \\
p &= \frac{1 - n}{1 - n + \rho_x n} \\
n' &= \mathcal{N}(z, \tau, n), h = \mathcal{H}(z, \tau, n).
\end{align*}
\]

The following indicator function describes the government’s choice on entering debt renegotiations:

\[
d(z, b, n) = \begin{cases} 
1 & \text{if } V^R(z, b, n) < V^D(z, b, n) \\
0 & \text{else.}
\end{cases}
\]

The set of productivity shocks \( z \in \mathcal{R}_z \) for which the government enters debt renegotiations is given by:

\[ \mathcal{D}(b, n) = \{ z \in \mathcal{R}_z : d(z, b, n) = 1 \}. \]

The probability of debt renegotiations reads as

\[ \eta(z, b', n') = \int_{\mathcal{D}(b', n')} \mu(z', z) dz'. \]

### 3.2.3 Debt Renegotiations

As in Yue (2010), debt renegotiations follow a generalized Nash bargaining game, in which an agreement implies a new debt level \( \tilde{b} \). The value of the agreement to the government is given by \( V^D(z, b, n) \). The creditors receive the present value of the reduced debt in terms of expectations \( q(z, \tilde{b}, n') \tilde{b} \). The government’s outside option to an agreement is permanent autarky, while external creditors would lose their investment. The expected continuation value of autarky reads as

\[ V^A(z, n) = u(c) + \beta \int_{z'} V^A(z', n') \mu(z', z) dz', \]
where

\[ g + T = \tau z (n' - \xi ph) \]
\[ c = \Pi + (n' - ph)w^H + phw^L + T \]
\[ \frac{g}{c} = \gamma \]
\[ p = \frac{1 - n}{1 - n + \rho x n} \]
\[ n' = \mathcal{N}(z, \tau, n), h = \mathcal{H}(z, \tau, n). \]

The borrower’s bargaining surplus is given by

\[ \Delta_B(z, \hat{b}, n) = V^D(z, b, n) - V^A(z, n), \]

where \( V^D(z, b, n) \) changes with the negotiated debt level \( \hat{b}(z, b, n) \), see Equation (12).

The bargaining surplus of the international creditors reads as

\[ \Delta_L(z, \hat{b}, n) = q(z, \hat{b}, n') \hat{b}. \]

Let \( \theta \) denote the borrower’s bargaining power. The bargaining problem solves

\[ \tilde{b} = \arg\max_{b \in [0, 1]} \left[ \left( \Delta_B(z, \hat{b}, n) \right)^\theta \left( \Delta_L(z, \hat{b}, n) \right)^{1-\theta} \right] \tag{15} \]

subject to

\[ \Delta_B(z, \hat{b}, n) \geq 0, \]
\[ \Delta_L(z, \hat{b}, n) \geq 0. \]

The recovery rate is given by the ratio of the debt stocks before and after the renegotiations \( \tilde{b} \in [0, 1] \).

### 3.2.4 International Creditors

International creditors are risk-neutral and internalize the risk of debt renegotiations and the expected return from an agreement. The bond price follows from the zero-profit condition:

\[ q(z, b', n') = \frac{1}{1 + r} \int_{z'} (1 - d(z', b', n') (\delta + (1 - \delta)(\psi + q(z', b'', n''))) \mu(z', z) dz' \]
\[ + \frac{1}{1 + r} \int_{z'} d(z', b', n') \tilde{b}' q(z', \tilde{b}, n'') \mu(z', z) dz'. \]

The spread is the difference between the internal rate of return \( i(z, b', n') \) and the risk-free rate \( r \), where \( i \) follows from \( q(z, b', n') = (\delta + (1 - \delta)\psi)/(\delta + i(z, b', n')) \). The bond price lies in the interval \([0, (\delta + (1 - \delta)\psi)/(\delta + r)]\).
4 Quantitative Analysis

4.1 Calibration

For the quantitative analysis, I calibrate the model to Portugal for the time period from 1995 to 2017. A period in the model corresponds to a year. The annual calibration implies that a low-skilled worker has been unemployed for at least one year and can be considered as long-term unemployed. Kroft et al. (2013) provide empirical evidence for the U.S. which suggests that the probability of receiving an interview call-back is decreasing in unemployment duration, where most of the decline occurs within the first eight months of unemployment. Thus, it seems reasonable to assume that most of the human capital depreciation occurs during the first year after job loss. Table 1 summarizes the parameter values. I employ seasonally adjusted annual series for real GDP, real private consumption, real government consumption, the unemployment rate and the long-term interest rates of Portugal and Germany which are taken from Eurostat. For external debt statistics, I rely on OECD data.

I assume that the per-period utility of households is specified by the following constant relative risk-aversion (CRRA) utility function

\[ u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \]

where \( \sigma \) is the relative risk aversion. I set \( \sigma \) to 2, which is a standard value in the literature. The annual risk-free interest rate \( r \) of 4.2 percent corresponds to the average German 10Y-bond yield.

Following Cuadra et al. (2010), I assume that production is linear in labor, \( f(n) = n \). Productivity shocks follow an AR(1) process:

\[ \ln(z') = \rho_z \ln(z) + \epsilon, \]

where \( \epsilon \) is i.i.d. \( N(0, \sigma^2_\epsilon) \). The values for the parameters \( \rho_z \) and \( \sigma_z \) are set to match the autocorrelation and standard deviation of the log-quadratically detrended annual Portuguese real GDP series between 1995 and 2017.

For simplicity, similar to Pries (2008), I assume the job separation rate \( \rho_x \) to be negatively correlated with the productivity shock \( z \). In particular, I choose the following relationship which ensures that the job separation rate is always non-negative:

\[ \rho_x = \bar{\rho}_x \exp(-\sigma_x \ln(z)), \]

where \( \bar{\rho}_x \) is a constant around which the separation rate fluctuates. \( \sigma_x \) is a parameter for the sensitivity of the job separation rate to productivity \( z \). Because of the choice of an annual calibration, I consider workers that lose their job at the beginning of a period and cannot find a new job in the same period as short-term unemployed. Since the separation rate determines the unemployment inflow, I set \( \bar{\rho}_x \) to match the mean of the short-term unemployment rate of 4.34%.
### Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Full model</th>
<th>No skill loss</th>
<th>Source / Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Externally calibrated parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk-free rate $r$</td>
<td>0.042</td>
<td>0.042</td>
<td>Mean German 10-Y bond rate</td>
</tr>
<tr>
<td>Relative risk aversion $\sigma$</td>
<td>2.0</td>
<td>2.0</td>
<td>Standard value</td>
</tr>
<tr>
<td>Public good weight $\gamma$</td>
<td>0.32</td>
<td>0.32</td>
<td>Mean of $g/c$</td>
</tr>
<tr>
<td>Maturing probability $\delta$</td>
<td>0.16</td>
<td>0.16</td>
<td>Mean residual maturity</td>
</tr>
<tr>
<td>Matching function elasticity $\alpha$</td>
<td>0.50</td>
<td>0.50</td>
<td>Petrongolo and Pissarides (2001)</td>
</tr>
<tr>
<td>Workers’ bargaining power $\omega$</td>
<td>0.50</td>
<td>0.50</td>
<td>Laureys (2021)</td>
</tr>
<tr>
<td>Size of transfers $\Omega$</td>
<td>0.50</td>
<td>0.50</td>
<td>Replacement rate 50%</td>
</tr>
<tr>
<td><strong>Internally calibrated parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor $\beta$</td>
<td>0.88</td>
<td>0.88</td>
<td>Mean external debt to GDP</td>
</tr>
<tr>
<td>Utility cost (intercept) $\Lambda_0$</td>
<td>0.91</td>
<td>0.87</td>
<td>90% quantile of spread</td>
</tr>
<tr>
<td>Utility cost (slope) $\Lambda_1$</td>
<td>9.2</td>
<td>8.7</td>
<td>$</td>
</tr>
<tr>
<td>Borrower’s bargaining power $\theta$</td>
<td>0.35</td>
<td>0.54</td>
<td>Market value of renegotiated debt 23%</td>
</tr>
<tr>
<td>Skill cost $\xi$</td>
<td>0.465</td>
<td>–</td>
<td>Wage ratio $w^L/w^H$ of 58.48%</td>
</tr>
<tr>
<td>Vacancy cost $\kappa$</td>
<td>3.16</td>
<td>2.65</td>
<td>Mean unemployment rate</td>
</tr>
<tr>
<td>Mean job separation rate $p_x$</td>
<td>0.096</td>
<td>0.096</td>
<td>Mean short-term unemployment rate</td>
</tr>
<tr>
<td>Separation rate sensitivity $\sigma_x$</td>
<td>9.2</td>
<td>9.8</td>
<td>Std. $u_{long-term}/u$</td>
</tr>
<tr>
<td>Productivity autocorrelation $\rho$</td>
<td>0.85</td>
<td>0.85</td>
<td>Autocorrelation of real GDP</td>
</tr>
<tr>
<td>Standard deviation of $\epsilon$</td>
<td>$\sigma_\epsilon$</td>
<td>0.0112</td>
<td>0.0113</td>
</tr>
</tbody>
</table>

1 $|\Delta s| = |s - s_{-1}|$ denotes the absolute change in the bond spread.

The sensitivity parameter $\sigma_x$ affects the volatility of the job separation rate and contributes to the fluctuations in the number of short-term unemployed. It thus influences the volatility of both the shares of short-term and long-term unemployed on total unemployment. I choose the volatility of long-term unemployment on total unemployment of 6.87 as target for $\sigma_x$.

In my model, the ratio of government consumption over private consumption is fixed. I set the parameter $\gamma$ to 0.32, which corresponds to the empirical mean ratio of government to private consumption.$^{13}$ I follow Bianchi et al. (2018) and assume the utility cost$^{14}$ to be specified by

$$\chi(z) = \max\{0, \Lambda_0 + \Lambda_1 \log(z)\}.$$ 

I set the utility cost parameters to match characteristics of the spread. In the data, the spreads are close to zero for a large share of periods and exhibit spikes after sequences of increasing default risk. In the model, spreads are permanently substantially larger than zero because the government

$^{13}$For the time period from 1995 to 2017, this ratio has a standard deviation of 0.90% and fluctuates between 30.59% and 33.65%.

$^{14}$In Bianchi et al. (2018) the cost depends on the realization of an endowment. I slightly deviate from this specification in considering a production economy such that the utility cost depends on productivity.
has a low discount factor and chooses large debt levels, which creates a trade-off when trying to match the mean and the volatility of the spread simultaneously. For instance, when matching the mean spread, the spikes cannot be reproduced without triggering debt renegotiations and the spreads are not volatile enough. I thus choose the 90%-quantile of the absolute change of the spread, $|\Delta s| = |s - s_{-1}|$, of 2.69% as a target for the slope $\Lambda_1$, which drives the steepness of the bond price curve. The intercept $\Lambda_0$ is set to match the 90% quantile of the spread (5.3%). With respect to the short length of the time series, I choose the 90%-quantile to ensure that the calibration does not target potentially large outliers. For reasons of completeness, I will also report the maximum values.

Since Portugal has not renegotiated its debt obligations over the past 100 years, I only use simulation episodes of at least 23 years without renegotiations to calibrate the model parameters. I only refer to the complete simulation sample to pin down the bargaining power of the borrower in debt renegotiations, $\theta$, which is set to match a market value of outstanding debt during debt renegotiations of 23%. This value corresponds to Greece in 2012 (Zettelmeyer et al., 2013), which has been the only case of a default on external debt during the European sovereign debt crisis. The analysis of Section 4.7 includes the results for alternative values of the bargaining power. The discount factor $\beta$ is set to match the average external debt-to-GDP ratio of 50.18%. The value of the maturing probability $\delta$ corresponds to an average residual maturity of 6.3 years. The minimum issuance price for a bond $q$ is set to 0.5748, equivalent to 60% of the mean default-free price. Thus, the maximum allowed annual yield to maturity is 13%, which is above the highest annual yield observed for issuances by European governments after 2008 (c.p. Hatchondo et al., 2016). The constraint is not binding in the simulations.

The matching function elasticity $\alpha$ is set to 0.5, which is in the range of estimates reported by Petrongolo and Pissarides (2001). I follow standard practices and set the bargaining power of workers $\omega$ to the same value, which ensures that the congestion externality, which follows from the search frictions, is internalized when all job seekers find a match with the same probability (Laureys, 2021). The Portuguese unemployment insurance system features a gross replacement rate of 65% and a net replacement rate of more than 90% (Esser et al., 2013). However, for 2011, Matsaganis et al. (2014) report shares of benefit recipients of only 37.4% on workers that are unemployed for 3 to 5 months as well as 42.9% and 23.2% for those with an unemployment duration of 6 to 11 months and more than 11 months, respectively. To take account of these factors, I set the size of transfers in terms of wages $\Omega$ to 0.5 and provide a robustness analysis on this parameter in Section 4.8. The vacancy cost $\kappa$ is set to match the empirical mean unemployment rate of 8.67%. For the calibration of the skill cost parameter, $\xi$, I refer to empirical evidence from Lopes (2021). She uses Portuguese social security data from 2005 to 2012 and finds a decline in

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15This value is based on monthly data from the Portuguese national statistics office for the time span from December 2000 to December 2017.

16This value ensures that there are no multiple steady states, see e.g. Pissarides (1992). Sterk (2016) studies multiple steady states using a calibration with $\alpha > 0.5$. 

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the re-employment wage of around 0.5% for each additional month of unemployment duration, which is significant for the first 5 years of reemployment. Workers in the sample are on average jobless for around 18 months. Since the complete wage loss in the model materializes in the first year of reemployment, I set $\xi$ such that the wage of a low-skilled worker equals 58.48% of the wage of a high-skilled worker. The wage gap is the result of 9% wage loss during 5 years, where future wage losses are discounted with the real interest rate.

### 4.2 Policy Functions

I first consider the optimal decision on entering debt renegotiations. The left panel of Figure 3 presents the combinations of productivity shocks $z$ and debt levels $b$ for which the government enters debt renegotiations. The solid line refers to an employment level $n$ of 80.5%. The dashed (dashed-dotted) line shows the combinations for an employment level of 91% (100%). The right panel presents the optimal decision for combinations of productivity shocks $z$ and employment $n$. The solid, dashed and dashed-dotted lines represent the choice for a high ($-0.59$), medium ($-0.45$), and low debt level ($-0.31$). In the areas to the left of the lines, the government enters debt renegotiations. Repayment is optimal for states to the right of the lines. The panels reveal that renegotiations are optimal when the realization of the productivity shock is low, debt is high, and employment is low.

**Figure 3: Renegotiation Decision**

Notes: The left (right) panel shows the combinations for the productivity shock $z$ and debt $b$ (employment $n$) for which the government just prefers to enter debt renegotiations. Right of the lines, the government prefers to serve its debt obligations. The lines in the left panel refer to an initial employment level of 80.5% (solid), 91% (dashed), and 100% (dashed-dotted). The lines in the right panel refer to initial debt of $-0.59$ (solid), $-0.45$ (dashed), and $-0.31$ (dashed-dotted).

Figure 4 presents the borrowing decisions, the bond price functions, the recovery rates, the tax policies, the employment decisions, and endogenous productivity for employment levels of 91% (solid lines) and 80.5% (dashed lines) as functions of the debt level. Endogenous productivity refers to the share of employed workers who are high-skilled. The first and third (second and
Figure 4: Policy Functions - Fixed Initial Employment Level

Notes: The solid (dashed) lines represent the debt policies $b'(b, n, z)$, the bond prices $q(b', n', z)$, the recovery rates $\tilde{b}(n', z)/b$, the tax policies $\tau(b, n, z)$, and the employment policies $n'(b, n, z)$ at different debt levels $b$ for an employment level $n$ of 91% (80.5%). Endogenous productivity denotes the share of employed workers who are high-skilled. In the first and third column, productivity is 3.1% below the trend. High productivity refers to a level 3.1% above the trend.
fourth) column refer to productivity realizations 3.1% below (above) the trend. The panels reveal that the bond price is decreasing in the debt level and increasing in exogenous productivity $z$. For small debt levels, the government never finds it optimal to enter renegotiations such that the bond price converges towards the risk-free bond price. With increasing debt, the government has more incentives to dilute debt in the future and the probability of debt renegotiations increases. For high debt levels, the government always enters renegotiations and the bond price equals the expected bond price at the bargained debt level weighted with the debt recovery rate. Since the outcome of bargaining $\tilde{b}$ is independent from the initial debt level $b$ by assumption for any $b < \tilde{b}$, the recovery rate $\tilde{\frac{b}{b}}$ has an upper bound of 1 and decreases in the debt level.\(^{17}\)

For a lower level of employment, the government faces lower bond prices. Lower employment implies a smaller tax base, higher expenditures for unemployment transfers, and a lower expected skill level of new hires such that the government is less reluctant to enter debt renegotiations. The recovery rates are marginally increasing with higher unemployment. The underlying reason is the increased benefit from access to financial markets in form of the ability to set lower tax rates and thus to improve hiring incentives.

The third and fourth column of Figure 4 present the tax policies, the employment decisions, and endogenous productivity. With increasing debt, the government has to impose higher sales taxes to pay the interest rates on its debt obligations. The higher tax rates reduce the firms’ benefit from hiring and the employment level decreases. Lower hiring implies a lower inflow of low-skilled workers such that the share of employed workers who are high-skilled increases. The employment level is increasing in initial employment and exogenous productivity $z$. The net change in employment decreases in the initial employment level because of increasing hiring costs per worker. Endogenous productivity is higher for low productivity shocks because lower hiring reduces the inflow of low-skilled workers and higher job separation rates increase the share of high-skilled job seekers and thus the share of high-skilled new hires.

When debt is low, the tax rate is increasing in the employment level. There are two underlying channels. First, higher taxes have a negative effect on the firms’ hiring incentives. As a reduction in unemployment implies higher production and a higher skill level of newly hired workers in the future, the government has an incentive to set lower tax rates when unemployment is high. The government can reduce the taxes and improve hiring incentives by higher debt issuance. Second, when employment is low, firms have higher expenditures for vacancy posting. It follows that firm profits drop and private consumption declines overproportionally compared to the change in firms’ output. Thus, the government needs a smaller share of total production to finance the public good which is proportional in size to private consumption.\(^{18}\) The reduction in public consumption

\(^{17}\)While the assumption of independence of initial debt is common to models with one-round bargaining, Cruces and Trebesch (2013) find that a higher debt-to-GDP ratio may imply longer durations of renegotiations. Benjamin and Wright (2013) provide evidence on the relation between renegotiation outcome and duration.

\(^{18}\)Assuming a constant government spending would extinguish this channel. However, tax rates would have to strongly increase in unemployment.
dominates increasing transfer payments. When debt is high, the government is more borrowing constrained such that the tax rate increases with higher unemployment rates. Additionally, higher debt service in the presence of lower employment necessitates a larger increase in sales taxes. It follows that high debt has a quantitatively smaller negative effect on the employment level when initial employment is high.

Figure 5: Policy Functions - Fixed Initial Debt Level

<table>
<thead>
<tr>
<th>New Debt Issuance</th>
<th>Tax Rate $\tau(n, z)$</th>
<th>Recovery Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The graphs show the amount of new debt issuance, the tax policies, and the recovery rates at different employment levels $n$ for a debt level of $-0.45$, which is close to the mean in simulations. Blue (red) lines refer to exogenous productivity 3.1% above (below) the trend. New debt issuance is defined as $b' - (1 - \delta)b$ in repayment and 0 if the government enters debt renegotiations. Blue and red lines refer to exogenous productivity 3.1% above and below the trend, respectively. The government enters debt renegotiations when exogenous productivity is low and repays its debt obligation when exogenous productivity is high (c.p. the dashed line in the right panel of Figure 3). During renegotiations, the government is excluded from international financial markets and has to impose higher tax rates at lower employment levels to finance rising unemployment benefits. When productivity is high, borrowing and tax rates are U-shaped in initial employment. At high employment levels, the government can react to an increase in unemployment with extended borrowing and tax cuts to increase the firms’ benefits from additional hiring. When employment is low, the government has to make higher total unemployment transfer payments and is more borrowing constrained such that it issues less new debt and has to set higher tax rates. Tax rates for high exogenous productivity exceed tax rates for low exogenous productivity at high employment levels where the government enters renegotiations for the worse productivity realization. The reason is that new debt issuance is lower than the amount of matured debt, which is about $-0.07$. High productivity also implies less job separation such that the hiring costs per worker are higher and firms post less vacancies. Firm profits are higher because of lower vacancy costs and private consumption increases. It follows that the share of total resources allocated to the proportional amount of
public spending increases. The increase of the recovery rate in unemployment is smaller when exogenous productivity is low. Since exogenous shocks are persistent and job separation is high, a large share of hiring is lost when productivity is low in the periods following a haircut. Thus, the employment increase after a haircut is short-lived which reduces the impact of initial employment on the recovery rate.

4.3 Cyclical Properties

In this section, I compare the cyclical properties of the Portuguese economy with the statistics from simulations of the model. The first column of Table 2 summarizes the statistics from the Portuguese data. I consider log-quadratically detrended series for the time period from 1995 to 2017. For the cyclicity of taxes, I take the correlation between the percentage changes of real GDP and the tax index from Vegh and Vuletin (2015) as a reference. In column (2), I report the cyclical properties of the model simulations for the benchmark calibration. Out of a simulation of 500,000 years where the first 100 observations are discarded, I consider episodes of at least 23 periods without renegotiations, preceded by at least four periods of good credit standing. Calibration targets are highlighted in bold.

Overall, the model describes the properties of the Portuguese economy well. In line with the sovereign debt literature, e.g. Arellano (2008) and Cuadra et al. (2010), private consumption is more volatile than output, sovereign bond yields are countercyclical and fiscal policy is procyclical, in particular tax rates and output are negatively correlated. The behavior of fiscal policy corresponds to Vegh and Vuletin (2015) who find that Portugal belongs to the industrial countries with the most procyclical fiscal policy and is similar to emerging economies in terms of tax policy cyclicity. The 90%-quantiles of the spread and the absolute changes of the spread are close to the data. Also the untargeted maximum of the spread, 8.05%, and the maximum of the absolute change of the spread, 4.38%, are close to their empirical counterparts of 9.05% and 4.97%. The model, however, does not match the distribution of spreads, especially the large number of spreads close to zero, such that the volatility is too low. The targeted statistics on the labor market are well matched, but the volatility of unemployment is too low. In line with the data, the model produces a larger autocorrelation for unemployment than for output.

4.4 The Dynamics of Unemployment and Sovereign Debt Renegotiations

In this section, I employ an event analysis to study the dynamics of unemployment and sovereign debt renegotiations. Out of the model simulations of 500,000 years, I consider episodes in which

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19Bocola et al. (2019) are able to generate higher shares of spreads close to zero by the introduction of domestic government debt.
Table 2: Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>Data (1)</th>
<th>Benchmark (2)</th>
<th>Model Simulations (3)</th>
<th>Model Simulations (4)</th>
<th>Model Simulations (5)</th>
<th>Model Simulations (6)</th>
<th>Model Simulations (7)</th>
<th>Model Simulations (8)</th>
<th>No Skill Loss (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ(y)</td>
<td>2.81</td>
<td>2.80</td>
<td>2.81</td>
<td>2.80</td>
<td>2.83</td>
<td>2.76</td>
<td>2.83</td>
<td>2.90</td>
<td>2.74</td>
</tr>
<tr>
<td>σ(c)/σ(y)</td>
<td>1.11</td>
<td>1.70</td>
<td>1.64</td>
<td>1.67</td>
<td>1.75</td>
<td>1.72</td>
<td>1.70</td>
<td>1.68</td>
<td>1.70</td>
</tr>
<tr>
<td>σ(s)</td>
<td>2.48</td>
<td>1.51</td>
<td>1.31</td>
<td>1.41</td>
<td>1.64</td>
<td>1.30</td>
<td>1.60</td>
<td>1.89</td>
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<td>σ(u)</td>
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<td>1.83</td>
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<td>1.71</td>
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<tr>
<td>ρ(c,y)</td>
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<td>0.96</td>
<td>0.97</td>
<td>0.96</td>
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<td>0.96</td>
<td>0.96</td>
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</tr>
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<td>ρ(τ,y)</td>
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<td>-0.55</td>
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<td>-0.40</td>
<td>-0.39</td>
<td>-0.37</td>
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<td>-0.70</td>
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<td>-0.67</td>
<td>-0.66</td>
<td>-0.64</td>
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</tr>
<tr>
<td>ρ(y,y')</td>
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<td>0.72</td>
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<tr>
<td>ρ(u,u')</td>
<td>0.93</td>
<td>0.87</td>
<td>0.87</td>
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<td>3.01</td>
<td>3.17</td>
<td>3.56</td>
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<td>5.75</td>
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<tr>
<td>90% quantile of</td>
<td>Δs</td>
<td></td>
<td>2.69</td>
<td>2.61</td>
<td>2.26</td>
<td>2.43</td>
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<td>E(u)</td>
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<td>8.74</td>
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<td>9.82</td>
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<td>4.29</td>
<td>4.77</td>
<td>4.52</td>
<td>3.96</td>
<td>4.48</td>
<td>4.44</td>
<td>4.45</td>
<td>4.29</td>
</tr>
<tr>
<td>σ(u_long-term/u)</td>
<td>6.87</td>
<td>6.65</td>
<td>5.99</td>
<td>6.36</td>
<td>7.01</td>
<td>6.53</td>
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<td>58.48</td>
<td>58.46</td>
<td>100.00</td>
<td>78.94</td>
<td>27.34</td>
<td>58.37</td>
<td>58.43</td>
<td>58.54</td>
<td>100.00</td>
</tr>
<tr>
<td>Market value of reneg. debt</td>
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<td>23.14</td>
<td>33.27</td>
<td>28.04</td>
<td>16.41</td>
<td>34.13</td>
<td>17.35</td>
<td>0.00</td>
<td>23.13</td>
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<tr>
<td>Reneg. Prob.</td>
<td>-</td>
<td>3.56</td>
<td>3.79</td>
<td>3.66</td>
<td>3.43</td>
<td>3.77</td>
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<td>-</td>
<td>-</td>
<td>2.92</td>
<td>2.00</td>
<td>-</td>
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Notes: Column (1) is based on annual Eurostat data for Portugal and considers the time period from 1995 to 2017. y and c denote log-quadratically detrended real output and real private consumption, respectively. s denotes the sovereign spread, calculated as the difference between the interest rates on Portuguese and German 10-year bonds. u refers to the unemployment rate. Shares and probabilities are given in %. The reference for tax cyclicality is the correlation between the percentage changes of real GDP and the tax index from Vegh and Vuletin (2015). Columns (2) to (9) are based on simulations of 500 000 years, where I omit the first 100 observations. I refer to episodes of at least 23 years without renegotiations, which are preceded by at least 4 years without renegotiations. Column (2) reports the statistics for the benchmark model, columns (3) to (5) and (6) to (8) consider variations of the skill loss parameter ξ and the bargaining power θ, respectively. Column (9) refers to a recalibrated version of the model without skill loss (for the parameters, see Table 1). Renegotiation probabilities and welfare equivalents refer to the complete series of 499 900 years.
the government enters debt renegotiations at $t = 0$. Similar to the procedure for the simulation statistics, I restrict to renegotiations which have been preceded by at least 27 periods of good credit standing. Figure 6 shows the dynamics around renegotiations at $t = 0$, denoted by the grey bars. Productivity, output, and private consumption are presented as percentage deviations from a log-linear trend, which I apply to the complete series. The tax rate, initial debt-to-GDP, unemployment, the sovereign spread and the probability of renegotiations are denoted in percent. Endogenous productivity is the percentage share of high-skilled employed workers. The panels show the variables 8 years before and 12 years after the event at $t = 0$. The solid lines refer to the benchmark model.

Renegotiations are preceded by a drop in exogenous productivity and output as well as periods of increasing unemployment and high debt-to-GDP ratios. Simultaneously, the tax rates and the spread increase, reflecting an increased probability of debt renegotiations. While higher tax rates dampen hiring, firms still post more vacancies because of increased returns following from the increase in the number of job seekers in response to higher separation rates. The inflow of low-skilled workers prevents an increase in the share of high-skilled employed workers. In the period of renegotiations, the government can reduce the tax rates since it does not serve the coupon payments and the maturing debt. There is a haircut such that the debt-to-GDP ratio drops. In the year after the haircut, the government reenters international financial markets and uses new borrowing for tax cuts. The lower tax burden and increasing productivity improve the firms’ hiring incentives such that the unemployment rate declines. Since unemployment is high in the previous period, the productivity of job seekers is low such that increased hiring is followed by a decline in endogenous productivity. The drop in unemployment is in line with the findings of Levy Yeyati and Panizza (2011) for sovereign defaults in emerging economies. In response to increasing output, private consumption peaks.

The decrease in the tax rate, however, is short-lived. Since the debt level rises again, the government has to increase taxes. As productivity has not yet fully recovered, the separation rate is still high. High separation rates and increasing taxes have a negative effect on hiring incentives such that the unemployment rate increases again in the medium run. Private consumption drops. In the long run, on average, productivity returns to the trend. Since the separation rate becomes lower, the unemployment rate falls and converges to its pre-crisis level. In a minor share of episodes, starting in the third period after the renegotiations, the government chooses to bargain on the remaining debt again.

I deviate from the procedure for the business cycle statistics because, for the complete series, the system is underdetermined in case of log-quadratic detrending. I use the complete series instead of short episodes to abstract from distortions on detrending around renegotiation episodes.
Figure 6: Event Analysis

Exogenous Productivity | Output | Private Consumption
---|---|---
[Graphs showing fluctuations in productivity, output, and private consumption over time.]

Tax | Unemployment Rate | Endogenous Productivity
---|---|---
[Graphs showing fluctuations in tax rate, unemployment rate, and endogenous productivity over time.]

Debt-to-GDP | Sovereign Spread | Renegotiation Probability
---|---|---
[Graphs showing fluctuations in debt-to-GDP ratio, sovereign spread, and renegotiation probability over time.]

Notes: The solid (dashed) lines present the statistics for episodes out of a model simulation of 500,000 years of the benchmark model (the model without skill loss), where the first 100 are omitted. I consider sovereign debt renegotiations which have been preceded by at least 27 years of repayment. The grey bars represent the event period. Productivity, output and private consumption are shown as percentage deviations from a log-linear trend. The tax rate, debt to GDP, unemployment, the sovereign spread and the probability of renegotiations are denoted in percent. Endogenous productivity denotes the percentage share of high-skilled employed workers. The panels show the variables 8 years before and 12 years after the event at $t=0$.

4.5 The Impact of the Human Capital Depreciation

The intensity of human capital depreciation determines the level of employment and the response of fiscal policy on cyclical fluctuations. The degree of skill loss is implied by the difference in skills between high-skilled and low-skilled workers. Figure 7 shows the policy functions for two different values of the skill loss parameter $\xi$, given an employment level of 91\%. The solid lines refer to the benchmark model with $\xi = 0.465$. The dashed lines present policy functions for the benchmark model without skill depreciation, $\xi = 0$. When human capital depreciates during unemployment spells, the share of low-skilled workers is higher and, for identical initial states, the economy has less resources to serve its debt obligations. The benefits from entering debt renegotiations
and avoiding higher tax rates to prevent higher employment losses are larger because the loss of productivity in the following periods can be avoided. Thus, the government is less reluctant to enter debt renegotiations and faces lower bond prices. Since lower bond prices imply a lower debt market value at any given debt level, the debt level after renegotiations is reduced and the debt recovery rate is lower.

The firms’ decision on employment is affected by two opposing forces. First, as skill loss occurs, the productivity of newly hired workers is lower which reduces the benefits from hiring. On the other hand, there are incentives for hiring to prevent skill depreciation of newly displaced workers. The second effect dominates such that firms increase their vacancy posting expenditures and the employment level is higher for any considered initial state of the economy. Note that this finding is in line with Laureys (2021).

In the economy with skill loss, the tax rate is lower when bond prices are high and the government is not budget-constrained. Lower tax rates increase the firms’ benefits from hiring and thus support higher employment levels. However, taxes are also lower during debt renegotiations when taxes cannot be reduced by debt issuance because higher employment levels imply lower expenditures on unemployment transfers. Additionally, since firms have higher expenditures on vacancy posting, firm profits are lower and the households have less resources to spend on private consumption. Due to the assumption of a constant ratio of public to private consumption, it follows that public consumption is also reduced and the government needs less income from sales taxes.21

The statistics in columns (3) to (5) of Table 2 confirm the reduction of unemployment and long-term unemployment in equilibrium. The per-period change of unemployment decreases as both firms and the government provide more resources to keep employment at a high level. Instead of entering debt renegotiations more often, the government, from an ex ante perspective, issues less debt and enters debt renegotiations less frequently when human capital depreciation is larger. By choosing lower debt levels, the government avoids higher interest spreads and reduces the likelihood of larger tax increases in case of bad realizations of the exogenous productivity shock. It follows that the tax rates become less procyclical in the intensity of the skill loss, i.e. the negative correlation between output and tax rates declines. Cyclical fluctuations in the size of total transfer payments to unemployed workers, which also depends on the degree of skill loss, may be an important driver of fiscal policy cyclicality. In Section 4.8, I provide a robustness analysis on the size of transfers \( \Omega \). Despite lower debt-to-GDP ratios, the market value of renegotiated debt is lower with increasing skill loss because of decreased recovery rates and lower bond prices. A recalibrated version of the model without skill loss in column (9) of Table 2 shows that the cyclicity of fiscal policy also remains higher than in the benchmark model when I readjust the parameters to get lower unemployment and a lower debt-to-GDP ratio (see Table 1). The results

\[21\] The tax rate would increase when public consumption is instead assumed to be constant because the tax base is reduced when human capital depreciation is larger.
Notes: The solid (dashed) lines represent the debt policies, the bond prices, the recovery rates, the tax policies and the employment policies at different debt levels $b$ for the benchmark model with $\xi = 0.465$ (the model without skill loss $\xi = 0$), given an employment level of 91%. In the left column, productivity is 3.1% below the trend. High productivity refers to levels 3.1% above the trend.

emphasize the strong motive of the government to avoid strong tax hikes which would be followed by a larger increase in unemployment and the loss of workers’ productivity.
The dashed lines in Figure 6 show the dynamics around renegotiations for the model without human capital depreciation. Qualitatively the patterns do not deviate from the benchmark model. In the absence of human capital depreciation, firms are more reluctant to post vacancies when separation rates are high because of the high risk of separation of new hires in the next period, the low exogenous productivity realization, and the lack of impact on future endogenous productivity. Unemployment exhibits a stronger increase despite higher vacancy yields. Due to lower vacancy expenditures, firm profits increase and there are more resources for private consumption. Since the employment enhancing effects are reduced, the government uses less of its fiscal capacities immediately after renegotiations such that the return of the debt-to-GDP ratio to a higher level and the reduction in the unemployment rate occur at a slower speed. As the government faces a lower debt-to-GDP ratio and thus a smaller debt service ratio in the model with skill loss, it can serve a stronger spread increase before renegotiations.

To study the dynamics of taxes, debt and unemployment, I show in Figure 8 the impulse responses to a 1% decline in exogenous productivity, where blue (red) lines represent the responses in the benchmark model (the parameter variation without skill loss, $\xi = 0$). Productivity, output, and debt are denoted in percentage deviations, unemployment rate, spread, and tax rate are in percentage point deviation from the long-run equilibrium. Productivity in the absence of skill loss (red) equals the variation in exogenous productivity. Productivity in the benchmark case additionally includes the endogenous variation, which depends on the mix of high-skilled and low-skilled employed workers.

For the benchmark model, the initial drop in output and the increase in unemployment are around 10% smaller than in the absence of skill loss ($\xi = 0$). Due to the high initial separation rate, which corresponds to the negative productivity shock, there is a high share of high-skilled job seekers, making hiring more attractive to avoid the costs from hiring a low-skilled worker with a high probability in the next period. The lower unemployment rate and thus higher output and lower transfer payments allow the government to simultaneously keep the tax rate lower by around 0.09 percentage points and reduce debt more strongly. Since the bond price curve in the benchmark model is steeper (c.p. Figure 4), the government nevertheless faces a stronger spread increase.

During the first periods after the shock, the government both reduces taxes, but also, in the presence of high separation rates, further reduces debt. The government does not yet use additional resources to reduce taxes because of the high anticipated risk of losses on new employment relationships following from high expected separation rates in the next period. When productivity improves, the government becomes less budget-constrained and reduces taxes more strongly in the presence of lower separation rates to improve hiring incentives. The tax reduction is more distinctive in the presence of human capital depreciation with taxes being up to 0.19 percentage points below taxes in the model when skill loss is shut down, $\xi = 0$. The weaker initial tax increase and the stronger tax reductions in the presence of an alleviating negative productivity shock reduce pro-cyclicality of taxation in the model with skill loss during unemployment. The
Notes: The figure shows the impulse responses to a 1% productivity shock in the benchmark model (blue) and in the model when skill loss is shut down, $\xi = 0$, (red). Productivity as a joint measure of exogenous and endogenous productivity, output, and debt are expressed in percentage deviations from their long-run equilibrium values in the absence of a shock, i.e. 1000 years with $\epsilon = 0$. Unemployment rate, spread, and tax rate are in percentage point deviations.

response in output in both models is quantitatively very similar after the initial period, with output in the benchmark model being around 0.06 percentage points lower.\textsuperscript{22} The small difference in output follows from lower unemployment in the benchmark model response, which peaks at around 0.14 percentage points less than in the model with $\xi = 0$, and higher productivity of

\textsuperscript{22}In comparison with the recalibrated model, the drop of output in the benchmark model is around 20% larger.
workers in the absence of skill loss. At its peak, the drop of worker productivity in the benchmark model response is around twice the drop in the model where the skill loss parameter is set to zero.

Figure 9: Impulse Responses - High and Low Debt

![Figure 9: Impulse Responses - High and Low Debt](image)

Notes: The figure shows the impulse responses to a 1% productivity shock in the benchmark model (blue) and in the model when skill loss is shut down, $\xi = 0$, (red). Initially, debt is 10% above (solid) / below (dashed) its long-run mean. Productivity as a joint measure of exogenous and endogenous productivity, output, and debt are expressed in percentage deviations from their long-run equilibrium values in absence of a shock, i.e. 1000 years with $\epsilon = 0$. Unemployment rate, spread, and tax rate are in percentage point deviations.

Figure 9 shows the impulse responses assuming an initial debt level 10% above (solid lines) and 10% below (dashed lines) the long-run mean, where blue (red) lines represent the benchmark model (benchmark with $\xi = 0$). When debt is low initially, the government faces a lower bond price increase and may even be able to increase its debt obligation and to set lower tax rates. In the benchmark model, the government initially reduces taxes less than in the absence of skill loss, but postpones the use of fiscal resources to cut taxes more strongly when the negative productivity shock and thus the increase in the separation rate has become weaker. Avoiding tax hikes reduces the increase of the unemployment rate significantly in both specifications. The difference in unemployment between the two models, also caused by the firms' incentives in the benchmark to hire relatively more when separation rates are high and there are more high-skilled job seekers, is less distinctive than in the case of higher debt levels. Unlike for low debt, when debt is high, the government increases taxes less in the benchmark economy than in the absence of skill loss to avoid the long-run productivity losses following from higher unemployment. The government more distinctively chooses the timing on tax cuts and debt reduction to avoid strong unemployment increases and to strengthen hiring incentives when external conditions are improving.

Figure 10 shows the impulse responses assuming unemployment 1 percentage point above (dashed lines) and below (solid lines) the long-run unemployment rate. When unemployment is low initially, unemployment increases more strongly because hiring incentives are particularly low due to low productivity in the presence of higher search costs following from the smaller number of job seekers. However, the government faces lower unemployment transfer payments and can thus simultaneously dampen the tax increase and reduce debt even more to have sufficient resources in the following periods to lower tax rates more strongly.
Figure 10: Impulse Responses - High and Low Unemployment

Notes: The figure shows the impulse responses to a 1% productivity shock in the benchmark model (blue) and in the model when skill loss is shut down, $\xi = 0$, (red). Initially, unemployment is 1 percentage point below (solid) / above (dashed) its long-run mean. Productivity as a joint measure of exogenous and endogenous productivity, output, and debt are expressed in percentage deviations from their long-run equilibrium values in absence of a shock, i.e. 1000 years with $\epsilon = 0$. Unemployment rate, spread, and tax rate are in percentage point deviations.

4.6 Case Study: Portugal 2008 - 2017

To evaluate the model predictions for Portugal, I employ the following experiment. I take the Portuguese unemployment rate in 2007 and assume that all workers are high-skilled in 2007. I calculate a value for debt based on the debt-to-GDP ratio in 2007 and define productivity to be on the trend in the initial period. Then, I run a simulation series of 10 years, for which I choose realizations of productivity $z$ such that the spread for Portuguese debt from 2008 to 2017 is matched.

Figure 11 shows the unemployment rate, the productivity realization, the spread and the debt-to-GDP ratio, where the blue solid and black dashed lines refer to the model outcome and the data series, respectively. The model matches the pattern of the Portuguese unemployment very well. However, the level of unemployment is too low. While doing very well in terms of targeted moments, i.e. the higher spread levels and spread increases, the model has difficulties in matching the very low spreads in the data. For the first years, very high productivity shocks are necessary to get close (in 2008)\(^23\) or to match (in 2009) the spread. High productivity implies a very low separation rate, such that the unemployment rate initially becomes very low. Increasing spreads follow from a very strong drop in productivity without triggering debt renegotiations. The model cannot reproduce the increase in debt-to-GDP after 2010. Note, however, that in 2011 Portugal agreed on a 3-year economic adjustment program, which provided the Portuguese government with generous credits from the European Financial Stabilisation Mechanism, the European Financial Stability Facility, and the International Monetary Fund. While the model abstracts from the effect of bailouts, e.g. Fink and Scholl (2016) find that official financial assistance can help to

\(^{23}\)Matching the spread in 2008 exactly would have required a productivity realization more than 4 standard deviations above the trend.
Figure 11: Model Prediction for Portugal

Notes: As initial situation, the figure considers the unemployment rate of Portugal in 2007, a productivity realization on the trend, and the debt to GDP ratio of Portugal in 2007 under the assumption that all workers are high-skilled. For the time period from 2008 to 2017, a series of productivity realizations is generated such that the model produces the respective spreads of Portuguese debt. Unemployment rate, sovereign spread and debt-to-GDP are in percentage values, exogenous productivity denotes the log realization of the productivity shock $z$. Blue solid lines refer to the model outcome, black dashed lines to the Portuguese data.

prevent sovereign defaults in the short run. Bailouts provide the government with new liquidity and reduce the risk premia such that the government can increase its indebtedness at lower interest rates. Thus, the absence of financial assistance can explain differences in the debt-to-GDP ratio between the data and the model as well as the need of very high productivity realizations followed by a very low level of the unemployment rate.\footnote{Matching the unemployment rate instead of the spread would trigger renegotiations in 2011, 2012, and 2013 because this type of model cannot explain the full unemployment volatility without triggering a very strong increase in spreads.}

Figure 12 shows the counterfactual model outcome for the case that Portugal would have entered debt renegotiations in 2011 (red dashed), 2012 (red dash-dotted), or 2013 (red dotted). Entry in debt renegotiations in 2011, 2012, and 2013 is accompanied by an immediate reduction of unemployment of $0.86, 0.86$ and $0.49$ percentage points, respectively. In the year after renegotiations, the government uses the low debt-to-GDP ratio at market reentry to issue new debt and set lower
Notes: The blue lines represent the model outcome for the years 2008 to 2017 for a model simulation where the exogenous productivity realizations are set to match the Portuguese bond spreads. The full series starts in 2008 and takes the unemployment rate of Portugal in 2007, productivity on the trend, and the debt to GDP ratio of Portugal in 2007 under the assumption that all workers are high-skilled as initial situation. The dashed, dash-dotted, and dotted red lines represent the counterfactual model outcome when the government enters debt renegotiations in the years 2011, 2012, and 2013, respectively. Welfare equivalent denotes the welfare improvement from moving from the economy without renegotiations to the counterfactual economy with debt relief in terms of the equivalent variation in consumption. Recovery rate refers to the percentage of debt remaining in case the government enters debt renegotiations.

tax rates. The lower tax rates improve the firms’ hiring incentives and unemployment is lowered by up to 2.81, 2.34, and 1.78 percentage points in the aftermath. In the following years, however, the government becomes more budget constrained because of increasing debt and has to increase taxes. Spread, debt, and tax level converge to the level observed without renegotiations in 2017. However, in 2017, unemployment is still 1.14 (2011), 1.30 (2012) and 1.48 (2013) percentage points lower than in the case without renegotiations. The unemployment rate is lowest in 2017 if the government renegotiates in 2013 because of the timing of expansionary fiscal policy and the realization of bad productivity shocks and high separation rates. Since the government tends to use new fiscal space after renegotiations directly to decrease tax rates, high separation rates in the following periods reduce the positive effect from renegotiations on employment.

The right panel in the second column shows for each year the recovery rate if the economy enters debt renegotiations. From 2011 to 2013, the haircuts are decreasing with recovery rates of 17.16% (2011), 19.35% (2012) and 21.74% (2013). The decline follows from the reduced renegotiated debt stock. Despite the reduction of GDP in 2012 following from increased unemployment and low
productivity realizations, the debt-to-GDP ratio decreases after 2011 because matured debt is not completely replaced by new debt issuance. At the same time, the debt level after renegotiations is lowest (−0.097) in 2011 when the government faces higher debt, low productivity, but also lower unemployment.

To evaluate the welfare effects of debt renegotiations, I compute the equivalent variation in consumption $\Lambda$. The use of GHH-preferences allows me to follow Durdu et al. (2013) and to calculate the welfare gain of moving from an economy with a high realization of the utility cost shock $\epsilon_X$ to the counterfactual economies in which the government enters debt renegotiations:

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left( (1 + \Lambda) e_t(\circ) \right) = E_0 \sum_{t=0}^{\infty} \beta^t u \left( e_t(\ast) \right).$$

‘$\circ$’ refers to the model with high realizations of the utility cost shock $\epsilon_X$, and ‘$\ast$’ denotes the counterfactual model in which the government enters debt renegotiations. The equivalent variation follows from this equation as

$$\Lambda = \left( \frac{V^0(\ast)}{V^0(\circ)} \right)^\frac{1}{1-\sigma} - 1,$$

where $V^0$ is the expected lifetime utility. For welfare calculations, I use the expected lifetime utility realized by the households, i.e. I abstract from the one-time utility cost which the policy maker suffers. Renegotiations in 2011 imply the largest welfare gain of 6.04%. The welfare gains for renegotiations in 2012 and 2013 are 5.56% and 4.48%, respectively. The reduction in the benefit of a debt relief over time results from both the larger percentage debt relief and the lower unemployment peak with the implication of a lower productivity loss of workers following early renegotiations. Since the debt-to-GDP ratio and the unemployment rate converge back to the levels in the model outcome without renegotiations, welfare gains of being in the counterfactual economy decline over time. However, in 2017, the welfare gains are still 1.40%, 1.69% and 2.13%, respectively, reflecting the differences in the unemployment level and thus also in the productivity of workers.

### 4.7 The Size and Welfare Effects of Haircuts

The size of the recovery rate is crucial for the incentives to enter renegotiations and for the amount of fiscal resources to stimulate employment via tax reductions after the debt restructuring. In the model, the size of haircuts is controlled by the borrower’s bargaining power $\theta$. Columns (6) to (8) of Table 2 provide the statistics for variations of $\theta$. For lower bargaining power, the average recovery rate increases such that the borrower receives a smaller debt relief, reflected in the increasing market value of renegotiated debt. As smaller haircuts reduce the incentives to enter debt renegotiations, the government serves its debt obligations at higher debt levels, becomes less

\[\text{If I account for the utility cost of entering renegotiations, there are initial welfare losses of 0.34% (2011), 0.13% (2012), and 1.09% (2013).}\]
budget constrained and accumulates more debt. For the chosen debt levels, the government faces higher bond prices where the increased market value of renegotiated debt dominates the negative effect of an increased probability of debt renegotiations. The recovery rate in terms of the market value of renegotiated debt of 34.13% for a bargaining power $\theta$ of 0.1 is slightly below the average of 40.01% found by Cruces and Trebesch (2013).

In the long run, employment on average shows very little variation in the bargaining power, but becomes slightly more volatile. There are two opposing forces at work. On the one hand, since the debt level is higher in the long run, debt service and thus tax rates are higher with decreasing bargaining power. Increased tax rates reduce the firms’ benefit from hiring such that employment marginally decreases. On the other hand, since recovery rates increase with declining bargaining power, bond prices increase and the government becomes less borrowing constrained. More fiscal space allows the government to reduce taxes and improve hiring incentives.

The last row of Table 2 shows the percentage equivalent variation in consumption across the full simulation series from the perspective of the policymaker. Welfare is decreasing in the government’s bargaining power. While a decreasing bargaining power implies smaller debt reductions after renegotiations, the government benefits from higher bond prices and access to larger amounts of debt. The benefit increases up to a value of 2.92% for a bargaining power $\theta$ of 0.1.

### 4.8 Robustness

In this section, I study the impact of the size of transfers to unemployed workers paid by the government $\Omega$ on the model dynamics. The first three columns of Table 3 present the statistics for variations of $\Omega$ given the benchmark choice of the skill loss parameter $\xi = 0.465$.

<table>
<thead>
<tr>
<th></th>
<th>$\xi = 0.465$</th>
<th>$\xi = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Omega = 0$</td>
<td>$\Omega = 0.3$</td>
</tr>
<tr>
<td>$\rho(\tau, y)$</td>
<td>-0.29</td>
<td>-0.36</td>
</tr>
<tr>
<td>$E(s)$</td>
<td>3.29</td>
<td>3.27</td>
</tr>
<tr>
<td>$E(b'/y)$</td>
<td>53.80</td>
<td>52.66</td>
</tr>
<tr>
<td>$E(u)$</td>
<td>11.06</td>
<td>10.07</td>
</tr>
<tr>
<td>Reneg. Prob.</td>
<td>4.00</td>
<td>3.82</td>
</tr>
</tbody>
</table>

Notes: All statistics are based on simulations of 500,000 years, where I omit the first 100 observations. I refer to episodes of at least 23 years without renegotiations, which are preceded by at least 4 years without renegotiations. The first three columns present variations of the size of transfers to unemployed workers $\Omega$ given a value for the skill loss parameter $\xi$ of 0.465. The last column shows the statistics for the model without skill loss and without transfer payments. Renegotiation probabilities and welfare equivalents refer to the complete series of 499,900 years.

The size of total transfer payments $T$ is highly sensitive to changes in the unemployment rate and cyclical fluctuations. It follows that with increasing transfer size, government expenditures become
more volatile and more negatively correlated with the realizations of the exogenous productivity shock. A stronger increase in total transfer payments during an economic downturn demands increasing sales tax rates such that the tax policy becomes more procyclical. External creditors incorporate an increased renegotiation probability at high debt levels such that the government faces higher bond spreads and issues less debt.

Column (4) of Table 3 displays the statistics for the model without skill loss and without transfers. The comparison with column (1) shows that more than half of the reduction in tax cyclicality and a quarter of the debt reduction observed for the benchmark model in Table 2 cannot be explained by cyclical fluctuations of transfer payments.

4.9 Discussion

In this section, I compare the main implications of the model to empirical observations. I implicitly use the empirical finding by Ortego-Marti (2017a) that the rate of human capital depreciation is higher for jobs which require more skills. To control for more procyclical fiscal policy in the presence of lower rates of skill loss during unemployment, I consider the relation between GDP per hour worked as a measure of human capital intensity and a measure of tax cyclicality. Figure 13 plots the correlation between percentage changes in real GDP and a tax index against the average GDP per hour worked between 1996 and 2013 for 27 OECD economies. Data on real GDP per hour worked is from the OECD. The tax index information is taken from Vegh and Vuletin (2015) and includes changes in corporate, personal, and value-added tax rates.

In line with the model outcome, countries with higher GDP per hour worked follow a less procyclical fiscal policy. While developed economies tend to have an acyclical or procyclical tax policy, fiscal policy in Spain, Portugal and Greece is particularly procyclical while both countries also feature lower GDP per hour worked. The two latter economies, however, feature exceptionally high debt to GDP ratios, where the model would ex ante predict lower debt levels. An explanation may be that the model abstracts from several features which determine the sovereign’s ability to issue debt. The literature has studied numerous aspects including political stability (Cuadra et al., 2010), the availability of bailout credits (Fink and Scholl, 2016), and domestic public debt (Bocola et al., 2019).

5 Conclusions

In this paper, I have introduced skill loss during unemployment in a dynamic stochastic model of sovereign debt with endogenous haircuts, and matching frictions to study optimal fiscal policy when human capital is lost during terms of unemployment. While contractionary fiscal policy may serve to avoid a default, it has a negative effect on employment. Since skills are lost during unemployment, the economy’s production potential may be reduced and there will be less resources to serve external debt obligations in the future. Thus, debt renegotiations may be considered as
Figure 13: Tax Cyclicality and GDP per hour worked in OECD economies

Notes: The figure plots the correlation between the percentage changes of real GDP and the tax index of Vegh and Vuletin (2015) against the average real GDP per hour worked in US $ between 1996 and 2013 for 27 OECD economies.

In a quantitative exercise, I have applied the model to Portugal. The policy functions reveal that the government enters debt renegotiations when employment and productivity are low and debt is high. If debt is sufficiently low, the government responds to rising unemployment with tax reductions to increase the firms’ hiring incentives. If debt is high, fiscal policy is contractionary and higher tax rates reduce the firms’ benefit from hiring such that the government may prefer to enter debt renegotiations to avoid higher unemployment and the resulting loss of productivity. From an ex ante perspective, a higher intensity of skill loss during unemployment reduces the procyclicality of fiscal policy. For larger human capital depreciation, the government issues less debt, faces lower debt-to-GDP ratios and enters debt renegotiations less frequently. With lower debt levels, the government can avoid large tax hikes during economic downturns such that fiscal policy becomes less procyclical. Similarly, firms increase hiring in the presence of larger human capital depreciation and average employment rises. In response to negative productivity realizations, the government avoids as strong tax hikes as in the absence of skill loss during unemployment. The government also saves resources to support hiring in later periods with stronger tax cuts when the negative external factors become weaker. When the bargaining power of the government is lower, haircuts become smaller such that the fiscal space after renegotiations to reduce taxes and to improve the firms’ benefit from hiring declines. However, since higher recovery rates imply lower
spreads, the government becomes less borrowing-constrained and enters renegotiations more frequently. In equilibrium, there is a welfare gain to the sovereign from lower bargaining power and smaller haircuts.

While the model finds for Portugal that repayment was optimal, renegotiations in 2011, 2012, and 2013 would have reduced unemployment in the medium run by up to 2.81, 2.34, and 1.78 percentage points, respectively. Early renegotiations imply a lower peak of the unemployment rate in the short run, but slightly higher unemployment rates in the medium run because persistently low productivity realizations reduce the long-term effects of expansionary fiscal policy. The model cannot match the debt increase after 2009 since it abstracts from bailouts. The question how financial assistance and conditionality should be tailored such that the persistent negative effects of austere fiscal policy are minimized is left for future research.
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References


A The Recursive Equilibrium

Definition. The recursive equilibrium for the small open economy is defined as

1. a set of policy functions for private consumption \( c^R(b, n, z), c^D(n, z) \),
2. a set of policy functions for hiring, vacancy posting and employment \( h^R(\tau, n, z), h^D(\tau, n, z), v^R(\tau, n, z), v^D(\tau, n, z), n'^R(\tau, n, z), n'^D(\tau, n, z) \),
3. a set of wages \( w^{H,R}(\tau, n, z), w^{H,D}(\tau, n, z), w^{L,R}(\tau, n, z), w^{L,D}(\tau, n, z) \) and transfers to unemployed workers \( T^{H,R}(\tau, n, z), T^{H,D}(\tau, n, z), T^{L,R}(\tau, n, z), T^{L,D}(\tau, n, z) \),
4. a set of value functions for the firms’ value of a high-skilled and low-skilled job \( J^H(\tau, n, z), J^{H,R}(\tau, n, z), J^{H,D}(\tau, n, z), J^L(\tau, n, z), J^{L,R}(\tau, n, z), J^{L,D}(\tau, n, z) \), for the value of a job to a high-skilled and a low-skilled worker \( W^H(\tau, n, z), W^{H,R}(\tau, n, z), W^{H,D}(\tau, n, z), W^L(\tau, n, z), W^{L,R}(\tau, n, z), W^{L,D}(\tau, n, z) \), and the value of being unemployed \( U^H(\tau, n, z), U^{H,R}(\tau, n, z), U^{H,D}(\tau, n, z), U^L(\tau, n, z), U^{L,R}(\tau, n, z), U^{L,D}(\tau, n, z) \),
5. a set of policy functions for the government’s borrowing choice \( b'(b, n, z) \), government consumption \( g^R(b, n, z), g^D(n, z) \), and the tax policy \( \tau^R(b, n, z), \tau^D(n, z) \),
6. a default set \( D(b, n) \),
7. the bond price function charged by international creditors, \( q(b, n, z) \),
8. the bond level determined in debt renegotiations, \( \tilde{b}(n, z) \),
9. a set of value functions for the government \( V(b, n, z), V^R(b, n, z), V^D(n, z) \),

such that

1. taking as given the public sector policies and the firms’ policy choices \( c^R(b, n, z) \) and \( c^D(n, z) \) satisfy the household’s budget constraint (6),
2. taking as given the public sector policies, hiring \( h^R(\tau, n, z), h^D(\tau, n, z) \) and vacancy posting \( v^R(\tau, n, z), v^D(\tau, n, z) \), and employment \( n'^R(\tau, n, z), n'^D(\tau, n, z) \) satisfy the optimality condition (9) as well as the transition equation, the equation of the vacancy yield \( a \), and the non-negativity constraint on hiring from the firms’ profit maximization problem (2),
3. given the public sector policies and the firms’ choices on vacancy posting and hiring, wages \( w^{H,L,R}(\tau, n, z) \) and \( w^{H,L,D}(\tau, n, z) \) solve the bargaining problems (23) and (24) and transfers to unemployed workers \( T^{H,L,R}(\tau, n, z) \) and \( T^{H,L,D}(\tau, n, z) \) equal a share \( \Omega \) of wages,
4. the value functions for firms $V^{H,L}(b,n,z)$, $V^{R}(b,n,z)$, and the value functions of employed workers $W^{H,L}R(b,n,z)$, $W^{H,L}D(b,n,z)$, and unemployed workers $U^{H,L}(b,n,z)$, $U^{R}(b,n,z)$, and $U^{H,L}D(b,n,z)$ fulfill equations (17), (18), (20), (21) and (22),

5. given the bond price functions $q(b',n',z)$, the debt level after renegotiations $\tilde{b}(n,z)$, and the private sector equilibrium, the government’s value functions $V(b,n,z)$, $V^{R}(b,n,z)$, $V^{D}(b,n,z)$, the default set $D(b,n)$, and the policy functions $b'(b,n,z)$, $g^{R}(b,n,z)$, $g^{D}(b,n,z)$, $\tau^{R}(b,n,z)$, $\tau^{D}(n,z)$ solve (10), (11), (12) and (13),

6. given the bond price function $q(b',n',z)$ and the value function $V^{D}(b,n,z)$, the renegotiated debt level $\tilde{b}(n,z)$ solves the bargaining problem (15),

7. given the expected debt level after renegotiations $\tilde{b}(n',z)$, bond prices $q(b',n',z)$ fulfill equation (16), such that risk-neutral international creditors earn zero expected profits.

### B Wage Determination

Following Den Haan et al. (2000) and Rendahl (2016), I abstract from complete insurance markets across households and consider each worker as a single risk-neutral entity. Thus, the surplus of a job to a worker is the difference between the expected discounted values of receiving a wage and being high-skilled at the beginning of the next period and receiving unemployment benefits and being or becoming low-skilled. As in Pissarides (2000), wages are determined by Nash bargaining on the total surplus of the job. The firm’s value $J$ of a high-skilled and low-skilled job are

\[
J^{H} = (1 - \tau)z - w^{H} + \beta \frac{u'(c')}{w'(c)} E((1 - \rho'_{x})J^{H}) \tag{17}
\]

\[
J^{L} = (1 - \tau)z(1 - \xi) - w^{L} + \beta \frac{u'(c')}{w'(c)} E((1 - \rho'_{x})J^{H}) \tag{18}.
\]

The equilibrium condition of the private sector on hiring implies that the value of a vacancy $V$ equals zero:

\[
V = -\frac{K}{a} + pJ^{L} + (1 - p)J^{H} = 0.
\]

The value of a job to a high-skilled and low-skilled worker are

\[
W^{H} = w^{H} + \beta \frac{u'(c')}{w'(c)} E((1 - \rho'_{x} + \rho'_{z}\rho'_{f})W^{R,H} + \rho'_{x}(1 - \rho'_{f})U^{H}) \tag{19}
\]

\[
W^{L} = w^{L} + \beta \frac{u'(c')}{w'(c)} E((1 - \rho'_{x} + \rho'_{z}\rho'_{f})W^{R,H} + \rho'_{x}(1 - \rho'_{f})U^{H}) \tag{20}.
\]

The value of being unemployed is

\[
U^{H} = T^{H} + \beta \frac{u'(c')}{w'(c)} E(\rho_{f}W^{L} + (1 - \rho_{f})U^{L}) \tag{21}
\]

\[
U^{L} = T^{L} + \beta \frac{u'(c')}{w'(c)} E(\rho_{f}W^{L} + (1 - \rho_{f})U^{L}), \tag{22}
\]

\[
47
\]
where $T^H = \Omega w^H$ ($T^L = \Omega w^L$).

Let the Nash bargaining power of workers be denoted by $\omega$. The equilibrium wages are given by

$$w^H = \operatorname{argmax}\{(J^H)^{1-\omega}(W^H - U^H)^{\omega})\}$$

$$w^L = \operatorname{argmax}\{(J^L)^{1-\omega}(W^L - U^L)^{\omega})\}$$

The surplus is divided between the firms and the workers such that the wages for high-skilled and low-skilled workers are respectively

$$w^H = \omega (1 - \tau) z + \frac{(1 - \omega)}{1 - \Omega} \beta \frac{u(c')}{u(c)} E \left[ \rho_f' W^{nL} + (1 - \rho_f') U^{nL} - \rho_f' \rho_f' W^{nH} - (1 - \rho_f' \rho_f') U^{nH} \right]$$

$$w^L = \omega (1 - \tau) z (1 - \zeta) + \frac{(1 - \omega)}{1 - \Omega} \beta \frac{u(c')}{u(c)} E \left[ \rho_f' W'^{nL} + (1 - \rho_f') U'^{nL} - \rho_f' \rho_f' W'^{nH} - (1 - \rho_f' \rho_f') U'^{nH} \right].$$

### C Numerical Algorithm

I solve the model using value function iteration. My algorithm closely follows Hatchondo et al. (2016) and employs cubic spline interpolations. I approximate the equilibrium as the equilibrium of the finite-horizon economy and execute simultaneously iterations on the value functions and the bond price function.

Given the budget constraints (6), (7), (8), and the relationship between private and public consumption, the tax rate $\tau$ can be written as a function of the employment policy $n'$ and the debt policy $b'$. For a given choice of the debt level $b'$, the optimal employment level $n'$ follows from the optimality conditional of the private sector (9).

To solve the model, I employ the following algorithm. I define evenly distributed grid points for international debt $b \in [b, \bar{b}]$, employment $n \in [n, \bar{n}]$, and productivity $z \in [z, \bar{z}]$. I set initial guesses for the value functions $V(0)(b, n, z)$, $V^R(0)(b, n, z)$ and $V^D(0)(n, z)$, the values for firms $J(0)(\cdot)$, employed workers $W(0)(\cdot)$, unemployed workers $U(0)(\cdot)$, the private and public sector policy functions and the bond price function $q(0)(b, n, z)$. Given the guesses for the value functions, I employ a global search procedure to find candidate values for $b'(0)(b, n, z)$ for every grid point $(b, n, z) \in [b, \bar{b}] \times [n, \bar{n}] \times [z, \bar{z}]$. Using these candidate values as initial guesses, I find optimal values with the FORTRAN optimization routine BCPOL from the IMSL library. The probability of debt renegotiations $\eta(0)(z, b'(0), n'(0))$ and the bond price $q(0)(z, b'(0), n'(0))$ follow from equations (14) and (16), respectively. The debt level after debt renegotiations $\tilde{b}(0)(n'(0), z)$ is determined by the bargaining problem (15). Expected continuation values and expected policies are evaluated.
with Gauss-Hermite quadrature points and weights. To compute values for policies and productivity realizations off the grid, I employ cubic spline interpolations, using the FORTRAN routine DCSAKM from the IMSL library for the computation of expectation values on the $z$-dimension and bidimensional Akima (1996) spline interpolation for $b$ and $n$.

Given the solutions found at each grid point, I update the value functions $V_0(b, n, z)$, $V^R_0(b, n, z)$ and $V^D_0(n, z)$ as well as the values for firms, workers, private and public sector policy functions and the bond price function. I iterate until the value functions converge.